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On Nano İ-pre-g-Open Set

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Abstract

In this work, the notion ŇÍ-pre -g -openset is defined by using nano topological space and some properties of this set are studied also, nano İ-pre-g-δ-set and nano İ-pre-g-μ-closedset are two concepts that are defined by using ŇÍ-pre -g -open set; many examples have been cited to indicate that the reverse of the propositions and remarks is not achieved. In addition, new application example of nano İ-pre -g -closed set was studied.

Keywords: Nano İ-pre-g-open set, nano İpgδ-set, nano İpgμ-closed set, nano-open, nano-closed, ideal.

1. Introduction

In1933, kuratowski [1]. Introduced the concept of an ideal İ on anon empty set X, where the hereditary and finite additively property were achieved.

In 1945, the notion of operator $(\ )^*: P(X) \longrightarrow P(X)$, was introduced by Vaidyanathaswamy [2]. And namely local function.

In 2013, Thivagar and Richard [3]. Introduced on X. nano forms of weakly open sets, Parimala and Jafari [4]. In 2018, introduced on some new notions in nano ideal topological spaces.

An ideal $İ \neq \emptyset$ such that $İ \subseteq P(X)$ was defined as the following:

i. if $A, B \in İ$, then $A \cup B \in İ$.

ii. if $A \in İ$ and $B \subseteq A$, then $B \in İ$ [1,2].

The closure operator $cl^*(\ )$ for a topology $T^*(İ, T)$, namely the $*$ topology, finer than T, is defined by $cl^*(A) = A \cup A^*(İ, T)$, and then, $T^*(İ, T) = \{A \subseteq X : cl^*(X - A) = (X - A)\}$.

The collection $B(\dot{I}, T) = \{A - B; A \in T \text{ and } B \in \dot{I}\}$ is a basis for $T^*(T, \dot{I})$, when there is no chance for confusion. The simple A^* write for $A^*(T, \dot{I})$ and T^* for $T^*(\dot{I}, T)$. The notion $(X, T, \dot{I})$ will denote to a topological space (X, T) with an ideal $\dot{I}$ on X with no separation properties assumed and called an ideal topological space or an ideal space for short.

The elements of T^* are namely T^*-open sets. If $(X - A)$ is T^*-open set, then A is namely T^*-closed and so it is closed in the space (X, T^*). A subset A of an ideal space $(X, T, \dot{I})$ is a T^*-closed if and only if $A^* \subseteq A$.

A subset A of an ideal space $(X, T, \dot{I})$ is said to be T^*dense if $cl^*(A) = X$, it is clear that, in a space $(X, T, \dot{I})$, if $\dot{I} = \{\emptyset\}$, then $T = T^*(\dot{I}, T)$. If $A \subseteq X$, $int^*(A)$ (respectively, $cl^*(A)$ will denote the interior (respectively , the closure) of A in (X, T^*), so the mapping $(\,)^*: P(X) \longrightarrow P(X)$, is used to generalize the concept of topology and create a new topology namely $T^* \subseteq T$ such that the shortcut $(X, T, \dot{I})$ is the ideal topological space [5-7].

By using lower and upper approximation with equivalence relation in 2013 [3,8]. A new space emerged, which is a nano topological space. In this research and by taking advantage of the previous concepts, another type of near nano open set is presented, which is the above space with ideal $\check{N}\dot{I}$-pre-g-closed set and will clarify the most important characteristics of these sets.

2. Preliminaries

Definition 2.1. [3, 8]. Let $X \neq \emptyset$, and $\hat{R}$ be an equivalence relation, where $\hat{R} \subseteq X \times X$ and $\hat{R}$ is reflexive, symmetric and transitive on X, $A \subseteq X$.

1. The upper approximation of A for $\hat{R}$ is symbolizes $\overline{\hat{R}(A)}$, which is, $\overline{\hat{R}(A)} = \cup_{x \in X} \{\hat{R}(x): \hat{R}(x) \cap A \neq \emptyset\}$.

2. The lower approximation of A for $\hat{R}$ is symbolizes $\underline{\hat{R}(A)}$, which is, $\underline{\hat{R}(A)} = \cup_{x \in X} \{\hat{R}(x): \hat{R}(x) \subseteq A\}$.

3. The boundary of A for $\hat{R}$ is symbolizes $B_{\hat{R}(A)}$, which is, $B_{\hat{R}(A)} = \overline{\hat{R}(A)} - \underline{\hat{R}(A)}$.

