HUNGARIAN JOURNAL
OF INDUSTRY AND CHEMISTRY
VESZPRÉM
Vol. 42(2) pp. 109–113 (2014)

STABILITY AND PARAMETER SENSITIVITY ANALYSES OF AN INDUCTION MOTOR

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A simple dynamical model of an induction motor is derived and analyzed in this paper based on engineering principles that describe the mechanical phenomena together with the electrical model. The used state space model consists of nonlinear state equations. The model has been verified under the usual controlled operating conditions when the speed is controlled. The effect of load on the controlled induction motor has been analyzed by simulation. The sensitivity analysis of the induction motor has been applied to determine the model parameters to be estimated.

Keywords: induction motor, stability analysis, sensitivity analysis

Introduction

Induction motors (IM) are the most commonly used electrical rotating machines in several industrial applications. Irrespective of size and the application area, these motors share the most important dynamical properties, and their dynamical models have a similar structure.

Because of the specialties and great practical importance of IMs in industrial applications, their modelling for control purposes is well investigated in the literature. Besides basic textbooks [1-3], there are several papers that describe the modelling and use the developed models for the design of different types of controllers: vector control [1, 4], sensorless vector control [5] and direct torque control (DTC) [6]. The aim of this paper is to build a simple dynamical model of the IM and to perform its parameter sensitivity analysis. The results of this analysis will be the basis of the next step since the final aim of our study is to estimate the parameters of the IM and design a controller that can control the speed and torque of the IM. The state space model has been implemented in the Matlab/Simulink environment which enables us to analyze the parametric sensitivity based on simulation experiments.

Nonlinear Model of an Induction Motor

In this section a state space model of an induction motor (IM) is presented. The model development is largely based on Refs.[1, 8-10]. For constructing the IM model, the following modelling assumptions are made:

1. a symmetrical triphase stator winding system is assumed,
2. the flux density is radial in the air gap,
3. the copper loss and the slots in the machine can be neglected,
4. the spatial distribution of the stator fluxes and apertures wave are considered to be sinusoidal,
5. stator and rotor permeability are assumed to be infinite with linear magnetic properties.

According to the above modelling conditions the mathematical description of the IM is developed through the space-vector theory. If the voltage of the stator is presumed to be the input excitation of the machine, then the spatial distribution along the stator of the x phase voltage can be described by the complex vector $v_{\mathrm{sx}}(t)$. We can determine the orientation of the voltage vector v_{s}, the direction of the respective phase axis and the voltage polarity.

$$\begin{aligned} i_{\mathrm{s}}(t) &= \frac{2}{3}\left(a^0 \cdot i_{\mathrm{sa}}(t) + a^1 \cdot i_{\mathrm{sb}}(t) + a^2 \cdot i_{\mathrm{sc}}(t)\right) \\ &= \sqrt{2} \cdot i_{\mathrm{eff}}(t) \cdot e^{j\omega_0 t + \frac{\pi}{2} + \varphi_i}, \end{aligned} \tag{1}$$

where a is the e^{j120^o} vector and i_{sa}, i_{sb} and i_{sc} are the following:

$$i_{\mathrm{sa}}(t) = \mathrm{Re}(a^0 \cdot i_{\mathrm{s}}(t)) = \mathrm{Re}(i_{\mathrm{s}}(t)),$$

$$i_{\mathrm{sb}}(t) = \mathrm{Re}(a^2 \cdot i_{\mathrm{s}}(t)), \mathrm{and}$$

$$i_{\mathrm{sc}}(t) = \mathrm{Re}(a^1 \cdot i_{\mathrm{s}}(t)).$$

In *Eq.(1)*, 2/3 is the normalizing factor. The flux density distribution can be obtained by integrating the current density wave along the cylinder of the stator. The flux linkage wave is a system variable, because it contains detailed information about the winding geometry.

