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FUZZY-BASED REAL-CODED GENETIC ALGORITHM FOR OPTIMIZING NON-CONVEX ENVIRONMENTAL ECONOMIC LOSS DISPATCH

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Abstract. *A non-convex Environmental Economic Loss Dispatch (NCEELD) is a constrained multi-objective optimization problem that has been solved for assigning generation cost to all the generators of the power network with equality and inequality constraints. The objectives considered for simultaneous optimization are emission, economic load and network loss dispatch. The valve-point loading, prohibiting operating zones and ramp rate limit issues have also been taken into consideration in the generator fuel cost. The tri-objective problem is transformed into a single objective function via the price penalty factor. The NCEELD problem is simultaneously optimized using a fuzzy-based real-coded genetic algorithm (GA). The proposed technique determines the best solution from a Pareto optimal solution set based on the highest rank. The efficacy of the projected method has been demonstrated on the IEEE 30-bus network with three and six generating units. The attained results are compared to existing results and found superior in terms of finding the best-compromise solution over other existing methods such as GA, particle swarm optimization, flower pollination algorithm, biogeography-based optimization and differential evolution. The statistical analysis has also been carried out for convex multi-objective problem.*

Key words: *Multi-objective optimization, non-convex Environmental Economic Loss Dispatch, price penalty factor, Pareto optimality, real-coded genetic algorithm, valve-point loading, prohibiting operating zones, ramp rate limit*

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List of Abbreviations:

CEED: Combined emission and economic dispatch
ED: Emission dispatch
ELD: Economic load dispatch
FPA: Flower Pollination Algorithm
FRCGA: Fuzzy-based real-coded genetic algorithm
GA: Genetic algorithm
N/w: Network
NCEELD: Non-convex Environmental Economic Loss Dispatch
NSGA: Non-dominated sorting genetic algorithm
POZs: Prohibiting operating zones
PPF: Price penalty factor
PSO: Particle swarm optimization
RCGA: Real-coded genetic algorithm
RRL: Ramp rate limit
VPL: Valve point loading

1. INTRODUCTION

1.1. Motivation

The electrical power networks traditionally functioned to minimize total generation fuel cost and were less bothered about the harmful emissions generated in the network [1-3]. After the *US clean air act* of 1990 (amended in 2010) and similar legislation in several other countries, the public concern towards the pollutants like CO_X, SO_2 and NO_X produced from the thermal power plant has grown. This, in turn, forces the utilities to deliver the power to the consumers with simultaneous minimum total generator fuel cost and total emission level [4-22]. A high degree of non-linearity and complexity is present in the modern generator's cost curve function because of the presence of valve point loading (VPL) effect and other effects, the resultant approximate solutions lead to a lot of revenue loss over time which is also affected by the network losses. To overcome this, the optimal amount of generated power of the thermal units are to be determined by minimizing emission, loss and cost simultaneously while satisfying all practical constraints, hence, generating a large-scale highly constrained non-linear multi-objective optimization problem.

1.2. Literature survey

The economic load dispatch (ELD) [1-3] is a real-world problem that, earlier, only considers the minimization of the generator fuel cost. Therefore, emission dispatch (ED) is considered in [4] for the very first time. Hence, both generator fuel cost and harmful environmental emissions should be treated as competing objectives. The combined emission and economic dispatch (CEED) minimize harmful emissions and generating unit cost simultaneously to obtain optimal generation for each network (N/w) unit satisfying various practical constraints. In [5-9], the authors presented weighted-sum or price penalty factor (PPF) based methods where all the considered objectives are treated as a unit function. Conventional genetic algorithm (GA) and differential evolution have been presented in [10] and [11], respectively to demonstrate the effect of VPL on the generators cost function but

GA requires large CPU time for the optimization. A fast initialization approach has been presented in [12] to solve non-convex economic dispatch problem but is usually stuck in local minima. A new whale optimization approach has been presented in [13] and have high computational efficiency. A flower pollination algorithm (FPA) is demonstrated in [14] for solving ELD and CEED problem in larger N/w.

Many evolutionary algorithms such as non-dominated sorting genetic algorithm (NSGA) [15], squirrel search algorithm [16], evolutionary programming [17] and NSGA-II [18] have been proposed for solving the bi-objective problem. The evolutionary programming has a slow convergence rate for large problem. A mine-blast algorithm has been developed in [19] to incorporate the valve point loading effect for solving the environmental economic load dispatch problem. A new global particle swarm optimization (PSO) is developed in [20] to solve bi-objective problem without and with transmission losses. A fuzzified PSO technique [21], harmony search [22] and Cuckoo search [23] is applied to optimize the solution for the CEED problem. The PSO approach deals with the problem of partial optimism.

1.3. Paper contributions

a) As most of the research has been carried out considering only two objectives (fuel cost and emissions), the authors have incorporated additional objective (network loss) to make the problem formulation more comprehensive and find better solution by merging two soft-computing techniques (RCGA and Fuzzy) for finding the best compromised solution out of the obtained Pareto solutions. Moreover, it has been found from the exhaustive literature review that the non-convex multi-objective optimization problem formulation with simultaneous minimization of three objective functions (emission, fuel cost and network loss) at different load demands has not been explored before.
b) The different non-linearities like valve-point loading, prohibiting operating zones (POZs) and ramp rate limit (RRL) are considered in this article for three conflicting objectives.
c) As all the considered objectives are competitive, the method generates multiple non-dominated Pareto optimal solutions rather than a single best solution from which the best-compromised solution is selected based on the highest fuzzy membership function value.
d) To validate the proposed methodology, three test cases have been considered at different load demands and the results are compared with already published methods based on GA [25], PSO [25, 26], FPA [27], Biogeography-based Optimization [28] and differential evolution [29].

