Ibn Al-Haitham Journal for Pure and Applied Sciences

Journal homepage: jih.uobaghdad.edu.iq

Estimation of a Parallel Stress-strength Model Based on the Inverse Kumaraswamy Distribution

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Article history: Received 3 Augest 2022, Accepted 29 November 2022, Published in January 2023.

doi.org/10.30526/36.1.2972

Abstract

The reliability of the stress-strength model attracted many statisticians for several years owing to its applicability in different and diverse parts such as engineering, quality control, and economics. In this paper, the system reliability estimation in the stress-strength model containing Kth parallel components will be offered by four types of shrinkage methods (Constant Shrinkage Estimation Method, Shrinkage Function Estimator, Modified Thompson Type Shrinkage Estimator, Squared Shrinkage Estimator). The Monte Carlo simulation study is compared among proposed estimators using the mean squared error. The result analyses of the shrinkage estimation methods showed that the shrinkage functions estimator was the best since it has a minor mean squared error than the other methods followed by the additional shrinkage estimator. The stress and strength belong to the In verse Kumaraswamy distribution.

Keywords: Invers Kumaraswamy distribution, Stress - Strength reliability, Shrinkage estimator, Mean Squared Error.

1. Introduction

The word 'Reliability' refers to the ability of a system to execute its stated purpose adequately for a specified time under the operational conditions encountered [1]. The stress -strength model is used to compute reliability. It is found to be helpful in situations where the reliability of a component or system is defined by the probability that a random variable of strength is more significant than a random variable (stress). At the same time, it makes intuitive sense that a component is deemed to have failed when its strength is lower than the applied stress [2-3]. Several researchers studied various lifetime distributions of the parallel system reliability in stress- strength model. [4] studied estimation, the system reliability consists of two parallel components, and the two strengths follow the Bivariate Pareto distribution subject to common stress, which follows the Pareto distribution. Also, [5] considered estimating the system of reliability in the S-S model in series and the parallel of K components when the stress and strengths follow exponential distributions. In 2013, [6] estimated S-S reliability consisting of two parallel components based on Exponential distribution. In addition, [7] offered estimations of system Reliability for one component, two parallel components, and s-out-of-k components in S-S system models with non-identical component strengths which are subjected to a common stress, based on Exponentiated Exponential distribution with common scale parameter. Also, a simulation study was used to compare the Maximum likelihood, Percentile, and Least squares estimators.

The device combination of k independent components with the strengths $X_1, X_2, \ldots, X_k$ and each component in the system is subject to common stress Y.it is termed parallel in the analogy with electric circuits when the system operates successfully whenever at least one of the components survives. Consequently, this model is called the parallel stress-strength model [8].

This paper aims to estimate the reliability of the system containing Kth parallel components that have strengths $X_1, X_2, \ldots, X_k$ subject to a common stress Y when the strengths and stress follow the Inverse Kumaraswamy distribution.

Hence, the organization of this paper: Section 2 displays Inverse Kumaraswamy distribution. Section 3 includes the formula of the system reliability in the S-S model. Section 4 contains shrinkage estimation methods, while simulation studies are in Section 5. The efficiency of the user method appeared in Section 6. Lastly, in Section 7, a conclusion is delivered.

2. Inverse Kumaraswamy Distribution

The Kumaraswamy distribution was defined by Ponndi Kumaraswamy in 1980 [9]. The Kumaraswamy distribution is like the beta distribution in many ways [10]. Then, [11] introduced the Inverse KumD using the $X = \frac{1-T}{T}$ transformation ; $T \sim$KumD (α, β).

The probability density function (pdf) and cumulative distribution function (CDF) of a r.v. X, $X \sim$ IKum (α, β) can be written, respectively as [12], [13]:

$$f(x, \alpha, \beta) = \alpha\beta(1+x)^{-(\alpha+1)}(1-(1+x)^{-\alpha})^{\beta-1} \ , x > 0 \, ; \ \alpha, \beta > 0 \qquad (1)$$

$$F(x; \alpha, \beta) = (1-(1+x)^{-\alpha})^{\beta} \ \ , x > 0 \, , \alpha, \beta > 0 \qquad (2)$$

where β and α are shape parameters.