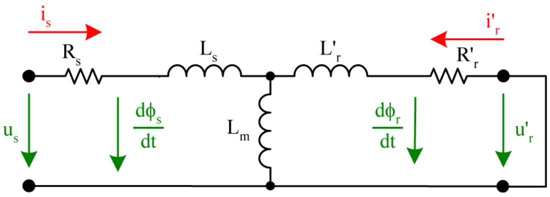

Figure 1: The equivalent circuit of the IM

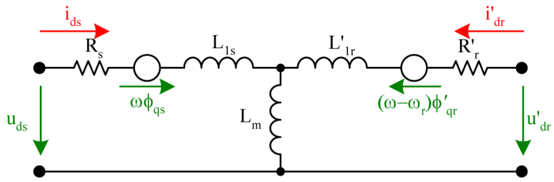

Figure 2: The equivalent circuit of the d-axis of the IM

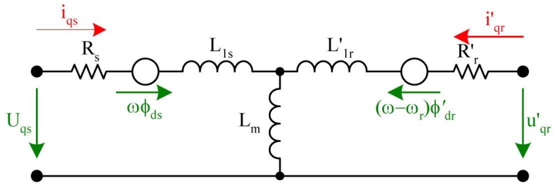

Figure 3: The equivalent circuit of the q-axis of the IM

The rotating flux density wave induces voltages in the individual stator windings. Thus stator voltage $v_s(t)$ can be represented as the overall distributed voltages in all phase windings:

$$\begin{aligned} v_s(t) &= \frac{2}{3}\left(a^0 \cdot v_{sa}(t) + a^1 \cdot v_{sb}(t) + a^2 \cdot v_{sc}(t)\right) \\ &= \sqrt{2} \cdot v_{eff}(t) \cdot e^{j\omega_0 t + \frac{\pi}{2} + \varphi_u}, \end{aligned} \quad (2)$$

where a is the e^{j120^o} vector and i_{sa}, i_{sb} and i_{sc} are the following:

$$v_{sa}(t) = \mathrm{Re}(a^0 \cdot u_s(t)) = \mathrm{Re}(u_s(t)),$$

$$v_{sb}(t) = \mathrm{Re}(a^2 \cdot u_s(t)), \text{and}$$

$$v_{sc}(t) = \mathrm{Re}(a^1 \cdot u_s(t)).$$

Considering the stator of the IM as the primer side of the transformer, then using Kirchoff's voltage law the following equation can be written (*Fig.1*):

$$v_s(t) = i_s(t) \cdot R_s + \frac{d\phi_s(t)}{dt}. \quad (3)$$

As for the secondary side of the transformer, it can be deduced that the same relationship is true for the rotor side space vectors:

$$v_r(t) = i_r(t) \cdot R_r + \frac{d\phi_r(t)}{dt} = 0. \quad (4)$$

Eqs.(3) and *(4)* describe the electromagnetic interaction as the connection of first order dynamical subsystems:

$$\phi_s(t) = i_s(t) \cdot L_s + i_r(t) \cdot L_m, \text{and} \quad (5)$$

$$\phi_r(t) = i_s(t) \cdot L_m + i_r(t) \cdot L_r. \quad (6)$$

Since four complex variables ($i_s(t)$, $i_r(t)$, $\phi_s(t)$, and $\phi_r(t)$) are presented in *Eqs.(5)* and *(6)*, flux equations are needed to complete the relationship between them.

The mechanical power ($P_{mech}(t)$) of the IM can be defined as:

$$P_{mech}(t) = \frac{W_{mech}(t)}{dt}, \quad (7)$$

where the mechanical energy ($P_{mech}(t)$) in the rotating system can be given by the following expression:

$$P_{mech}(t) = \frac{W_{mech}(t)}{dt} = T_{mech}(t) \cdot \omega_r, \quad (8)$$

where $T_e(t)$ is the torque and ω_r is the angular velocity of the IM. Afterwards the energy balance of the IM is as follows:

$$W_e = W_{mech} + W_R + W_{Field}, \quad (9)$$

where

$$P_e = \frac{W_e(t)}{dt} = \frac{3}{2}\mathrm{Re}\left(u_s \cdot i_s + u_r \cdot i_r\right) \quad (10)$$

is the input electrical power,

$$P_R = \frac{W_R(t)}{dt} = \frac{3}{2}\mathrm{Re}\left(R_s \cdot |i_s|^2 + R_r \cdot |i_r|^2\right) \quad (11)$$

represents the resistive power losses and

$$P_{Field} = \frac{W_{Field}(t)}{dt} = \frac{3}{2}\mathrm{Re}\left(\frac{d\phi_s}{dt} i_s + \frac{d\phi_r}{dt} i_r\right) \quad (12)$$

is the air gap power. Using *Eqs.(8-12)*,

$$P_{mech}(t) = T_{mech}(t) \cdot \omega_r = \frac{3}{2} \cdot \frac{L_m}{L_r} \cdot \phi_s(t) \times i_s(t), \text{and} \quad (13)$$

$$T_e = 1.5p \cdot (\phi_{ds} i_{qs} - \phi_{qs} i_{ds}). \quad (14)$$

The equivalent circuit of the IM can be decomposed to direct axis and quadratic axis components by Park's transformation as shown in *Figs.2* and *3*.

The actual terminal voltage v of the windings can be written in the form

$$v = \pm \sum_{j=1}^{J} (R_j \, i_j) \pm \sum_{j=1}^{J} \left(\frac{\mathrm{d}\phi_j}{\mathrm{d}t}\right), \quad (15)$$

where i_j are the currents, R_j are the winding resistances, and ϕ_j are the flux linkages. The positive directions of the stator currents point out of the IM terminals.

By composing the d and q axes of the IM the following equations can be written:

$$v_{qs} = R_s \cdot i_{qs} + \frac{\phi_{qs}}{dt} + \omega \cdot \phi_{ds}, \quad (16)$$

$$v_{ds} = R_s \cdot i_{ds} + \frac{\phi_{ds}}{dt} - \omega \cdot \phi_{qs}, \quad (17)$$

$$v_{qr} = R'_r \cdot i'_{ds} + \frac{\phi'_{dr}}{dt} + (\omega - \omega_r) \cdot \phi'_{dr}, \text{and} \quad (18)$$

$$v_{dr} = R'_r \cdot i'_{dr} + \frac{\phi'_{dr}}{dt} - (\omega - \omega_r) \cdot \phi'_{qr}. \quad (19)$$

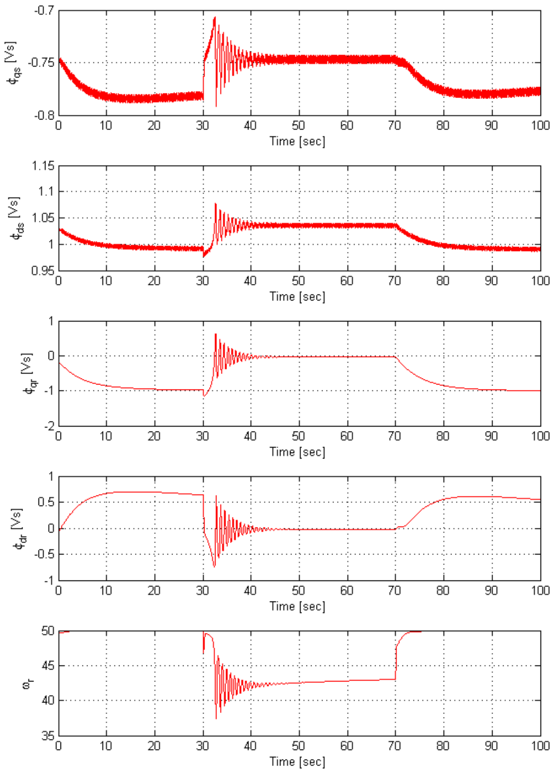

Figure 4: Response to the step change of the speed-controlled IM

$$\frac{d\omega_{\rm m}}{dt} = \frac{1}{2H}(T_{\rm e} - F\cdot\omega_{\rm m} - T_{\rm mech}) \quad (20)$$