2. MATHEMATICAL MODELING

The practical non-convex EELD problem has three conflicting objectives which aim to minimize generating cost, amount of harmful emissions and losses of the complex and non-linear network. To formulate a non-convex EELD problem following objectives and operating constraints are given below:

2.1. Non-convex economic load dispatch

It is more practical for fossil fuel-based generators to introduce the steam valve-point loading effect in a turbine by adding a rectified sinusoidal term to the quadratic cost equation which leads to non-smooth and non-convex function having manifold minimas [10]. Total generator fuel cost based on active power output can be represented as [14]

$$Minimize\ f1 = F_T = \sum_{i=1}^{N}(a_i P_i^2 + b_i P_i + c_i) + |e_i \times \sin(f_i \times (P_{min} - P_i))| \quad (1)$$

where P_i represents the output power generation of ith unit. a_i, b_i, c_i, e_i, and f_i are the generator fuel cost coefficients.

2.2. Emission dispatch (ED)

The goal of ED is to minimize the total environmental degradation due to fossil fuel burning to produce power. The total pollution level of the environment that needs to be minimized is given as [14]:

$$Minimize\ f2 = E_T = \sum_{i=1}^{N} 10^{-2} \times (\alpha_i + \beta_i P_i + \gamma_i P_i^2) + \xi_i \exp(\lambda_i P_i) \quad (2)$$

where α_i, β_i, γ_i, ξ_i, λ_i represents the pollution coefficients of the i^{th} generating unit.

2.3. Loss dispatch

The loss dispatch aims to minimize power loss without considering the generator cost and harmful emission of the network. To minimize loss [14]

$$Minimize\ f3 = P_L = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{io} P_i + B_{oo} \quad (3)$$

where B_{ij}, B_{io} and B_{oo} represents the line loss coefficients.

2.4. Non-convex Environmental Economic Loss Dispatch (NCEELD)

The NCEELD problem is to be formulated having an economy, harmful emissions and losses of the network as competing objectives. The proposed complex problem can be written as

$$Minimize\ C = f1 + (pfe) * f2 + (pfl) * f3 \quad (4)$$

where $'Pfe'$ and $'Pfl'$ are the PPF for emission and loss respectively. $f1$ represents total generator fuel cost, $f2$ represents total emission and $f3$ represents total N/w loss. The ratio of the max value of f1 to the max value of f2 gives PPF for emission, whereas, the ratio of the max value of f1 to the max value of f3 of the corresponding generator gives PPF for loss.

The procedure for finding PPF for emission and loss can be given as:

(a) The generator fuel cost ($/hr) is calculated at its maximum output using (1) for the convex and non-convex problems.

(b) The emission release from every generator (lb/hr or kg/hr) is calculated at its maximum output using (2).

(c) The losses of each are calculated at its maximum output using (3).

(d) $Pfe[i], Pfl[i]\ (i = 1,2\ldots n)$ for each generator is determined as in (5) and (6).

$$pfe[i] = \frac{\sum_{i=1}^{N}(a_i + b_i P_i^{max} + c_i P_i^{max\,2}) + |e_i \times \sin\{f_i \times (P_{imin}^{max} - P_i^{max})\}|}{\sum_{i=1}^{N} 10^{-2} \times (\alpha_i + \beta_i P_i^{max} + \gamma_i P_i^{max\,2}) + \xi_i \exp(\lambda_i P_i^{max})}\ (\$/lb) \quad (5)$$

$$pfl[i] = \frac{\sum_{i=1}^{N}(a_i + b_i P_i^{max} + c_i P_i^{max\,2}) + |e_i \times \sin\{f_i \times (P_{imin}^{max} - P_i^{max})\}|}{\sum_{i=1}^{N}\sum_{j=1}^{N} P_i^{max} B_{ij} P_j^{max} + \sum_{i=1}^{N} B_{io} P_i^{max} + B_{oo}}\ (\$/pu) \quad (6)$$

where P_i^{max} is the maximum capacity of the unit.

(e) $Pfe[i]$ and $Pfl[i]$ (i=1, 2... n) are sorted in ascending order.
(f) P_i^{max} is added starting from the generator unit with the smallest $Pfe[i]$ for harmful emissions and the generator unit with the smallest $Pfl[i]$ for the loss until $\sum P_i^{max} \geq P_D$.
(g) The $Pfe[i]$ and $Pfl[i]$ linked with the last generator unit is the PPF for emission and loss, respectively for a given load P_D.
(h) The $Pfe[i]$ and $Pfl[i]$ for particular load are determined. Eq. (4) is optimized subject to constraints in case of the tri-objective minimization problem.

For the convex EED problem, the $'Pfe'$ selected is 43.55981 \$/Kg and 44.07915 \$/Kg [27] for three generator unit network at 400 MW and 500 MW respectively. For non-convex problem considering standard IEEE 30-bus network, Pfe' and $'Pfl'$ calculated for load P_D of 2.834 p.u is 5932.9377 \$/lb & 10445.0680 \$/p.u and for load P_D = 4.32 p.u is 10949.4251 \$/lb & 19612.6323 \$/p.u respectively using method given in reference [8].