3. Reliability System of Parallel S-S Model

We assume a system having K^{th} components with strengths $X_1, X_2, \dots, X_k$ independently distributed Inverse Kumaraswamy random variables with unknown shape parameter β_i , $i=1,2,\dots,k$ and known other shape parameter α subjected to stress random variable Y such that $Y \sim IKumD(\alpha, \beta_{k+1})$ with unknown shape parameter β_{k+1} and known α.
The formula of Parallel S-S model can be derived as fellows [14-16]:
$R = P(Y < max\ (X_1 X_2, \dots, X_k))$
Let
$Z = max\ X_1 X_2, \dots, X_k$
Therefore,

$$R = \int_0^\infty \bar{F}_z(y) f(y)\, dy \tag{3}$$

$$F_z(Z) = P(Z < z)$$
$$= P(x_1 < z) P(x_2 < z) \dots P(x_k < z)$$
$$= (1-(1+z)^{-\alpha})^{\beta_1}(1-(1+z)^{-\alpha})^{\beta_2} \dots (1-(1+z)^{-\alpha})^{\beta_k}$$
$$= (1-(1+z)^{-\alpha})^{\sum_{i=1}^{k}\beta_i}$$

This implies that a random variable Z follows IKumD with the parameters α and $\sum_{i=1}^{k}\beta_i$.

Then,

$$\bar{F}_z(y) = \left(1-(1-(1+y)^{-\alpha})^{\sum_{i=1}^{k}\beta_i}\right) \tag{4}$$

In equation (3), substitute equation (4) and equation (1), we get:

$$R = \int_0^\infty \left(1-(1-(1+y)^{-\alpha})^{\sum_{i=1}^{k}\beta_i}\right) \alpha\beta_{k+1}(1-y)^{-(\alpha+1)}(1-(1+y)^{-\alpha})^{\beta_{k+1}-1}\, dy$$

$$R = \int_0^\infty \alpha\, \beta_{k+1}(1-y)^{-(\alpha+1)}(1-(1+y)^{-\alpha})^{\beta_{k+1}-1}\, dy - \int_0^\infty \alpha\beta_{k+1}(1-y)^{-(\alpha+1)}(1-(1+y)^{-\alpha})^{\sum_{i=1}^{k+1}\beta_i - 1} dy$$

$$R = 1 - \frac{\beta_{k+1}}{\sum_{i=1}^{k+1}\beta_i}$$

$$R = \frac{\sum_{i=1}^{k}\beta_i}{\sum_{i=1}^{k+1}\beta_i} \tag{5}$$

4. Estimation Methods for System Reliability

4.1 Shrinkage Estimation Method (Sh)

In 1968, [17] proposed to shrink the usual estimator $\hat{\beta}$ (ex. MLE or Unbiased estimator) of the parameter β to prior information β_0 using shrinkage weight factor $\psi(\hat{\beta})$, such that $0\leq \psi(\hat{\beta}) \leq 1$. Thompson says that:
"We are estimating β and we believe β_0 is closed to the true value of β and something bad happens when $\beta_0 \approx \beta$ and we do not use β_0". Thus, Thompson gave the form of shrinkage estimator of β say $\hat{\beta}_{Sh}$ as below:

$$\hat{\beta}_{Sh} = \psi(\hat{\beta})\hat{\beta}_{ub} + (1-\psi(\hat{\beta}))\beta_0 \tag{6}$$

Unbiased estimator $\hat{\beta}_{ub}$ was applied as the usual estimator of β, and β_0 is a very closed value of β as prior information due to previous studies or experience and $\psi(\hat{\beta})$ denote the shrinkage weight factor as we mentioned above such that $0 \leq \psi(\hat{\beta}) \leq 1$, which may be a function of $\hat{\beta}_{ub}$: a function of sample size or may be constant. Also, it is possible to find $\psi(\hat{\beta})$ through minimizing the mean square error of $\hat{\beta}_{sh}$ (ad hoc basis) [18-20].
Note that $\hat{\beta}_{iub}$ of the strengths $X_1, X_2, \dots, X_k$ can be found depending on observation X_{ij}, $i = 1,2,\dots,k$ and $j = 1,2,\dots,n_i$ as below:

$$\hat{\beta}_{i_{ub}} = \frac{n_i - 1}{-\sum_{j=1}^{n_i} \ln\left(1-(1+x_{ij})^{-\alpha}\right)} \tag{7}$$