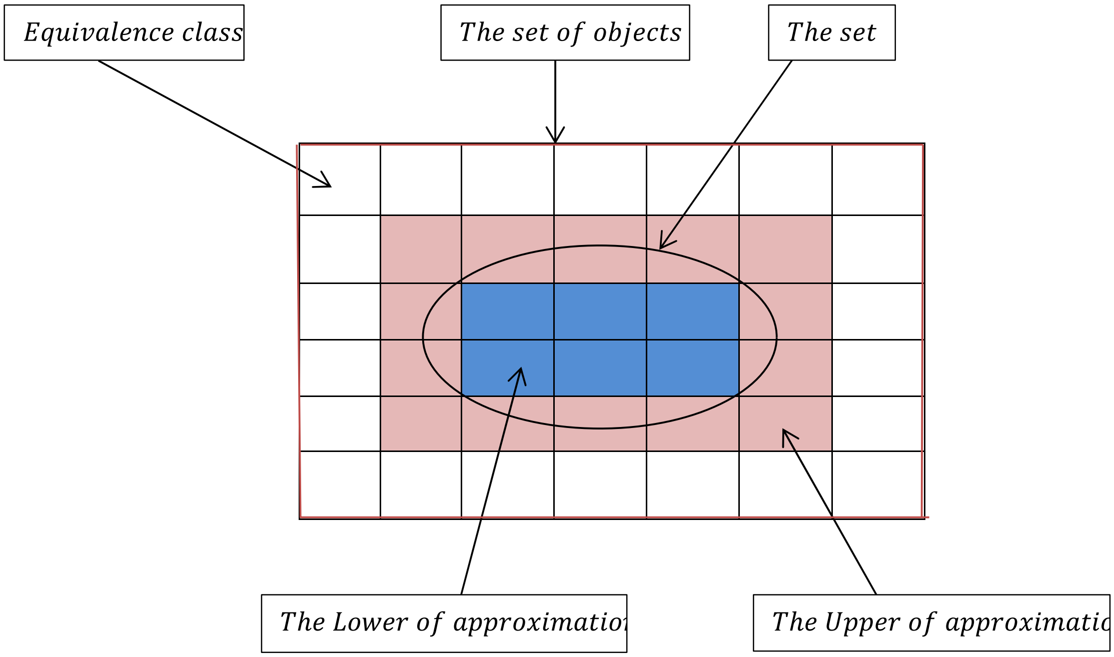

Figure 1. Approximation of ₳.

Definition2.2. [3]. Let $X \neq \emptyset$, $\hat{R}$ be an equivalence relation on X, $Ṭ_{\hat{R}\,(₳)} = \{X, \emptyset, \underline{\hat{R}(₳)}, \overline{\hat{R}}(₳), ฿_{\hat{R}(₳)}\}$ such that $₳ \subseteq X$. Then $Ṭ_{\hat{R}\,(₳)}$ is a topology on X namely nano topology of ₳ and $(X, Ṭ_{\hat{R}\,(₳)})$ is namely nano topological space. The elements of $Ṭ_{\hat{R}\,(₳)}$ are namely nano-open sets symbolize Ň-open sets.The complement of an Ň-open set is namely nano-closed symbolize Ň-closed. A nano-interior of a sub set ₳ of X symbolizes Ň-int(₳) and nano-closure of a subset ₳ of X symbolizes Ň-cl(₳).

We can find all nano topological spaces $(X, Ṭ_{\hat{R}\,(₳)})$, for any $X \neq \emptyset$, $₳ \subseteq X$ and $\hat{R}$ be an equivalence relation on X, by the following example:

Example2.3. Let $X = \{x_1, x_2, x_3\}$, $₳ \subseteq X$, $\hat{R} = \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_1, x_2), (x_2, x_1)\}$. Then $\hat{R}_{(x_1)} = \{x_1, x_2\} = \hat{R}_{(x_2)}$, $\hat{R}_{(x_3)} = \{x_3\}$.

Table 1. Nano topological spaces .

A	$\overline{\hat{R}(A)}$	$\underline{\hat{R}(A)}$	$B_{\hat{R}(A)}$	$T_{\hat{R}\,(A)}$
$\{\emptyset\}$	$\{\emptyset\}$	$\{\emptyset\}$	$\{\emptyset\}$	$\{X, \emptyset\}$
X	$\{X\}$	$\{X\}$	$\{\emptyset\}$	$\{X, \emptyset\}$
$\{x_1\}$	$\{x_1, x_2\}$	$\{\emptyset\}$	$\{x_1, x_2\}$	$\{X, \emptyset, \{x_1, x_2\}\}$
$\{x_2\}$	$\{x_1, x_2\}$	$\{\emptyset\}$	$\{x_1, x_2\}$	$\{X, \emptyset, \{x_1, x_2\}\}$
$\{x_3\}$	$\{x_3\}$	$\{x_3\}$	$\{\emptyset\}$	$\{X, \emptyset, \{x_3\}\}$
$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{\emptyset\}$	$\{X, \emptyset, \{x_1, x_2\}\}$
$\{x_1, x_3\}$	$\{X\}$	$\{x_3\}$	$\{x_1, x_2\}$	$\{X, \emptyset, \{x_3\}, \{x_1, x_2\}\}$
$\{x_2, x_3\}$	$\{X\}$	$\{x_3\}$	$\{x_1, x_2\}$	$\{X, \emptyset, \{x_3\}, \{x_1, x_2\}\}$