The optimization process is subjected to the following constraints:

a) The active power output of a generating unit is constrained by its bounds for a stable operation and is given as:

$$P_i^{min} \leq P_i \leq P_i^{max} \qquad i = 1,2,\ldots.,N \tag{7}$$

b) The total generated power balances the sum of the active power loss (P_L) and total load demand (P_D). Therefore,

$$\sum_{i=1}^{N} P_i - (P_D + P_L) = 0 \tag{8}$$

where P_L is denoted as B-coefficients. The error in loss coefficients is considered to be constant as in ref [14].

c) Generator ramp rate limits: The inclusion of ramp rate limits changes the operating limits of the generator as [24]

$$Max(P_i^{min}, P_i^o - DR_i) \leq P_i \leq Min(P_i^{max}, P_i^o + UR_i) \tag{9}$$

where, P_i^o is the previous operating point of ith generator and DR_i & UR_i are the down and up ramp rate limits respectively.

e) Prohibited operating zones: If any power plant works in these zones, some faults might occur for the machines or accessories such as pumps or boilers. Therefore, to prevent theses faults, the power generation limits must be changed so that they satisfy the POZ constraint. This feature can be included in the non-convex multi-objective problem formulation as [24]

$$\boldsymbol{P_i} \in \begin{cases} \boldsymbol{P_i^{\min}} \leq \boldsymbol{P_i} \leq \boldsymbol{P_{i1}^L} \\ \boldsymbol{P_{ik-1}^U} \leq \boldsymbol{P_i} \leq \boldsymbol{P_{ik}^L} \\ \boldsymbol{P_{izi}^U} \leq \boldsymbol{P_i} \leq \boldsymbol{P_i^{\max}} \end{cases} \tag{10}$$

Here z_i are the number of prohibited zones in *ith* generator curve, k is the index of prohibited zone of *ith* generator, P_{ik}^L is the lower limit of *kth* prohibited zone, and P_{ik-1}^U is the upper limit of *kth* prohibited zone of *ith* generator.

3. SOLUTION METHODOLOGY

The paper implemented FRCGA on three- and six generator networks, to identify the best-compromised solution amongst the available set of Pareto optimal solutions. The techniques used in the algorithm are as follows:

3.1. Pareto optimality

It is defined as the degree of efficacy in multi-objective and multi-criteria solutions and represents a condition where economic resources and its output have been assigned in such a manner that no objective can be made better without losing the well-being of the other. There is no way to improve one part of a Pareto optimal solution set without making another part worse. A state U will dominate state V if U is superior to V in at least one objective function and not worse in regard to the other objective functions. A decision vector 'u' will dominate another vector 'v' (as $m<n$) if

$$f_j(u) \leq f_j(v) \qquad \forall\, j = 1,2,3,,,i \tag{11}$$

and $$f_j(u) < f_j(v) \quad \text{for at least one } j \tag{12}$$

where j shows a total number of objectives considered for simultaneous optimization. The reduction in fuel cost of generator increases the environmental emissions and vice-versa. As the considered objectives are conflicting in nature so instead of getting an optimal solution a set of non-dominated (Pareto-optimal) solutions have been obtained, hence, Pareto-optimal solution has been considered.

3.2. Real-Coded Genetic algorithm

In a real-coded genetic algorithm (RCGA) for optimization, the output of each generator in the system is illustrated as a floating point rather than a binary number resulting in high precision solution [30]. For discontinuous, non-differentiable and discrete objective functions the algorithm is proved to be effective and superior to binary coded genetic algorithm. The outputs of all the generating units generate a solution string known as chromosome. The initial population is randomly generated in a given search space. The RCGA loop comprises pre-processing, three genetic operations and post-processing. It performs a global optimization to identify the best solution to the formulated problem and iterates until the convergence criteria is met. To estimate the fitness value for each individual to optimize NCEELD problem mentioned by (4) for a given load while satisfying limits shown in (7) and (8):

$$Min\ C = (f_1 + \alpha[\textstyle\sum_{i=1}^{N} P_i - (P_D + P_L)])^2 + ([pfe * (f_2 + \alpha[\textstyle\sum_{i=1}^{N} P_i - (P_D + P_L)]^2]) + \\ ([pfl * (f_3 + \alpha[\textstyle\sum_{i=1}^{N} P_i - (P_D + P_L)]^2]) \tag{13}$$

where α represents the penalty parameter that occurs if N/w load demand is not satisfied. This guarantees that a feasible solution gets higher fitness as compared to an infeasible solution.

3.3. Fuzzy approach based on min-max proposition

To optimize three conflicting objectives (fuel cost, emission and N/w loss) simultaneously is a tedious task as there are no single criteria to finalize the merit of the available non-dominated solutions. Due to the conflicting nature of the objectives, it is hard to find the best solution. Every objective is assigned a degree of satisfaction based on the membership functions provided by the fuzzy method. The membership functions represent the degree of membership in fuzzy sets in the range [0,1]. $\mu(F_i)$ is monotonically decreasing function given as [9]:

$$\mu(F_i) = \begin{cases} 1; F_i \le F_i^{\min} \\ \dfrac{F_i^{\max} - F_i}{F_i^{\max} - F_i^{\min}}; F_i^{\min} < F_i < F_i^{\max} \\ 0; F_i \ \ge F_i^{\max} \end{cases} \tag{14}$$

where $F_i^{\min}$ represents the expected minimum value and $F_i^{\max}$ represents the expected maximum value of objective function i.