Likewise
$\hat{\beta}_{k+1_{ub}}$ of the stress Y can be found depends on observation Y_r, r$= 1,2,\dots,n$ as below:

$$\hat{\beta}_{k+1_{ub}} = \frac{m-1}{-\sum_{r=1}^{m} \ln(1-(1+y_r)^{-\alpha})} \tag{8}$$

4.1.1 Constant Shrinkage Estimation Method (Sh1)

As we mention that $X_i (i = 1,2,\dots,k)$ follows $IKumD\ (\alpha,\beta_i)$ and $Y \sim IKumD\ (\alpha,\beta_{k+1})$. A constant shrinkage weight factor $\Psi(\hat{\beta}_i) = \mathrm{h} = 0.01$: $i = 1,2,\dots,k+1$ will be suggested in this subsection. Then, the constant shrinkage estimator for β_i, $i = 1,2,\dots,k+1$ will be:

$$\hat{\beta}_{i_{Sh1}} = \mathrm{h}\hat{\beta}_{i_{ub}} + (1-\mathrm{h})\beta_{i_0}$$
$$\text{for } i = 1,2,\dots,k+1 \tag{9}$$

Put the equation (9) in equation (5), and then the constant shrinkage estimator $\hat{R}_{Sh1}$ for system reliability R in S-S model consist K^{th} parallel components become:

$$\hat{R}_{Sh1} = \frac{\sum_{i=1}^{k} \hat{\beta}_{i_{Sh1}}}{\sum_{i=1}^{k+1} \hat{\beta}_{i_{Sh1}}} \tag{10}$$

4.1.2 Shrinkage Function Estimator

We suggest the shrinkage function estimator $(Sh2)$ for estimating the parameters $\beta_i (i = 1,2,\dots,k+1)$, and the reliability system R based on the shrinkage weight function depending on the sample size n_i and m respecting, as below:

$\Psi\left(\hat{\beta}_{i_{ub}}\right)=e^{-n_i}$ for i= 1, 2,…, k and $\Psi\left(\hat{\beta}_{k+1_{ub}}\right)=e^{-m}$

Thus, based on equation (6), the shrinkage function estimators of β_i ($i=1,2,\dots,k+1$) using the above shrinkage weight functions will be:

$$\hat{\beta}_{i_{Sh2}}=\Psi\left(\hat{\beta}_{i_{ub}}\right)\hat{\beta}_{i_{ub}}+\left(1-\Psi\left(\hat{\beta}_{i_{ub}}\right)\right)\beta_{i_0}\quad i=1,2,\dots,k \tag{11}$$

$$\hat{\beta}_{k+1_{Sh2}}=\Psi\left(\hat{\beta}_{k+1_{ub}}\right)\hat{\beta}_{k+1_{ub}}+\left(1-\Psi\left(\hat{\beta}_{k+1_{ub}}\right)\right)\beta_{k+1_0} \tag{12}$$

Substitute $\hat{\beta}_{i_{Sh2}}$ i= 1, 2,…, $k+1$ in equation (5), we get the shrinkage function estimator of reliability system R based on shrinkage weight function, as follows:

$$\hat{R}_{Sh2}=\frac{\sum_{i=1}^{k}\hat{\beta}_{i_{Sh2}}}{\sum_{i=1}^{k+1}\hat{\beta}_{i_{Sh2}}} \tag{13}$$