where ω is the reference frame angular velocity, $\omega_{\rm r}$ is the electrical angular velocity,

$$\phi_{\rm qs} = L_{\rm s}\cdot i_{\rm qs} + L_{\rm m} i'_{\rm qr}, \quad (21)$$

$$\phi_{\rm ds} = L_{\rm s}\cdot i_{\rm ds} + L_{\rm m} i'_{\rm dr}, \quad (22)$$

$$\phi_{\rm qr} = L_{\rm r}\cdot i'_{\rm qr} + L_{\rm m} i'_{\rm qs}, \text{and} \quad (23)$$

$$\phi_{\rm dr} = L_{\rm r}\cdot i'_{\rm dr} + L_{\rm m} i'_{\rm ds}. \quad (24)$$

The above model can be written in state space form by expressing the time derivative of the fluxes and ω from the voltage and swing equations *Eqs.(21-24)*. The nonlinear state space model of the IM is given by *Eqs.(25-29)*:

$$\frac{d\phi_{\rm qs}}{dt} = \frac{-R_{\rm s}}{L_{\rm s} - \frac{L_{\rm m}^2}{L'_{\rm r}}}\cdot\phi_{\rm qs}\frac{R_{\rm s}\cdot L_{\rm m}}{L'_{\rm r}(L_{\rm s} - \frac{L_{\rm m}^2}{L'_{\rm r}})}\cdot\phi'_{\rm qr} - \omega\cdot\phi_{\rm ds} + v_{\rm qs}, \quad (25)$$

$$\frac{d\phi_{\rm ds}}{dt} = \frac{-R_{\rm s}}{L_{\rm s} - \frac{L_{\rm m}^2}{L'_{\rm r}}}\cdot\phi_{\rm ds} + \frac{R_{\rm s}\cdot L_{\rm m}}{L'_{\rm r}(L_{\rm s} - \frac{L_{\rm m}^2}{L'_{\rm r}})}\cdot\phi'_{\rm dr} - \omega\cdot\phi_{\rm qs} + v_{\rm ds}, \quad (26)$$

$$\frac{d\phi'_{\rm qr}}{dt} = \frac{-R'_{\rm r}}{L_{\rm m} - \frac{L'_{\rm r}\cdot L_{\rm s}}{L_{\rm m}}}\cdot\phi_{\rm qs} + \frac{-R'_{\rm r}\cdot L_{\rm s}}{L_{\rm m}\cdot(L_{\rm m} - \frac{L'_{\rm r}\cdot L_{\rm s}}{L_{\rm m}})}\cdot\phi'_{\rm qr} - \omega\cdot\phi'_{\rm dr} + \omega_{\rm r}\cdot\phi'_{\rm dr} + v_{\rm qr}, \quad (27)$$

$$\frac{d\phi'_{\rm dr}}{dt} = \frac{-R'_{\rm r}}{L_{\rm m} - \frac{L'_{\rm r}\cdot L_{\rm s}}{L_{\rm m}}}\cdot\phi_{\rm ds} + \frac{-R'_{\rm r}\cdot L_{\rm s}}{L_{\rm m}\cdot(L_{\rm m} - \frac{L'_{\rm r}\cdot L_{\rm s}}{L_{\rm m}})}\cdot\phi'_{\rm dr} + \omega\cdot\phi'_{\rm qr} - \omega_{\rm r}\cdot\phi'_{\rm qr} + v_{\rm dr}, \text{and} \quad (28)$$

$$\frac{d\omega_{\rm r}}{dt} = \frac{1}{2H}\cdot\frac{1.5\cdot p\cdot L_{\rm m}}{L_{\rm m}\cdot(L_{\rm s} - \frac{L_{\rm m}^2}{L'_{\rm r}})}\cdot\phi'_{\rm dr}\cdot\phi_{\rm qs} - \frac{1}{2H}\cdot\frac{1.5\cdot p\cdot L_{\rm m}}{L_{\rm m}\cdot(L_{\rm s} - \frac{L_{\rm m}^2}{L'_{\rm r}})}\cdot\phi'_{\rm qr}\cdot\phi_{\rm ds} - \frac{F}{2H}\cdot\omega_{\rm r} - \frac{T_{\rm m}}{2H}. \quad (29)$$