The membership function value signifies how much a solution satisfies F_i on a scale of 0 to 1. The fuzzy min-max proposition to nominate the best solution amongst many solutions can be given as [9]

$$\mu_{bestsolution} = Max\{\min\,[\mu(F_j)]^k\} \tag{15}$$

where k is the number of Pareto-optimal solutions.

Each objective is expected to attain higher satisfaction for each solution. The best-compromised solution is identified based on the highest rank among k solutions. The pseudo-code to solve NCEELD problem is shown below

Step I: Initialise the cost coefficients, generator limits, load demand and the min-max values of each objective.

Step II: Create a random population to define the number of generators within specified limits.

Step III: Evaluate the fitness of the constrained tri-objective problem of the network with prohibiting operating zones and ramp rate limits.

Step IV: Single point crossover is used for pairing and mating of the selected chromosomes.

Step V: Mutant is created on a random basis.

Step VI: Create new chromosomes and offspring for convergence check.

Step VII: Select the fittest individual for the next generation.

Step VIII: Check the convergence criteria. If the maximum counter is reached, jump to Step IX. Else, Step IV.

Step IX: Calculate the membership value of the Pareto optimal solutions using (14). The F_{min} and F_{max} value of each objective are determined by optimizing all the objectives independently to determine the endpoints of the obtained Pareto front.

Step X: The degree of satisfaction attained for each objective is used to find the best-compromise solution based on min-max proposition as given in (15).

4. RESULTS AND DISCUSSION

To validate the performance, FRCGA has been employed to solve NCEELD problem on two networks having 3 and 6 generators satisfying all the operational network constraints at various power demands. The network data for 3 and 6 generating units is given in the appendix (Table 13, Table 14, Table 15 and Table 16). A program to imitate results for both the test N/w is written on MATLAB 7.10. The standard IEEE-30 bus network with six generator units is presented in Fig.1.

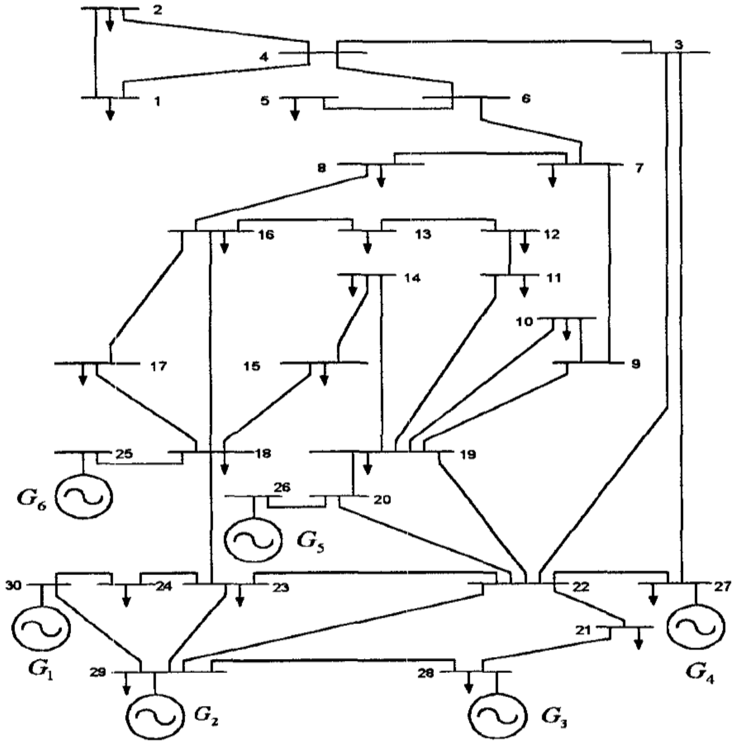

Fig. 1 One-line diagram of 30-bus network

To demonstrate the superiority of the FRCGA, three different test cases have been identified at different network complexity. The convergence test was carried out employing the same evaluation function for the same no. of iterations for convex case. The results for one trial of 250 iterations are shown in Fig. 2, Fig. 3 and Fig. 4 for optimized cost, emission and loss function respectively. It can be seen that FRCGA converges faster for the population size of 500.

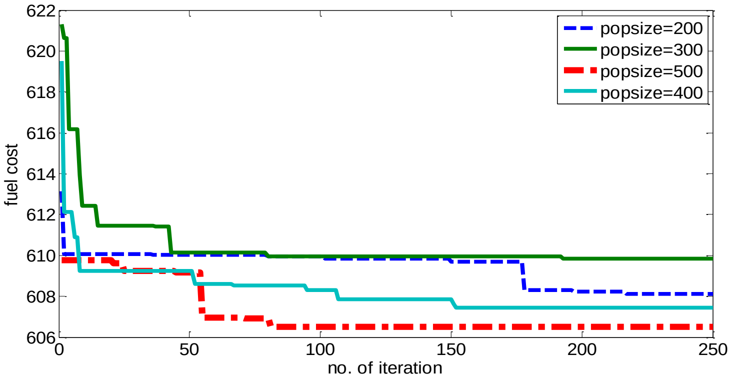

Fig. 2 Convergence characteristic for best fuel cost solution for different pop sizes