4.1.3 Modified Thompson Type Shrinkage Estimator (Th)

The shrinkage weight factor considered by Thompson in 1968 will be modified and used in shrinkage estimator to estimate the reliability system for the proposed of S-S models. The modified shrinkage weight factor (Th) has the form, as below:

$$\varphi\left(\hat{\beta}_{i_{ub}}\right)=\frac{\left(\hat{\beta}_{i_{ub}}-\hat{\beta}_{i_0}\right)^2}{\left(\hat{\beta}_{i_{ub}}-\hat{\beta}_{i_0}\right)^2+var\left(\hat{\beta}_{i_{ub}}\right)}\,(0.001),\ \text{for } i=1,2,\dots,k+1 \tag{14}$$

We can estimate the parameters β_i ($i=1,2,\dots,k+1$) by the modified Thompson type shrinkage estimator, as follows:

$$\hat{\beta}_{i_{Th}}=\varphi\left(\hat{\beta}_{i_{ub}}\right)\hat{\beta}_{i_{ub}}+\left(1-\varphi\left(\hat{\beta}_{i_{ub}}\right)\right)\beta_{i_0}\quad ,\text{for } i=1,2,\dots,k+1 \tag{15}$$

When substituting the equation (15) in equation (5), the reliability system for estimating of R using the modified Thompson type shrinkage estimator become:

$$\hat{R}_{Th}=\frac{\sum_{i=1}^{k}\hat{\beta}_{i_{Th}}}{\sum_{i=1}^{k+1}\hat{\beta}_{i_{Th}}} \tag{16}$$

4.1.4 Squared Shrinkage Estimator (Sq)

In this subsection, we propose the squared shrinkage estimator (Sq) for $\hat{\beta}_i$ for the parameters β_i($i=1,2,\dots,k+1$), as below:

$$\hat{\beta}_{i_{Sq}}=\gamma\left(\hat{\beta}_{i_{ub}}\right)\hat{\beta}_{i_{ub}}+\left(1-\gamma\left(\hat{\beta}_{i_{ub}}\right)\right)\beta_{i_0}\ ,\ i=1,2,\dots,k+1 \tag{17}$$

Substituting (17) in (5), we conclude the squared shrinkage estimator of the reliability system R which consist K^{th} Parallel component, as below:

$$\hat{R}_{Sq}=\frac{\sum_{i=1}^{k}\hat{\beta}_{i_{Sq}}}{\sum_{i=1}^{k+1}\hat{\beta}_{i_{Sq}}} \tag{18}$$

5. Simulation Study

To verify the performance of the proposed estimation methods introduced to estimate the reliability system of K^{th} components (R), Mote Carlo simulation was used. The proposed estimation methods in S-S models have been implemented using various samples (20, 30, 50, and 100). In addition, the statistical outcomes for every sample are based on Mean Squared Errors criteria with 1000 replicates. Therefore, the following subsections explain the steps of Monte Carlo simulation.

Step1: Generate random samples following the continuance uniform distribution defined on the interval (0,1) for the strength X_i, stress Y as $u_{i1}, u_{i2}, \dots, u_{in_i}$; i= 1,2, …, k. and $v_1, v_2, \dots, v_m$, respectively.

Step2: convert the uniform random samples using the cumulative distribution function for random samples of $X_i\sim$ IKumD (α, β_i) for i= 1,2, … k, as follow:

$$F(x_{i_t}) = \left(1-(1+x_{i_t})^{-\alpha}\right)^{\beta_1}$$

$$u_{i_t} = \left(1-(1+x_{i_t})^{-\alpha}\right)^{\beta_1}$$

$$x_{i_t} = \left[1-(u_{i_t})^{\frac{1}{\beta_1}}\right]^{-\frac{1}{\alpha}} - 1 \;\; ; i = 1,2,\dots,k.$$

Using the same method for $Y\sim$ IKumD (α, β_{k+1}):

$$y_j = \left[1-(v_j)^{\frac{1}{\beta_2}}\right]^{-\frac{1}{\alpha}} - 1$$

Step3: Recall R in equation (5)

Step4: Compute Shrinkage estimators of reliability using equations (10), (13), (16) and (18)

Step5*:* Based on L=1000 replication, MSE will be calculated as follows;

$$\text{MSE} = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{R}_i - R\right)^2$$

6. Numerical Results

In this model the reliability system $R = P(max\ (X_1, X_2, \dots, X_k))$ in the S-S model when k=3 and $\alpha = 5$ will be estimated. Simulation results based on the four parameters (β_1, β_2,β_3,β_4), clarified in tables 1 to 8 to show the order rank of the proposed estimators as follows; $\hat{R}_{\text{Sh2}}$,$\hat{R}_{Th}$, $\hat{R}_{\text{Sq}}$ *and* $\hat{R}_{\text{Sh1}}$ depending on the mean square error.