Definition 2.4. [3,4]. Let $A \subseteq X$, $(X, T_{\hat{R}\,(A)})$ be a nano topological space. Then A is a namely nano pre-open set if $A \subseteq$ Ň-int(Ň-cl(A)), the complements of A is namely nano pre closed set. The shortcuts Ň-pO(X) respectively Ň-pC(X) is for the collection of each Ň-pre-open(respectively Ň-pre-closed)sets. A space $(X, T_{\hat{R}\,(A)}, I)$ is namely ideal nano topological space , whenever I is an ideal on X.

Definition4.1. [9]. Let $(X, T_{\hat{R}\,(A)})$ be a nano topological spaces and $B \subseteq X$. Then a nano-kernal of $B = \cap \{U : B \subseteq U,\ U \in T_{\hat{R}\,(A)}\}$ and symbolizes Ň-Ker(B).

From **Table 2.** We can calculate and note all nano pre-open set and nano pre closed set:

For $(X, T_{\hat{R}\,(A)})$, where $X = \{x_1, x_2, x_3\}$, $\hat{R} = \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_1, x_2), (x_2, x_1)\}$.

,

Table 2. Nano pre-open set.

Ⱥ	$Ṭ_{\hat{R}(Ⱥ)}$	ŇpO(X)	ŇpC(X)
$\{\phi\}$	$\{X, \emptyset\}$	p(X)	p(X)
X	$\{X, \emptyset\}$	p(X)	p(X)
$\{x_1\}$	$\{X, \emptyset, \{x_1, x_2\}\}$	,$\{X, \emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}$ $\{x_1, x_3\}, \{x_2, x_3\}\}$	,$\{X, \emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_3\}$ $\{x_2, x_3\}\}$
$\{x_2\}$	$\{X, \emptyset, \{x_1, x_2\}\}$	$\{X, \emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\},$ $\{x_1, x_3\}, \{x_2, x_3\}\}$	,$\{X, \emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_3\}$ $\{x_2, x_3\}\}$
$\{x_3\}$	$\{X, \emptyset, \{x_3\}\}$	$\{X, \emptyset, \{x_3\}, \{x_1, x_3\}, \{x_2, x_3\}\}$	$\{X, \emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$
$\{x_1, x_2\}$	$\{X, \emptyset, \{x_1, x_2\}\}$	$\{X, \emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\},$ $\{x_1, x_3\}, \{x_2, x_3\}\}$	$\{X, \emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_3\},$ $\{x_2, x_3\}\}$
$\{x_1, x_3\}$	$\{X, \emptyset, \{x_3\}, \{x_1, x_2\}\}$	p(X)	p(X)
$\{x_2, x_3\}$	$\{X, \emptyset, \{x_3\}, \{x_1, x_2\}\}$	p(X)	p(X)

3. Nano İ-pre-g open set.

In this section and by using the notion of nano ideal topological space and Ň-pre-open set we will study Ň-İpg-closed set with some of it is properties.

Definition3.1. In $(X, Ṭ_{\hat{R}(Ⱥ)}, İ)$, let $Ⱥ \subseteq X$. Then Ⱥ is namely nano İ-pre-g-closed set symbolize Ňİpg- closed if $cl(Ⱥ)-Ų \in İ$ whenever $Ⱥ-Ų \in İ$ and Ų is a nano pre-open subset of X.

$Ⱥ^c$ is namely nano İ-pre-g-open set symbolize Ň-İpg-open. The collection of all nano İpg-closed sets respectively nano İpg-open sets in $(X, Ṭ_{\hat{R}(Ⱥ)}, İ)$ symbolizes Ň-İpgC(X) respectively Ň-İpgO(X).

From **Table 3.** We can calculate and note that all nano İ-pre-g-closed set and it is complement nano İ-pre-g-open set from the space$(X, Ṭ_{\hat{R}(Ⱥ)}, İ)$, where $X = \{x_1, x_2, x_3\}$, $\hat{R} = \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_1, x_2), (x_2, x_1)\}$, $İ = \{\emptyset, \{x_1\}\}$.

Table 3. Nano İ-pre-g-closed set.