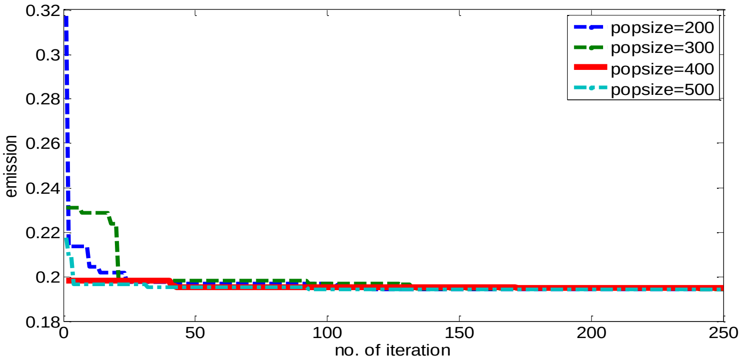

Fig. 3 Convergence characteristic for best emission solution for different pop sizes

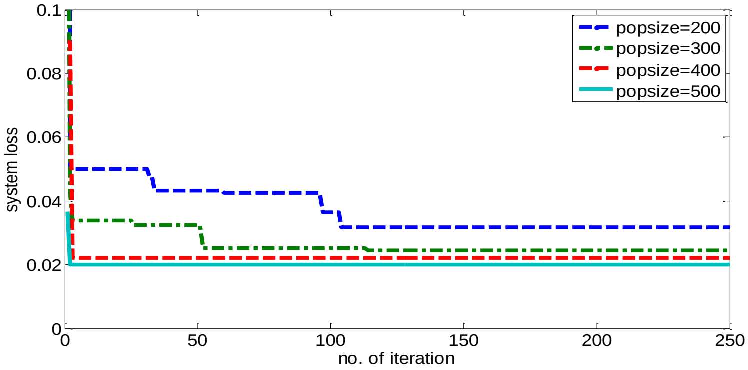

Fig. 4 Convergence characteristic for best n/w loss solution for different pop sizes

Hence, the optimal settings for both cases are the same, with the exception of population size and are mentioned in Table 1

Table 1 FRCGA parameters for different case studies

Parameters	Selected value
population size	200 (case 1) 500 (case 2 & 3)
Selection rate	0.3
Mutation rate	0.2
Trials	60
Iterations	250

4.1. Environmental Economic Dispatch

Three and six generator networks have been tested without considering the effect of VPL in the network. Table 2 illustrates the best cost and emission linked with the network at two different power demands of 400 MW and 500 MW. When cost minimization is performed, the generating fuel cost and N/w emissions are 20792.88 $ and 206.3426 Kg, respectively, but the cost of the generator increases to 20846.60 $, and the network harmful emission reduces to 200.1578 Kg in ED case at power demand of 400 MW. For 500 MW, the generator cost and N/w emissions are 25453.26 $ and 319.5089 Kg when cost minimization is performed, but the cost rises to 25500.40 $ and emission reduces to 311.0776 Kg. Using min and max values of each objective function, the membership value of the non-dominated solutions is determined.

Table 2 Best solution for ELD and ED of 3-unit N/w at P_D=400 MW and 500 MW

	Load demand			
	400 MW		500 MW	
	ELD	ED	ELD	ED
P1(MW)	81.4957	106.4685	103.5167	130.8372
P2(MW)	175.8190	151.1246	217.1612	190.1187
P3(MW)	149.8137	149.7724	190.9736	190.7181
Fuel cost ($)	20792.88	20846.60	25453.26	25500.40
Emission (Kg)	206.3426	200.1578	319.5089	311.0776
Loss (MW)	7.5560	7.3865	11.9239	11.6800

The simultaneous optimization of the environmental emission and the generator fuel cost is carried out to determine a best-compromise solution. In Table 3 and Table 4, five intermediate Pareto solutions are listed from the attained Pareto solution set using the presented approach with its membership values. Solution 5 is selected as the best solution having the highest rank of 0.1584 and 0.1110 at 400 MW and 500 MW respectively.

Table 3 Pareto optimal solutions for the convex-EED problem at P_D=400 MW (3-unit N/w)

Solution number	Cost ($)	Emission (Kg)	μ_1	μ_2	μ_{min}
1	20845.74	203.7849	0.0160	0.4135	0.0160
2	20843.59	200.6626	0.0560	0.9184	0.0560
3	20812.80	205.3911	0.6293	0.1539	0.1539
4	20838.31	200.3850	0.1544	0.9633	0.1544
5	20838.09	200.2123	0.1584	0.9912	0.1584

Table 4 Pareto optimal solutions for the convex-EED problem at P_D=500 MW (3-unit N/w)

Solution number	Cost ($)	Emission (Kg)	μ_1	μ_2	μ_{min}
1	25497.79	312.3221	0.0553	0.8524	0.0553
2	25497.63	311.0877	0.0586	0.9988	0.0586
3	25497.56	312.2660	0.0602	0.8590	0.0602
4	25496.93	311.1103	0.0737	0.9961	0.0737
5	25495.17	311.1194	0.1110	0.9950	0.1110

The summarized result for a best-compromised solution for three generating unit network is tabulated in Table 5 and is compared with the other methods such as GA [25], PSO [25] and FPA [27].