Shrinkage estimator ($\hat{R}_{Sh2}$) was the best of others estimators followed by the modified Thompson type shrinkage weight factor ($\hat{R}_{Th}$). The Squared Shrinkage Estimator ($\hat{R}_{Sq}$) has the third rank, and finally was constant shrinkage estimator $\hat{R}_{Sh1}$.

The following **Tables (1-8**) will be presented the simulation results.

Table 1: Estimation value of $R = 0.802259$, when $\alpha = 5$, $\beta_1 = 4$, $\beta_2 = 5.2$, $\beta_3 = 5$ and $\beta_4 = 3.5$

(n, m_1, m_2, m_3)	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$	$\hat{R}_{Sq}$
(20,20,20,20)	0.802262	0.8022716997468	0.8022598	0.802268
(20,50,100,50)	0.802264	0.8022716998468	0.8022595	0.802272
(20,30,20,50)	0.802295	0.8022716998151	0.8022600	0.802277
(20,100,100,30)	0.802246	0.8022716998033	0.8022599	0.802266
(30,50,100,100)	0.802278	0.8022716998191	0.8022602	0.802282
(30,30,50,30)	0.802249	0.8022716998191	0.8022595	0.802270
(30,20,50,50)	0.802279	0.8022716997659	0.8022597	0.802282
(30,100,20,30)	0.802238	0.8022716997839	0.8022579	0.802243
(50,50,50,50)	0.802252	0.8022716998191	0.8022589	0.802262
(50,20,30,100)	0.802283	0.8022716997827	0.8022587	0.802289
(50,100,100,50)	0.802245	0.8022716998191	0.8022593	0.802263
(50,30,100,20)	0.802204	0.8022716998490	0.8022586	0.802238
(100,20,30,30)	0.802263	0.8022716997906	0.8022602	0.802280
(100,30,50,20)	0.802205	0.8022716997592	0.8022584	0.802252
(100,100,20,50)	0.802272	0.8022716997937	0.8022602	0.802271
(100,50,20,20)	0.802218	0.8022716997321	0.8022588	0.802245

Table 2. MSE value of $R = 0.802259$, when $\alpha = 5$, $\beta_1 = 4$, $\beta_2 = 5.2$, $\beta_3 = 5$ and $\beta_4 = 3.5$

(n, m_1, m_2, m_3)	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$	$\hat{R}_{Sq}$
(20,20,20,20)	2.0732E-07	1.39541E-010	1.4043E-09	6.0409E-08
(20,50,100,50)	8.1335E-07	1.39543E-09	5.8360E-09	2.4349E-07
(20,30,20,50)	1.0351E-07	1.39542E-010	7.3684E-010	9.7220E-08
(20,100,100,30)	1.1580E-07	1.39542E-010	8.0165E-010	3.9204E-08
(30,50,100,100)	4.6577E-07	1.39542E-09	3.4071E-09	1.2336E-07
(30,30,50,30)	1.2119E-07	1.39543E-010	8.4560E-010	2.7734E-08
(30,20,50,50)	9.1350E-07	1.39541E-09	6.6416E-09	2.7654E-07
(30,100,20,30)	1.3840E-07	1.39542E-010	9.9736E-010	1.4193E-07
(50,50,50,50)	7.2987E-07	1.39542E-09	5.3090E-09	3.0470E-07
(50,20,30,100)	6.2701E-08	1.39542E-010	4.5194E-010	2.6724E-08
(50,100,100,50)	6.6753E-07	1.39542E-09	4.7857E-09	1.6291E-07
(50,30,100,20)	1.8216E-07	1.39543E-010	1.2248E-09	2.3098E-07
(100,20,30,30)	1.3797E-09	1.39542E-010	9.7206E-010	8.5443E-8
(100,30,50,20)	1.6688E-07	1.39541E-010	1.0929E-09	6.2777E-08
(100,100,20,50)	8.2541E-07	1.39541E-09	5.9453E-09	3.5773E-07
(100,50,20,20)	1.9497E-07	1.39541E-010	1.3397E-09	1.6416E-07