Ⱥ	$Ṭ_{Ȓ (Ⱥ)}$	ŇpO(X)	Ň-İ-pre-g-closedset	Ň-İ-pre-g-openset
{∅}	{ X, ∅}	p(X)	{ X, ∅, {x_2, x_3}}	{ X, ∅, {x_1}}
X	{ X, ∅}	p(X)	{ X, ∅, {x_2, x_3}}	{ X, ∅, {x_1}}
{x_1}	{ X , ∅, {x_1, x_2}}	,{X, ∅, {x_1}, {x_2}, {x_1, x_2} {x_1, x_3}, {x_2, x_3}}	{ X, ∅, {x_3}, {x_2, x_3}}	{ X, ∅, {x_1}, {x_1, x_2
{x_2}	{ X , ∅, {x_1, x_2}}	{X, ∅, {x_1}, {x_2}, {x_1, x_2}, {x_1, x_3}, {x_2, x_3}}	{ X, ∅, {x_3}, {x_2, x_3}}	{ X, ∅, {x_1}, {x_1, x_2
{x_3}	{ X, ∅ , {x_3}}	{X, ∅, {x_3}, {x_1, x_3},{x_2, x_3}}	{ X, ∅, {x_2}, {x_1, x_2}, {x_2, x_3}}	{ X, ∅, {x_1}, {x_3}, {x_1, x_3}}
{x_1, x_2}	{ X, ∅, {x_1, x_2}}	{X, ∅, {x_1}, {x_2}, {x_1, x_2}, {x_1, x_3}, {x_2, x_3}}	{ X, ∅, {x_3}, {x_2, x_3}}	{ X, ∅, {x_1}, {x_1, x_2
{x_1, x_3}	{ X, ∅, {x_3}, {x_1, x_2	p(X)	{ X, ∅, {x_2}, {x_3}, {x_1, x_2}, {x_2, x_3}}	{ X, ∅, {x_1}, {x_3} {x_1, x_2}, {x_1, x_3}}
{x_2, x_3}	{ X, ∅, {x_3}, {x_1, x_2	p(X)	{ X, ∅, {x_2}, {x_3}, {x_1, x_2}, {x_2, x_3}}	{ X, ∅, {x_1}, {x_3}, {x_1, x_2}, {x_1, x_3}}

All nano İ-pre-g-closed set and nano İ-pre-g-open set from the space(X, $Ṭ_{Ȓ (Ⱥ)}$, İ), where X = {x_1, x_2, x_3}, Ȓ = {(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_1, x_2), (x_2, x_1)},İ = {∅, {x_1}}

Remark3.2: For (X, $Ṭ_{Ȓ (Ⱥ)}$, İ)

i. Every nano closed set is an Ň-İpg-closed.

ii. Every nano open set is an Ň-İpg-open.

Proof **(i):** let Ⱥ be any nano closed set in (X, Ṭ , İ) and Ụ be a nano-pre-open set such that Ⱥ-Ụ ∈ İ, but cl(Ⱥ) = Ⱥ, so cl(Ⱥ)- Ụ = Ⱥ- Ụ ∈ İ. This implies, Ⱥ is an nano-İ-pre-g- closed set.
Proof **(ii):** let Ụ be any nano open set in (X, Ṭ , İ), then $Ụ^c$ is a nano closed set. This implies that $Ụ^c$ is an nano- İ-pre -g -closed set, thus, Ụ is an nano-İ- pre -g -open set.
Reverse of Remark3.2 is not correct from **Table 3.** If Ⱥ = {x_3} then {x_2, x_3} is Ň-İpg-closed not nano closed and {x_1, x_3} is Ň-İpg-open not nano open.

4. Nano-İ- pre- g- Kernal of Set.

In this section and by using the topics described earlier as nano ideal space and Ň-İpg-open set, many of the topological properties will be presented.

Definition 4.1. Let (X, $Ṭ_{Ȓ (Ⱥ)}$, İ) be a nano ideal topological space and Ƀ ⊆ X. Then nano İ-pre-g-kernal of Ƀ is symbolized by **Ň**-İpg-Ker(Ƀ)= ∩ {Ụ: Ƀ ⊆ Ụ, Ụ ∈ Ň-İpg-O(X)}. It is clear that Ƀ = Ň-İpg-Ker(Ƀ) whenever Ƀ ∈ Ň-İpgO(X).

Remark 4.2. If Ƀ ⊆ X of a space (X, $Ṭ_{Ȓ (Ⱥ)}$, İ). Then nano İ-pre-g-kernal(Ƀ) ⊆ nano kernal(Ƀ) .

Proof: Let $\mathrm{x} \notin$ ŇKer(Ƀ), $\mathrm{x} \in$ Ҳ then $\mathrm{x} \notin \cap\{$Ų : Ƀ $\subseteq$ Ų, Ƀ and Ų $\in$ Ţ$_{\hat{R}(Ⱥ)}\}$, then $\exists$ Ų $\in$ Ţ$_{\hat{R}(Ⱥ)}$, Ƀ $\subseteq$ Ų, $\mathrm{x} \notin$ Ų. Since every Ň-open set in (Ҳ, Ţ$_{\hat{R}(Ⱥ)}$) is Ň-ίpgŎopen in (Ҳ, Ţ$_{\hat{R}(Ⱥ)}$, ί), then $\exists$ Ų $\in$ Ň-ίpgŎ(Ҳ), Ƀ $\subseteq$ Ų; $\mathrm{x} \notin$ Ų, then $\mathrm{x} \notin \cap\{$Ų : Ƀ $\subseteq$ Ų, and Ų $\in$ Ň-ίpg-O(Ҳ)$\}$.Thus $\mathrm{x} \notin$ Ň-ίpg-Ker(Ƀ).