Table 5 Best solution for the convex-EED problem at P_D=400 MW and 500 MW (3-unit N/w)

	Best-Compromised solution							
	400 MW				500 MW			
	FRCGA	GA [25]	PSO [25]	FPA [27]	FRCGA	GA [25]	PSO [25]	FPA [27]
P1 (MW)	102.8514	102.617	102.612	102.4468	129.3252	128.997	128.984	128.8074
P2 (MW)	154.0217	153.825	153.809	153.8341	192.4745	192.683	192.645	192.5906
P3 (MW)	150.5278	151.011	150.991	151.1321	189.8764	190.11	190.063	190.2958
Fuel cost ($)	20838.09	20840.10	20838.30	20838.10	25495.17	25499.40	25495.00	25494.70
Emission (Kg)	200.2123	200.256	200.221	200.2238	311.1194	311.273	311.15	311.155
Loss (MW)	7.4090	7.41324	7.41173	7.4126	11.6882	-	-	-
Total cost ($)	29559.59	29563.20	29559.90	29559.81	39209.7	39220.10	39210.20	39210.15

The comparison depicts that the total generation cost incurred in solving EED problem from the FRCGA approach is lower than that incurred using other optimization approaches in both test cases. Thus, FRCGA succeeds to obtain the global minimum solution and performs superior to these algorithms in respect of all parameters. The total network losses for the best-compromised solution are 7.4090 MW and 11.6882 MW for power demand of 400 MW and 500 MW, respectively. For 30-bus N/w, the best-compromised solution attained has the value of 0.1999 lb/hr and 619.90 $/hr respectively for harmful environmental emission and cost, respectively at load demand of 2.834 p.u and is in close agreement with 0.1969 lb/hr and 623.87 $/hr as mentioned in [20]. Fig. 5 is the Pareto front drawn between the fuel cost and the emission points which was found to have an inverse relationship between the two objectives.

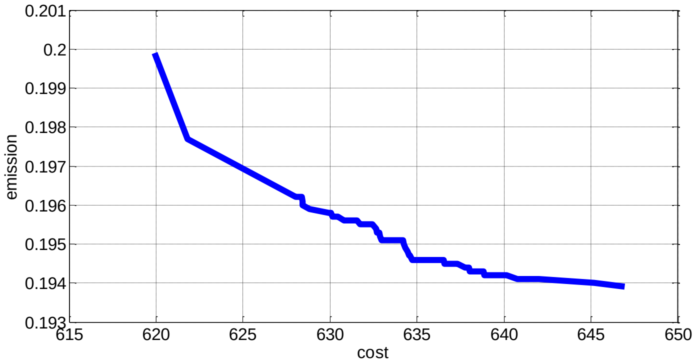

Fig. 5 Pareto front between generator fuel cost (\$/hr) and emission (lb/hr) for convex EED

4.2. Environmental Economic Loss Dispatch with valve-point loading

The performance of the FRCGA on the NCEELD problem is examined for the first time on the IEEE 30-bus network at two different loading conditions. Three objectives (fuel cost, environmental emission and losses) are simultaneously considered and optimized to obtain minimum network generation cost. The total generation cost comes out to be 1810.10 \$/hr at 2.834 p.u load demand which is found to be superior to published results at 2.834 p.u.

The minimum-maximum limits for fuel cost with VPL effect, harmful environmental emissions and losses for load demand of 2.834 p.u and 4.32 p.u are given in Table 6. For the load of 2.834 p.u, the values attained for cost and emission is 608.02 \$/hr and 0.1938 lb/hr that is found to be less when compared to 626.96 \$/hr & 0.2110 lb/hr [26], 613.342 \$/hr & 0.2028 lb/hr [28] and 613.338 \$/hr & 0.1953 lb/hr [29], respectively. The membership values of all the Pareto optimal solutions for the NCEELD problem are obtained. Five intermediate solutions are tabulated in Table 7 and Table 8 for P_D=2.834 p.u and P_D=4.32 p.u respectively.

Table 6 Min-max limit for fuel cost with VPL effect, emission and loss at 2.834 p.u and 4.32 p.u

		Load (p.u)	
		2.834	4.32
Cost (\$/hr)	Minimum	608.02	965.93
	Maximum	646.19	980.67
Emission (lb/hr)	Minimum	0.1938	0.2263
	Maximum	0.2211	0.2422
Loss (p.u)	Minimum	0.0209	0.0514
	Maximum	0.0379	0.0612

Table 7 Pareto optimal set of NCEELD problem with VPL effect for load P_D=2.834 p.u

Solution Number	Cost (\$/hr)	Emission (lb/hr)	Loss (p.u)	μ_1	μ_2	μ_3	μ_{min}
1	622.74	0.1973	0.0262	0.6144	0.8704	0.6894	0.6144
2	622.62	0.2001	0.0228	0.6174	0.7697	0.8862	0.6174
3	621.53	0.2022	0.0228	0.6461	0.6911	0.8863	0.6461
4	619.84	0.2021	0.0268	0.6905	0.6961	0.6558	0.6558
5	614.99	0.2027	0.0255	0.8174	0.6742	0.7284	0.6742