The state vector of the above model is $\mathbf{x}=[\phi_{\rm qs},\ \phi_{\rm ds},\ \phi'_{\rm qr},\ \phi'_{\rm dr},\ \omega_{\rm r}]^T \in \mathbb{R}^5$, and the input variables are organized in terms of the the input vector $\mathbf{u}=[v_{\rm q},\ v_{\rm d},\ -T_{\rm mech}]^T \in \mathbb{R}^3$. It is assumed that all the state variables can be measured i.e. $\mathbf{y} = \mathbf{x}$.

Model Validation

The dynamical properties of the IM have been investigated. The response of the speed-controlled motor has been tested under step-like changes. The simulation results are shown in *Fig.4*, where the fluxes ($\phi_{\rm qs}$, $\phi_{\rm ds}$, $\phi'_{\rm qr}$, and $\phi'_{\rm dr}$) and the angular velocity (ω) are shown.

Model Analysis

The above model *Eqs.(21-24)* has been verified by simulation against engineering intuition.

Local Stability Analysis

As the final aim of our research is to estimate the parameters of a particular Grundfos IM, first of all the resistances ($R_{\rm s}$ and $R_{\rm r}$) of the IM were measured. Afterwards the values of the inductances ($L_{\rm ls}$, $L_{\rm lr}$ and $L_{\rm m}$) and the mechanical parameters (H and F) of a similar IM with similar $R_{\rm s}$ and $R_{\rm r}$ found in the literature have been used. The parameters used during the model and the sensitivity analyses are the following:

$$\begin{aligned} R_{\rm s} &= 0.196\ \text{Ohm} \\ R_{\rm r} &= 0.0191\ \text{Ohm} \\ L_{\rm ls} &= 0.0397\ \text{H} \\ L_{\rm lr} &= 0.0397\ \text{H} \\ L_{\rm m} &= 1.354\ \text{H} \\ H &= 0.095 \\ F &= 0.0548 \\ p &= 1 \end{aligned} \quad (30)$$