From **Table 3.** let Ⱥ=$\{x_1\}$, Ţ$_{\hat{R}(Ⱥ)} = \{$ Ҳ, $\emptyset, \{x_1, x_2\}\}$, Ň-ίpg-O(Ҳ) $= \{$ Ҳ, $\emptyset, \{x_1\}, \{x_1, x_2\}\}$, if Ƀ $= \{x_1\}$, then Ň- ker(Ƀ) $= \cap$ {Ų: Ƀ $\subseteq$ Ų and Ų $\in$ Ţ$_{\hat{R}(Ⱥ)}$} $= \{x_1, x_2\}$, but Ň-ί-pre-g-kernal $= \cap$ {Ų: Ƀ $\subseteq$ Ų and Ų $\in$ Ň-ίpg-O(Ҳ)} $= \{x_1\}$, So Ň- ker (Ƀ) $\nsubseteq$ Ň-ίpg-Ker (Ƀ).

Definition4.4. For any Ƀ $\subseteq$ Ҳ of (Ҳ, Ţ$_{\hat{R}(Ⱥ)}$, ί). Ƀ is namely nano ίpgδ- set if Ƀ = Ňίpg-Ker(Ƀ).

Theorem4.5. The union of any two nano-ίpg-closed sets is an nano-ίpg-closed set.

Proof: Let Ⱥ and Ƀ are two nano-ίpg-closed set in (Ҳ, Ţ , ί) and Ų is a nano-pre-open subsets of Ҳ, where (Ⱥ ∪ Ƀ) - Ų ∈ ί, then Ⱥ - Ų ∈ ί and Ƀ - Ų ∈ ί, so nano cl(Ⱥ) - Ų ∈ί and nano cl(Ƀ) - Ų ∈ ί, therefore, (nano cl(Ⱥ) - Ų) ∪ (nano cl(Ƀ) - Ų) ∈ ί, so nano cl(Ⱥ ∪ Ƀ) - Ų ∈ ί. Hence, Ⱥ ∪ Ƀ is anano-ί pg -closed set.

Corollary 4.6. The intersection of any two nano-ί pg-open sets is a nano-ί pg-open set.

Proof: Let Ⱥ and Ƀ be two nano-ί-pre-g-open sets in (Ҳ,Ţ , ί), so Ⱥ^c, Ƀ^c are nano-ί-pre-g-closed sets, therefore, Ⱥ^c ∪ Ƀ^c is anano-ί-pre-g- closed set by Theorem 4.5, Hence (Ⱥ ∩ Ƀ)^c is a nano-ί-pre-g- closed set, so Ⱥ ∩ Ƀ is a nano-ί-pre-g- open set.

Remark4.7. If Ҳ is a finite set then Ƀ = Ňίpg-Ker(Ƀ) iff Ƀ ∈ Ň-ίpgO(Ҳ).
The prove of Remark 4.7 by using definition 4.4 and corollary 4.6.

Definition4.8. For any Ƀ $\subseteq$ Ҳ of a space (Ҳ, Ţ$_{\hat{R}(Ⱥ)}$, ί), the set Ƀ is namely nano ίpgμ-closed if Ƀ = M ∩ W, where M is nano ίpgδ-set and W is a nano ίpg-closedset.

From **Table 4.** We can calculate and note that Ň-ίpg-Ker(Ƀ) for a subset of Ҳ where Ⱥ = $\{x_1, x_2\}$, Ţ$_{\hat{R}(Ⱥ)} = \{$ Ҳ, $\emptyset, \{x_1, x_2\}\}$, Ň-ί-pre-g-open $= \{$ Ҳ, $\emptyset, \{x_1\}, \{x_1, x_2\}\}$ and Ň-ί-pre-g-closed $\{$ Ҳ, $\emptyset, \{x_3\}, \{x_2, x_3\}\}$

Table 4. Nano Ί-pre-g-kernal.