Table 3. Estimation value of $R = 0.659863$, when $\alpha = 5,\ \beta_1 = 3.5,\ \beta_2 = 2.5\ ,\beta_3 = 3.7\ and\ \beta_4 = 5$

$(n\ ,m_1, m_2, m_3)$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$	$\hat{R}_{Sq}$
(20,20,20,20)	0.659867	0.6598394124288	0.6598644	0.659847
(20,50,100,50)	0.659883	0.6598394120692	0.6598645	0.659851
(20,30,20,50)	0.659922	0.6598394120750	0.6598648	0.659865
(20,100,100,30)	0.659856	0.6598394120572	0.6598647	0.659833
(30,50,100,100)	0.659915	0.6598394120849	0.6598666	0.659854
(30,30,50,30)	0.659830	0.6598394120849	0.6598621	0.659820
(30,20,50,50)	0.659889	0.6598394121347	0.6598640	0.659850
(30,100,20,30)	0.659866	0.6598394120528	0.6598641	0.659828
(50,50,50,50)	0.659875	0.6598394120849	0.6598651	0.659847
(50,20,30,100)	0.659923	0.6598394121063	0.6598653	0.659860
(50,100,100,50)	0.659849	0.6598394120849	0.6598640	0.659832
(50,30,100,20)	0.659812	0.6598394122460	0.6598646	0.659805
(100,20,30,30)	0.659838	0.6598394121219	0.6598625	0.659837
(100,30,50,20)	0.659817	0.6598394121855	0.6598653	0.659815
(100,100,20,50)	0.659885	0.6598394120706	0.6598647	0.659846
(100,50,20,20)	0.659822	0.6598394121940	0.6598641	0.659795

Table 4. MSE value of $R = 0.659863$, when $\alpha = 5,\ \beta_1 = 3.5,\ \beta_2 = 2.5,\ \beta_3 = 3.7\ and\ \beta_4 = 5$

(n, m_1, m_2, m_3)	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$	$\hat{R}_{Sq}$
(20,20,20,20)	4.0707E-07	6.01875E-010	2.8366E-09	1.9030E-07
(20,50,100,50)	1.5669E-07	6.01893E-010	1.1314E-09	3.3125E-08
(20,30,20,50)	2.0169E-07	6.01893E-010	1.4103E-09	5.1564E-08
(20,100,100,30)	2.4504E-07	6.01894E-010	1.7792E-09	1.2937E-07
(30,50,100,100)	9.3516E-08	6.01892E-010	6.7290E-010	4.1893E-08
(30,30,50,30)	2.4678E-07	6.01892E-010	1.8055E-09	1.1665E-07
(30,20,50,50)	1.6287E-07	6.01890E-010	1.1784E-09	6.3491E-08
(30,100,20,30)	2.8990E-07	6.01894E-010	2.1492E-09	1.0802E-07
(50,50,50,50)	1.4567E-07	6.01892E-010	1.0915E-09	33611E-08
(50,20,30,100)	1.2185E-07	6.01891E-010	8.5750E-010	3.6065E-08
(50,100,100,50)	1.2732E-07	6.01892E-010	9.5287E-010	3.8116E-08
(50,30,100,20)	3.3889E-07	6.01884E-010	2.4187E-09	2.1102E-08
(100,20,30,30)	2.6669E-07	6.01890E-010	1.9410E-09	2.7221E-07
(100,30,50,20)	3.3352E-07	6.01887E-010	2.3632E-09	1.2724E-07
(100,100,20,50)	1.5450E-07	6.01893E-010	1.1495E-09	4.3320E-08
(100,50,20,20)	3.8060E-07	6.01887E-010	2.7720E-09	7.0661E-07