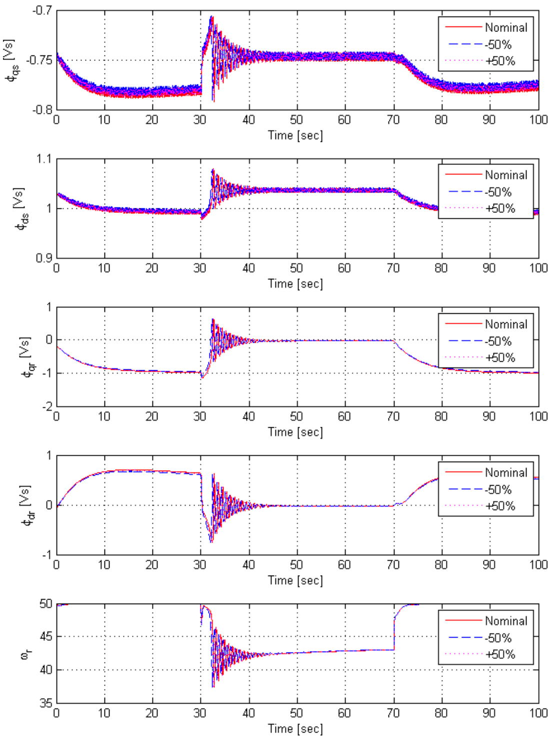

Figure 5: The model state variables for a $\pm 50\%$ change of L_{m}

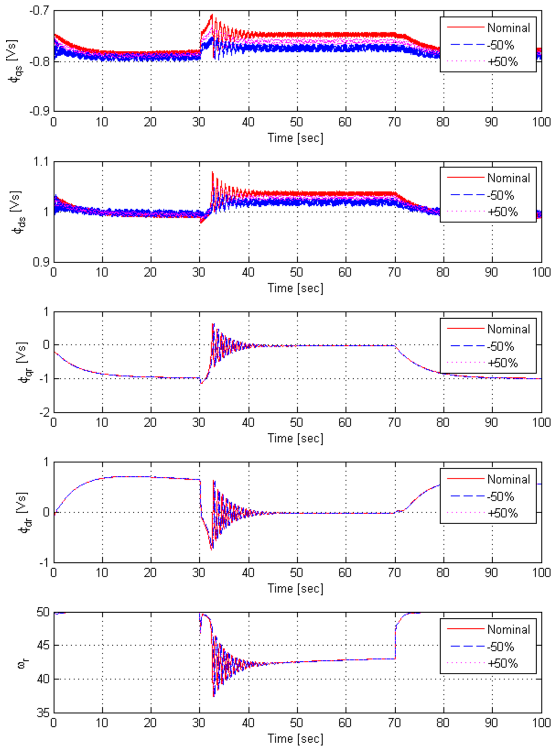

Figure 6: The model state variables for a $\pm 50\%$ change in R_{s}

It can easily be seen that the IM model is nonlinear since there are products of two state variables in the following equations:

- *Eq.(27)*: ϕ'_{dr} is multiplied by ω_{r}
- *Eq.(28)*: ϕ'_{qr} is multiplied by ω_{r}
- *Eq.(29)*: ϕ'_{dr} is multiplied by ϕ_{qs}
- *Eq.(29)*: ϕ'_{qr} is multiplied by ϕ'_{ds}

For the local stability analysis we have to calculate the eigenvalues of the system. The examined equilibrium point is

$$\begin{aligned} \phi_{\mathrm{qs}} &= -0.744 \ \ \mathrm{Vs} \\ \phi_{\mathrm{ds}} &= 1.0287 \ \ \mathrm{Vs} \\ \phi'_{\mathrm{qr}} &= -0.174 \ \ \mathrm{Vs} \\ \phi'_{\mathrm{dr}} &= -0.087 \ \ \mathrm{Vs} \\ \omega_{\mathrm{r}} &= 48.477 \ \ \mathrm{s}^{-1} \end{aligned} \tag{31}$$

The numerical value of the Jacobian of the nonlinear model (i.e. the state matrix of the locally linearized model) is as follows:

$$\begin{bmatrix} -2.504 & -50 & 2.4328 & 0 & 0 \\ 50 & -2.504 & 0 & 2.4328 & 0 \\ 0.2370 & 0 & -0.244 & -1.522 & 0 \\ 0 & 0.2370 & 1.5222 & -0.244 & 0 \\ -8.617 & 17.077 & 0 & 0 & -0.288 \end{bmatrix}. \tag{32}$$

The eigenvalues of the state matrix of the linearized systems are:

$$\begin{aligned} \lambda_{1,2} &= -2.504 \pm \mathrm{j}49.98, \\ \lambda_{3,4} &= -0.243 \pm \mathrm{j}1.534, \text{ and} \\ \lambda_{5} &= -0.288 \end{aligned} \tag{33}$$

It is apparent that the real parts of the eigenvalues are negative with small magnitudes.

Parameter Sensitivity Analysis

the sensitivity of the nonlinear model to the mutual inductance has been investigated. The steady state value of the system variables does not change (as is apparent in *Fig.5*) even for a considerably large change in L_{m}. Sensitivity analysis of the inductances L_{lr}, L_{ls} of the stator and rotor, resistances of the stator R_{s} and the damping constant F has also been investigated. In this investigation it can be seen that the values of the state variables were changed a bit, as shown in *Fig.6*.