Ƀ	**Ň-Ker(Ƀ)set**	**Ň-Ίpg-Ker(Ƀ)set**
$\{\emptyset\}$	$\{\emptyset\}$	$\{\emptyset\}$
Ҳ	Ҳ	Ҳ
$\{x_1\}$	$\{x_1, x_2\}$	$\{x_1\}$
$\{x_2\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$
$\{x_3\}$	Ҳ	Ҳ
$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$
$\{x_1, x_3\}$	Ҳ	Ҳ
$\{x_2, x_3\}$	Ҳ	Ҳ

From **Table 4.** The sets Ҳ, $\emptyset$, $\{x_1\}$ and $\{x_1, x_2\}$ are Ň-Ίpgδ-sets. And Ҳ, $\emptyset$, $\{x_1\}$, $\{x_2\}$, $\{x_3\}$,$\{x_1, x_2\}$, $\{x_2, x_3\}$ are Ň-Ίpgμ-closed sets but $\{x_1, x_3\}$ is not Ň-Ίpgμ-closed since ∄M and W such that M is a Ň-Ίpgδ-sets and W is a Ň-Ίpg-closed and $\{x_1, x_3\} = M \cap W$.

Remark 4.9. For any space (Ҳ, $T_{\hat{R}(A)}$, Ί):

i. Every nano Ίpg-closed set is nano Ίpg μ-closed.
ii. Every nano Ίpg-*o*pen set is nano Ίpgμ-closed.
iii. Every nano Ίpgδ-set is nano Ίpg-μclosed.

Proof:

(i): (⇒) Let Ƀ be an Ň-Ίpgclosed set. Since Ҳ = Ň-Ίpg-Ker(Ҳ) and Ƀ = Ҳ ∩ Ƀ such that Ҳ is Ň-Ίpgδ-set and Ƀ is Ň-Ίpg-closed set, hence Ƀ is nano Ίpg-μclosed.

(ii): (⇒) Let Ƀ is nano Ίpg-open set. Then Ƀ = nano Ίpg-Ker(Ƀ) by Remark 4.7, Ҳ finite. Then Ƀ is an Ň-Ίpgδ-set, so Ƀ is Ň-Ίpgμ-closed, by (part i).

(iii): (⇒) Let Ƀ = Ň-ΊpgKer(Ƀ). But Ƀ = Ƀ ∩ Ҳ and Ҳ is Ň-Ίpg-closed. So Ƀ is a Ň-Ίpg-μclosed.

Example 4.10. From **Table 3.** And **Table 4.**

(i) where $A = \{x_1, x_3\}$, $T_{\hat{R}(A)} = \{Ҳ, \emptyset, \{x_3\}, \{x_1, x_2\}\}$,Ň-Ίpg-O(Ҳ) = $\{Ҳ, \emptyset, \{x_1\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}\}$ and Ň-Ίpg-C(Ҳ) = $\{Ҳ, \emptyset, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_2\}\}$, where Ƀ = $\{x_1\}$, then $\{x_1\}$ is Ň-Ίpg-μclosed. And since $\{x_1\}$ is Ň-Ίpgδ-set but it is not Ň-Ίpg-closed set.

(ii) **If** $\AA = \{x_1, x_2\}$, $T_{\hat{R}(\AA)} = \{X, \emptyset, \{x_1, x_2\}\}$, Ň-ḯpg-O(X) $= \{X, \emptyset, \{x_1\}, \{x_1, x_2\}\}$ and Ň-ḯpg-C(X) $= \{X, \emptyset, \{x_3\}, \{x_2, x_3\}\}$, where Ƀ $= \{x_3\}$ then Ƀ is Ň-ḯpg-µclosed since $\{x_3\} = \{x_3\} \cap X$ such that $\{x_3\} \in$ Ň-ḯpg-C(X) and X is a Ň-ḯpgδ-set but it is not Ň-ḯpg-open set and it is not Ň-ḯpgδ-set.

Proposition 4.11: For $(X, T_{\hat{R}(\AA)}, \acute{I})$ if X is a finite set and Ƀ ⊆ X; Ƀ is an Ň-ḯpg-µclosed set, then Ƀ = Ň-ḯpg-ker(Ƀ) ∩ W, where W is Ň-ḯpg-closedset.

Proof: since Ƀ is Ň-ḯpg-µclosed, then Ƀ = M ∩ W such that M is Ň-ḯpgδ-set and W is a Ň-ḯpg-closed set this implies that Ƀ ⊆ M = Ň-ḯpg-ker(M) and Ƀ ⊆ Ň-ḯpg-ker(Ƀ) which is the smallest Ň-ḯpg-open set containing Ƀ. So Ň-ḯpg-ker(Ƀ) ⊆ Ň-ḯpg-ker(M) and Ƀ = M ∩ W there for Ƀ = Ň-ḯpg-ker(Ƀ) ∩ W , since Ƀ ⊆ Ň-ḯpg-ker(Ƀ) and M ⊆ Ň-ḯpg-ker(M).

5. The Application in Ň-ḯpg-closed set.

Example 5.1.

Tonsillitis is a common disease in children and adults. People get inflammation that causes them difficulty in eating and sometimes unable to chew food. Also, you may experience a high temperature with a change in the body with diarrhea and joint pain if the inflammation is very strong. Treatment lasts one to two weeks. To detect the most common symptoms of tonsillitis, we can take advantage of the concept of nano ḯpg-open set according to the following table, which shows the most common symptoms that may be associated with tonsillitis.