Table 8 Pareto optimal set of NCEELD for load P_D=4.32 p.u

Solution Number	Cost ($/hr)	Emission (lb/hr)	Loss (p.u)	Total cost ($/hr)	μ_1	μ_2	μ_3	μ_{min}
1	973.38	0.2326	0.0555	4490.2	0.4944	0.6013	0.5774	0.4944
2	972.85	0.2329	0.0563	4515.4	0.5301	0.5831	0.4959	0.4959
3	972.96	0.2335	0.0541	4489.12	0.5228	0.5451	0.7296	0.5228
4	972.76	0.2332	0.0545	4475.29	0.5365	0.5666	0.6788	0.5365
5	972.22	0.2335	0.0543	4497.8	0.5728	0.5480	0.7045	0.5480

The results reveal that the best-compromise solution for load demand of 2.834 p.u is 2099.20 $/hr and for load P_D=4.32 p.u is found to be 4497.82 $/hr with the highest rank of 67.42% and 54.80% respectively depending upon its membership value of each objective. Fig. 6 depicts the convergence criteria of 30-bus network on two different loads which reveal that the convergence of load P_D= 2.834 p.u and P_D= 4.32 p.u is attained faster even for the complex multi-objective minimization problem.

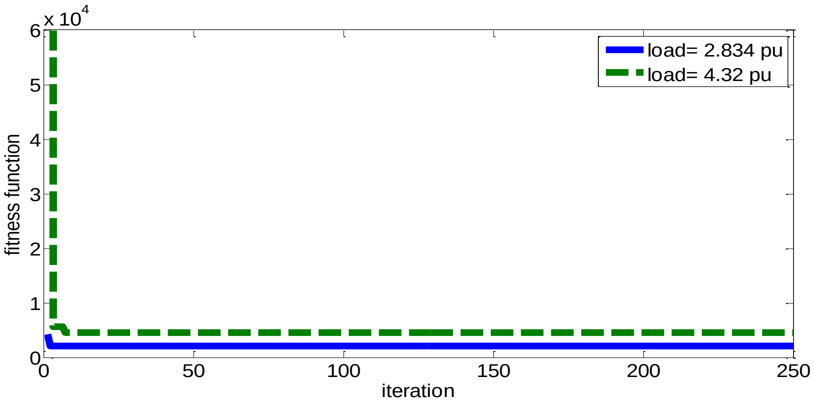

Fig. 6 Convergence characteristic for total generation cost for different load conditions

4.3. Environmental Economic Loss Dispatch with valve-point loading, POZs and RRL

For this test case, all the mentioned practical constraints and non-linear characteristic of non-convex multi-objective problem are considered. Due to which this test case is more complex than other test cases considered above. Data for the ramp rate limits and POZs has been taken from appendix (Table 15 and Table 17). The generator ramp rate limit needs to be satisfied as generator output cannot change (increase or decrease its output) arbitrarily to any value, the change has to within the up/down ramp rate limits. The inclusion of ramp rate limits changes the operating limits of the generator. The minimum-maximum limits of fuel cost, emission and loss evaluated for the six-unit system with POZs and RRL are given in Table 9 with load demand 2.834 pu. The results presented in Table 10 provides the intermediate solutions obtained using RCGA. The best solution is ranked on the basis of its performance for all the objectives considered. Therefore, overall rank for extreme points is zero. The rank of best solution is found to be 0.6685 which indicated that all three objectives are satisfied at least 66.85 % for load of 2.834 p.u.

Table 9 Min-max limit for fuel cost with VPL effect, emission and loss with POZs and RRL at 2.834 p.u

Cost($/h)		Emission(lb/h)		Loss(pu)	
minimum	maximum	minimum	maximum	minimum	maximum
611.2998	645.3562	0.1942	0.2073	0.0256	0.0358

Table 10 Pareto optimal set of NCEELD with POZs and RRL for load P_D=2.834 p.u

	Cost ($/h)	Emission (lb/h)	Loss (pu)	μ_1	μ_2	μ_3	μ_{min}
SOL.1	624.7335	0.1975	0.0257	0.6055	0.7473	0.9901	0.6055
SOL.2	623.7816	0.1989	0.0242	0.6335	0.6421	1.0000	0.6335
SOL.3	623.3781	0.1979	0.0291	0.6453	0.7208	0.6589	0.6453
SOL.4	621.4747	0.1987	0.0283	0.7012	0.6598	0.7379	0.6598
SOL.5	620.3646	0.1985	0.0256	0.7338	0.6685	1.0000	0.6685

The results clearly showed that all the constraints, such as VPL effect, POZs, RRL, generation limits and power balance constraints were fully satisfied for all considered test cases of tri-objective optimization problem. Due to the non-convexity constraints introduced in test system, the cost increases from 608.0296 $/hr to 611.2998 $/hr, emission increases from 0.1938 lb/hr to 0.1942 lb/hr and system loss from 0.0209 p.u to 0.0256 p.u.

4.4. Statistical Analysis

Table 11 lists the comparison of different approaches for cost and emission minimization in terms of their minimum, maximum, mean and median values, respectively, for IEEE 30-bus N/w. The cost minimum (C_{min}), cost mean (C_{mean}), cost median (C_{median}), emission minimum (E_{min}), emission mean (E_{mean}) and emission median (E_{median}) values obtained for the ELD and ED problem, respectively, are found to be lowest as compared to other published work. The statistical comparison of CEED problem has also been shown in Table 12 in terms of their mean and standard deviation. The values of C_{mean} and E_{mean} obtained from solving convex CEED problem also demonstrates the superiority of the method. The value of cost standard deviation (C_{std}) and emission standard deviation (E_{std}) attained from the proposed approach of FRGCA are 7.127 and 0.0057, respectively which is less than that obtained from other approaches. This clearly shows that the obtained results lie close to its mean value as compared to other published methods.