Table 5. Estimation value of $R = 0.791666$, when $\alpha = 5,\ \beta_1 = 4,\ \beta_2 = 3,\ \beta_3 = 2.5\ and\ \beta_4 = 2.5$

(n, m_1, m_2, m_3)	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$	$\hat{R}_{Sq}$
(20,20,20,20)	0.791656	0.7916805603479	0.7916659	0.791682
(20,50,100,50)	0.791684	0.7916805601685	0.7916667	0.791690
(20,30,20,50)	0.791716	0.7916805602060	0.7916675	0.791711
(20,100,100,30)	0.791676	0.7916805601476	0.7916682	0.791690
(30,50,100,100)	0.791697	0.7916805601867	0.7916676	0.791697
(30,30,50,30)	0.791672	0.7916805601867	0.7916674	0.791681
(30,20,50,50)	0.791706	0.7916805602331	0.7916679	0.791698
(30,100,20,30)	0.791651	0.7916805602568	0.7916654	0.791678
(50,50,50,50)	0.791673	0.7916805601867	0.7916672	0.791683
(50,20,30,100)	0.791714	0.7916805602663	0.7916679	0.791704
(50,100,100,50)	0.791649	0.7916805601867	0.7916659	0.791675
(50,30,100,20)	0.791606	0.7916805599520	0.7916649	0.791659
(100,20,30,30)	0.791655	0.7916805602083	0.7916663	0.791678
(100,30,50,20)	0.791613	0.7916805600171	0.7916656	0.791658
(100,100,20,50)	0.791650	0.7916805602617	0.7916654	0.791677
(100,50,20,20)	0.791600	0.7916805600296	0.7916642	0.791655

Table 6.MSE value of $R = 0.791666$, when $\alpha = 5,\ \beta_1 = 4,\ \beta_2 = 3,\ \beta_3 = 2.5\ and\ \beta_4 = 2.5$

(n, m_1, m_2, m_3)	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$	$\hat{R}_{Sq}$
(20,20,20,20)	2.3171E-07	1.93034E-010	1.4743E-09	1.0633E-07
(20,50,100,50)	9.3801E-07	1.93029E-09	6.2256E-09	54754E-07
(20,30,20,50)	1.1352E-07	1.93030E-010	7.2320E-010	1.7540E-07
(20,100,100,30)	1.3498E-07	1.93029E-010	8.7464E-010	4.6398E-08
(30,50,100,100)	5.7235E-07	1.93029E-09	3.8307E-09	1.3931E-07
(30,30,50,30)	1.3215E-07	1.93030E-010	8.5837E-010	4.5733E-08
(30,20,50,50)	1.1198E-06	1.93031E-09	7.4709E-09	4.2545E-07
(30,100,20,30)	1.4282E-07	1.93032E-010	9.2913E-010	3.9023E-08
(50,50,50,50)	7.6742E-07	1.93029E-09	5.0665E-09	1.8428E-07
(50,20,30,100)	6.3415E-08	1.93032E-010	4.0528E-010	2.0606E-08
(50,100,100,50)	7.2895E-07	1.93029E-09	4.8046E-09	2.1633E-07
(50,30,100,20)	1.8009E-07	1.93023E-010	1.0874E-09	6.0397E-07
(100,20,30,30)	1.2715E-07	1.93031E-010	7.9716E-010	3.1972E-08
(100,30,50,20)	1.8568E-07	1.93025E-010	1.1338E-09	1.0036E-07
(100,100,20,50)	7.9998E-07	1.93031E-09	5.2898E-09	3.4745E-07
(100,50,20,20)	2.0092E-07	1.93026E-010	1.2253E-09	7.0238E-08

Table 7. Estimation value of $R = 0.764705$, when $\alpha = 5,\ \beta_1 = 6,\ \beta_2 = 7.5\ ,\beta_3 = 6\ and\ \beta_4 = 6$