The following table gives information about four patient people $\{x_1, x_2, x_3, x_4\}$, we will refer to the symbol ¥ if the symptoms are clear to the person and indicate the symbol ₦ if the symptoms do not appear:

Table 5. Information of Tonsillitis.

patient person	Temperature (T)	Emaciation (N)	Diarrhea (D)	Inability to swallow(I)	Joint pin (J)	Tonsillitis (S)
x_1	High	¥	¥	¥	¥	¥
x_2	Very High	¥	₦	¥	₦	¥
x_3	High	¥	₦	₦	₦	₦
x_4	Normal	¥	₦	₦	₦	₦

Let $X = \{x_1, x_2, x_3, x_4\}$ be the set of patient person with tonsillitis, let $A = \{x_2, x_4\}$ and $\hat{R}$ be the equivalence relation on X, Such that $\hat{R} = \{(x_i, x_j): x_i, x_j$ have the same appear symptoms$\}$.Then the set of equivalence classes corresponding to R is given by $X/\hat{R} = \{\{x_1\},\{x_2\},\{x_3\},\{x_4\}\}$, $T_{\hat{R}(A)} = \{X, \emptyset, \{x_2, x_4\}\}$, $\dot{I} = \{\emptyset, \{x_1\}\}$. $\check{N}pO(X) = \{X, \emptyset, \{x_2\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}\}$,$\check{N}\acute{I}pgC(X) = \{X, \emptyset, \{x_3\}, \{x_1, x_3\}, \{x_2, x_3, x_4\}\}$, $\check{N}\acute{I}pgO(X) = \{X, \emptyset, \{x_1\}, \{x_2, x_4\}, \{x_1, x_2, x_4\}\}$

If we delete (Temperature(T)), then we get $X/\hat{R} - (T) = \{\{x_3, x_4\}, \{x_1\}, \{x_2\}\}$. Hence $T_{\hat{R}(A)-(T)} = \{X, \emptyset, \{x_2\}, \{x_3, x_4\}, \{x_2, x_3, x_4\}\}$. $\check{N}pO(X) = \{X, \emptyset, \{x_2\}, \{x_3\}, \{x_4\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_4\}, \}$. $\check{N}\acute{I}pgC(X) = \{X, \emptyset, \{x_1, x_2\}, \{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}\}$. $\check{N}\acute{I}pgO(X) = \{X, \emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}, \{x_3, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}\}$.

If we delete (Joint pain (J)), then we get $X/\hat{R} - \{J\} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$. Hence $T_{\hat{R}(A)-(J)} = \{X, \emptyset, \{x_2, x_4\}\} = T_{\hat{R}(A)}$. $\check{N}pO(X) = \{X, \emptyset, \{x_2\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}\}$, $\check{N}\acute{I}pgC(X) = \{X, \emptyset, \{x_3\}, \{x_1, x_3\}, \{x_2, x_3, x_4\}\}$, $\check{N}\acute{I}pgO(X) = \{X, \emptyset, \{x_1\}, \{x_2, x_4\}, \{x_1, x_2, x_4\}\}$.

If we delete (Diarrhea (D)), then we get $X/\hat{R} - D = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$ and hence $T_{\hat{R}(A)-(D)} = \{X, \emptyset, \{x_2, x_4\}\} = T_{\hat{R}(A)}$.

$\check{N}pO(X) = \{X, \emptyset, \{x_2\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}\}$, $\check{N}\acute{I}pgC(X) = \{X, \emptyset, \{x_3\}, \{x_1, x_3\}, \{x_2, x_3, x_4\}\}$, $\check{N}\acute{I}pgO(X) = \{X, \emptyset, \{x_1\}, \{x_2, x_4\}, \{x_1, x_2, x_4\}\}$

If we delete (Inability to swallow (I)), then we get $X/\hat{R} - (I) = \{\{x_1\}, \{x_4\}, \{x_2, x_3\}\}$. Hence $T_{\hat{R}(A)-(I)} = \{X, \emptyset, \{x_4\}, \{x_2, x_3\}, \{x_2, x_3, x_4\}\}$, $\check{N}pO(X) = \{X, \emptyset, \{x_2\}, \{x_3\}, \{x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}\}$.

$\check{N}\acute{I}pgC(X) = \{X, \emptyset, \{x_1, x_2, x_3\}, \{x_1, x_4\}, \{x_1\}, \{x_4\}, \{x_2, x_3\}, \{x_2, x_3, x_4\}\}$.

$\check{N}\acute{I}pgO(X) = \{X, \emptyset, \{x_1\}, \{x_4\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_4\}\}$.