The analysis of the resistances of the rotor R'_{r} and the inertia H of the rotor showed that every value of the state variables changed significantly, as shown in *Fig.7*.

As a final result of the sensitivity analysis, we can define the following groups of parameters:

- *Not sensitive*: Mutual inductance L_{m}. Since the state space model of interest is insensitive in this respect, the values of this parameter cannot be reliably determined from measurement data using any parameter estimation method.
- *Sensitive*: These sensitive parameters are candidates for parameter estimation.
  - Less: inductances L_{lr} and L_{ls}, resistance of the stator R_{s} and the damping constant F.
  - More: resistances of the rotor R'_{r} and the inertia H of the rotor.
- *Critically sensitive*: none.

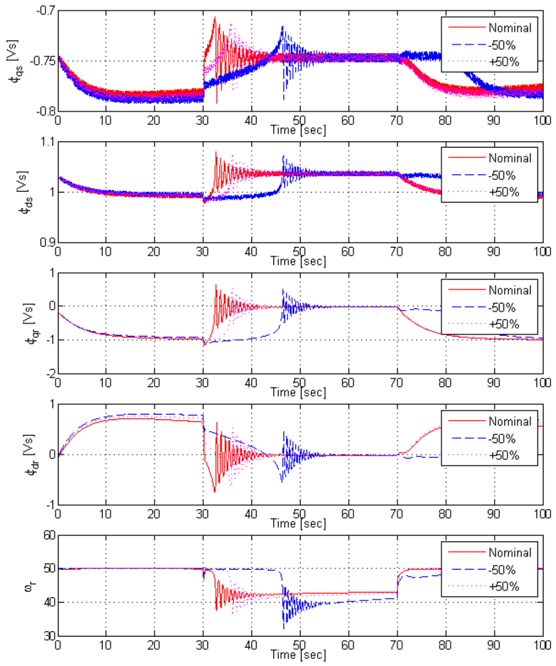

Figure 7: The model state variables for a $\pm 50\%$ change in H

Conclusion

The simple nonlinear dynamical model of an IM has been investigated in this paper. It has been shown that the model is locally asymptotically stable with regards to a physically meaningful equilibrium state. The effect of the controlled generator has been analyzed by simulation using a traditional PI controller. It has been found that the controlled system is stable and can follow the set-point changes. Seven parameters of the model were selected for sensitivity analysis, and the sensitivity of the state variables has been investigated. As a result, the parameters were partitioned into three groups. Based on the results presented here, a future aim is to estimate the parameters of the model for a real system from measurements. The sensitivity analysis enables us to select the candidates for estimation that are inductances L_{lr} and L_{ls}, resistances R_{s} and R_{r}, the damping constant F and the inertia H. An additional future aim is to develop a model for control purposes and investigate different controllers before applying them on real systems.

ACKNOWLEDGEMENT

We acknowledge the financial support of this work by the Hungarian State and the European Union under the TAMOP-4.2.2.A-11/1/KONV-2012-0072 project. We acknowledge also the investigated IM of Grundfos Hungary Producer Ltd.

SYMBOLS

C_1, C_2, C_3	constant
F	damping constant
ϕ_{qs} and ϕ_{ds}	q and d components of the stator flux
ϕ'_{qr} and ϕ'_{dr}	q and d components of the reduced rotor flux
H	inertia constant
i_{qs} and i_{ds}	q and d components of the stator current
i'_{qr} and i'_{dr}	q and d components of the reduced rotor current
L_{m}	mutual inductance
p	number of pole pairs
R_{r} and L_{r}	rotor resistance and inductance
R'_{r} and L'_{r}	reduced value of rotor resistance and inductance
R_{s} and L_{s}	stator resistance and inductance
v_{d} and v_{q}	q and d components of the stator voltage
ω	angular velocity of the magnetic field
ω_{r}	angular velocity of the rotor
T_{mech}	mechanical torque

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