$(n\ ,m_1, m_2, m_3)$	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$	$\hat{R}_{Sq}$
(20,20,20,20)	0.764741	0.7647081895378	0.7647086	0.764707
(20,50,100,50)	0.764729	0.7647081895554	0.7647068	0.764714
(20,30,20,50)	0.764747	0.7647081895190	0.7647059	0.764728
(20,100,100,30)	0.764693	0.7647081895096	0.7647062	0.764695
(30,50,100,100)	0.764735	0.7647081895199	0.7647068	0.764721
(30,30,50,30)	0.764707	0.7647081895199	0.7647064	0.764701
(30,20,50,50)	0.764743	0.7647081894968	0.7647068	0.764722
(30,100,20,30)	0.764704	0.7647081895328	0.7647062	0.764708
(50,50,50,50)	0.764711	0.7647081895199	0.7647063	0.764704
(50,20,30,100)	0.764747	0.7647081894660	0.7647058	0.764727
(50,100,100,50)	0.764690	0.7647081895199	0.7647056	0.764701
(50,30,100,20)	0.764639	0.7647081895568	0.7647039	0.764676
(100,20,30,30)	0.764701	0.7647081894775	0.7647055	0.764718
(100,30,50,20)	0.764664	0.7647081894843	0.7647063	0.764684
(100,100,20,50)	0.764713	0.7647081895303	0.7647062	0.764705
(100,50,20,20)	0.764673	0.7647081894824	0.7647065	0.764698

Table 8. MSE value of $R = 0.764705$, when $\alpha = 5,\ \beta_1 = 6,\ \beta_2 = 7.5,\ \beta_3 = 6\ and\ \beta_4 = 6$

(n,m_1, m_2, m_3)	$\hat{R}_{Sh1}$	$\hat{R}_{Sh2}$	$\hat{R}_{Th}$	$\hat{R}_{Sq}$
(20,20,20,20)	2.6414E-07	5.323E-012	1.9644E-09	9.9664E-08
(20,50,100,50)	9.9863E-07	5.3231E-011	7.7581E-09	2.0165E-07
(20,30,20,50)	1.2929E-07	5.323E-012	9.9643E-010	5.8344E-08
(20,100,100,30)	1.5517E-07	5.323E-012	1.2063E-09	1.0039E-07
(30,50,100,100)	5.8353E-08	5.323E-012	4.5743E-010	1.0984E-08
(30,30,50,30)	1.5375E-07	5.323E-012	1.1708E-09	4.8042E-08
(30,20,50,50)	1.1503E-07	5.323E-012	8.9958E-010	3.0406E-08
(30,100,20,30)	1.5690E-07	5.323E-012	1.1992E-09	7.4901E-08
(50,50,50,50)	9.2584E-07	5.3230E-011	7.3307E-09	3.3730E-07
(50,20,30,100)	7.9172E-08	5.323E-012	6.0768E-010	2.7153E-08
(50,100,100,50)	9.0335E-07	5.3230E-011	7.1707E-09	2.3430E-07
(50,30,100,20)	2.1534E-07	5.323E-012	1.5579E-09	1.8696E-07
(100,20,30,30)	1.6145E-07	5.323E-012	1.2280E-09	5.0486E-08
(100,30,50,20)	2.3598E-07	5.323E-012	1.7385E-09	1.8249E-07
(100,100,20,50)	1.0578E-06	5.3230E-011	83475E-09	5.8070E-07
(100,50,20,20)	2.1892E-07	5.323E-012	1.6044E-09	8.6640E-08

7. Conclusion

The estimation of S-S reliability for two parameters Invers Kumaraswamy distribution was introduced in this paper using different shrinkage methods and estimation methods, namely: Constant Shrinkage Estimation Method, Modified Thompson Type Shrinkage Estimator, Shrinkage Function Estimator, and Squared Shrinkage Estimator. The simulation was exhibited. Based on the results, the performance of the Shrinkage Function Estimator was appropriate behavior, and it is an efficient estimator than the others in the sense of MSE based on four parameters (β_1, β_2 , β_3, β_4). While Modified Thompson Type Shrinkage Estimator had the second rank and followed by Squared Shrinkage Estimator and Constant Shrinkage Estimation Method, respectively.

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