If we delete the attribute Emaciation (N), then we get, $X/\hat{R} - \{N\} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$, hence $T_{\hat{R}(A)-(J)}\{X, \emptyset, \{x_2, x_4\}\} = T_{\hat{R}(A)}$. $\check{N}pO(X) = \{X, \emptyset, \{x_2\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}\}$, $\check{N}\acute{I}pgC(X) = \{X, \emptyset, \{x_3\}, \{x_1, x_3\}, \{x_2, x_3, x_4\}\}$, $\check{N}\acute{I}pgO(X) = \{X, \emptyset, \{x_1\}, \{x_2, x_4\}, \{x_1, x_2, x_4\}\}$.

Therefore, from table above we get a core$(\hat{R}) = \{T, I\}$, we investigate that, (temperature(T)) and (Inability to swallow(I)) are the sufficient and necessary to say that a patient have tonsillitis(S), since $\check{N}\acute{I}pgO(X) = \{X, \emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}, \{x_3, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}$, where $T_{\hat{R}(A)-(T)} = \{X, \emptyset, \{x_2\}, \{x_3, x_4\}, \{x_2, x_3, x_4\}\}$ and $\check{N}\acute{I}pgO(X) = \{X, \emptyset, \{x_1\}, \{x_4\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_3\}\}$.Where, $T_{\hat{R}(A)-(I)} = \{X, \emptyset, \{x_4\}, \{x_2, x_3\}, \{x_2, x_3, x_4\}\}$.

Table 6. explains the difference for the $nano\ \acute{I}pgO(X)$ according to difference equivalent classes.

Table 6. Effective symptoms.

Equivalent cla	***Nano topolog***	***Nano*** $\dot{I}pgC(X)$	***Nano*** $\dot{I}pgO(X)$
$X/\hat{R}$ $=\{\{x_1\},\{x_2\},\{x_3$	$T_{\hat{R}(A)}$ $=\{X,\emptyset,\{x_2,x_4\}$	$\{X,\emptyset,\{x_3\},\{x_1,x_3\},\{x_2,x_3,x_4\}\}$	$\{X,\emptyset,\{x_1\},\{x_2,x_4\},\{x_1,x_2,x_4\}\}$
$X/\hat{R}-(T)$ $=\{\{x_3,x_4\},\{x_1\},\{$	$T_{\hat{R}(A)-(T)}=$ $\{X,\emptyset,\{x_2\},\{x_3,x$ $\{x_2,x_3,x_4\}\}$	$\{X,\emptyset,\{x_1\},\{x_2\},\{x_1,x_2\},\{x_3,x_4\},$ $\{x_1,x_3,x_4\},\{x_2,x_3,x_4\}\}$	$\{X,\emptyset,\{x_1\},\{x_2\},\{x_1,x_2\},\{x_3,x_4\},$ $\{x_2,x_3,x_4\},\{x_1,x_3,x_4\}\}$
$X/\hat{R}-(J)$ $=\{\{x_1\},\{x_2\},\{x_3$	$T_{\hat{R}(A)-(J)}$ $=$ $\{X,\emptyset,\{x_2,x_4\}\}=$	$\{X,\emptyset,\{x_3\},\{x_1,x_3\},\{x_2,x_3,x_4\}\}$	$\{X,\emptyset,\{x_1\},\{x_2,x_4\},\{x_1,x_2,x_4\}\}$
$X/\hat{R}-(I)$ $=\{\{x_1\},\{x_4\},\{x_2,$	$T_{\hat{R}(A)-(I)}$ $=$ $\{$ $X,\emptyset,\{x_4\},\{x_2,x_3$ $\{x_2,x_3,x_4\}\}$	$\{X,\emptyset,\{x_1\},\{x_4\},$ $\{\{x_2,x_3\},\{x_1,x_2,x_3\},$ $\{x_2,x_3,x_4\},\{x_1,x_4\}\}$	$\{X,\emptyset,\{x_4\},\{x_1\}$ $\{x_2,x_3\},\{x_1,x_2,x_3\},$ $\{x_2,x_3,x_4\},\{x_1,x_4\}\}.$
$X/\hat{R}-(D)$ $=\{\{x_1\},\{x_2\},\{x_3$	$T_{\hat{R}(A)-(D)}=$ $\{X,\emptyset,\{x_2,x_4\}\}=$	$\{X,\emptyset,\{x_3\},\{x_1,x_3\},\{x_2,x_3,x_4\}\}$	$\{X,\emptyset,\{x_1\},\{x_2,x_4\},\{x_1,x_2,x_4\}\}$
$X/\hat{R}-(N)$ $=\{\{x_1\},\{x_2\},\{x_3$	$T_{\hat{R}(A)-(N)}=$ $\{X,\emptyset,\{x_2,x_4\}\}=$	$\{X,\emptyset,\{x_3\},\{x_1,x_3\},\{x_2,x_3,x_4\}\}$	$\{X,\emptyset,\{x_1\},\{x_2,x_4\},\{x_1,x_2,x_4\}\}$

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