Table 11 Statistical comparison of ELD and ED minimization for IEEE 30-bus N/w at load P_D=2.834 p.u

		C_{min}	C_{max}	C_{mean}	C_{median}
Fuel cost minimization	Proposed approach	601.31	610.07	603.20	602.23
	GQPSO [31]	606.38	611.86	609.49	609.66
	SAIWPSO [32]	605.99	606.00	605.99	605.99
	NGPSO [20]	605.99	605.99	605.99	605.99
		E_{min}	E_{max}	E_{mean}	E_{median}
Emission minimization	Proposed approach	0.1938	0.2295	0.1941	0.1940
	GQPSO [31]	0.1942	0.1946	0.1944	0.1944
	SAIWPSO [32]	0.1941	0.1941	0.1941	0.1941
	NGPSO [20]	0.1941	0.1941	0.1941	0.1941

Table 12 Statistical comparison of CEED minimization for IEEE 30-bus N/w at load P_D=2.834 p.u

	C_{mean}	C_{std}	E_{mean}	E_{std}
Proposed approach	622.62	7.127	0.2012	0.0057
GQPSO [31]	644.09	12.2	0.2109	0.0095
SAIWPSO [32]	623.76	-	0.1970	-
NGPSO [20]	623.86	-	0.1969	-

5. CONCLUSION

The fuzzy-based RCGA is demonstrated to solve multi-objective environmental economic loss dispatch problem considering non-convex and non-smooth fuel cost function. The multi-objective minimization problem is transformed into the constrained single-objective problem by the use of price penalty factor which blends all competing objectives (generator cost, environmental emission and system losses). Because the objectives are inversely related, a set of Pareto optimal solutions are attained rather than a single optimal solution for a given objective. Furthermore, a fuzzy approach is exploited to extract best-compromised solution as per the highest rank based on their membership values. The convergence of the NCEELD problem at different load demand is also analyzed considering the different practical operating limits (POZs, RRL and VPL) of the network. The total generation cost of the network attained from the proposed method for different test cases has been compared to the other techniques which validate the solution to NCEELD problem for small and large networks. The statistical analysis also validates the FRGCA approach. The percentage reduction in C_{std} and E_{std} values are 41.5% and 40% as compared to ref. [31]. The proposed work can further be extended for the study of integration of renewable energy sources and for practical transmission networks considering dynamic non-convex CEELD problem.

APPENDIX

Table 13 Generator cost, emission coefficients & generation constraints for three generating unit network

		G1	G2	G3
Cost Coefficients	a_i	0.03546	0.02111	0.01799
	b_i	38.30553	36.32782	38.27041
	c_i	1243.5311	1658.5696	1356.6592
Emission Coefficients	α_i	0.00683	0.00461	0.00461
	β_i	-0.54551	-0.5116	-0.5116
	γi	40.2669	42.89553	42.89553
Unit limits	P_{min} (p.u)	35	130	125
	P_{max}(p.u)	210	325	315

Table 14 B-coefficients for three generating unit network

	0.71	0.3	0.25
B_{ij} * 0.0001	0.3	0.69	0.32
	0.255	0.32	0.8

Table 15 Generator fuel cost, emission coefficients and N/w generation constraints for 30-bus N/w

		G1	G2	G3	G4	G5	G6
Cost coefficients	a_i	100	120	40	60	40	100
	b_i	200	150	180	100	180	150
	c_i	10	10	20	10	20	10
	e_i	200	200	200	200	200	200
	f_i	0.0050	0.0060	0.0010	0.0009	0.0009	0.0015
Emission coefficients	α_i	4.091	2.543	4.258	5.326	4.258	6.131
	β_i	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
	γi	6.490	5.638	4.586	3.380	4.586	5.151
	ζi	0.0002	0.0005	0.00001	0.002	0.000001	0.00001
	λi	2.857	3.333	8.000	2.000	8.000	6.667
Generator unit constraints	P_{min} (p.u)	0.05	0.05	0.05	0.05	0.05	0.05
	P_{max} (p.u)	0.5	0.6	1.0	1.2	1.00	0.60
Ramp rate limits	DR_i(up)/h	0.08	0.11	0.15	0.18	0.15	0.18
	DR_i(dn)/h	0.08	0.11	0.15	0.18	0.15	0.18

Table 16 B-coefficients for six generating unit network

B_{ij}	0.1382	-0.0299	0.0044	-0.0022	-0.0010	-0.0008
	-0.0299	0.0487	-0.0025	0.0004	0.0016	0.0041
	0.0044	-0.0025	0.0182	-0.0070	-0.0066	-0.0066
	-0.0022	0.0004	-0.0070	0.0137	0.0050	0.0033
	-0.0010	0.0016	-0.0066	0.0050	0.0109	0.0005
	-0.0008	0.0041	0.0066	0.0033	0.0005	0.0244
B_o	-0.0107	0.0060	-0.0017	0.0009	0.0002	0.0030
B_{oo}	0.00098573					

Table 17 POZs of units for IEEE-30 bus N/w

Unit	1	2	5
POZ	[0.10 0.15]	[0.25 0.30]	[0.50 0.55]

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