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Meanings are acquired from experiencing differences against a background of sameness, rather than from experiencing sameness against a background of difference: Putting a conjecture to the test by embedding it in a pedagogical tool

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Abstract

In helping learners to make a novel meaning their own, such as when helping children to understand what a word means or teaching students a new concept in school, we frequently point to examples that share the aimed-at meaning but differ otherwise. This type of approach rests on the assumption that novel meanings can be acquired through the experience of sameness against a background of difference. This paper argues that this assumption is unfounded and that the opposite is the case: we make novel meanings our own through the experience of differences against a background of sameness. We put this conjecture to the test in an experimental study by embedding it in a computer game and the results support the conjecture.

Keywords: Variation Theory; Phenomenography; Discernment; Critical experiment

1. The Conjecture

This paper is about a conjecture and how it is put to the test. The conjecture is actually the title of the paper and we first briefly describe the theory that elaborates its implications, together with some previous results. After that we report on a study which is meant to be a critical test of it. We call this conjecture—and the system of corollaries that it implies—somewhat immodestly the *Variation Theory (of Learning)* (Marton, forthcoming; Marton & Tsui, 2004).

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1.1 The origin of meaning

It is commonly believed that a child - or an adult for that matter - can learn the meaning of a word by observing a number of examples of what the word refers to, that share this meaning but differ in other ways. For example, we point to a dog and say *dog*, point to another dog and say *dog*, point to a third dog and say *dog*, and then expect the child to understand what the word "dog" means (refers to), i.e., a certain kind of animal. In an experimental context such a learning event could look like the following: "…children might be shown a red fuzzy triangle labeled "wug", a blue bumpy triangle labeled "wug", a green scratchy triangle labeled "wug" and then at the test be asked to pick out a "wug" (a yellow squishy triangle) among two or three objects" (Vlach et al, 2008). Now, if a child has noticed previously that there are different geometric forms of which triangle is one, it is most likely that she will see that the three things are different, but they are all triangles regardless if she has learned that they are called "triangle". Hence she will identify the yellow squishy triangle in the test as a "wug". But if she has never noticed triangles previously, or geometric forms in general, she will not see any triangles at all. In consequence, she will not be able to see what the different cases have in common. There is no way of learning the idea of triangle in such an experimental context, if you have not come across that idea earlier. But if you have, you might be able to learn that triangles are called "wug" in the actual context. In the same way, no child can learn that dogs are a kind of animals, without coming across other animals than dogs. The idea of triangle derives from how it differs from other geometric forms, and the idea of dog derives from how it differs from other animals.

Providing different examples of the same thing is not only the most common method of helping young children to build a vocabulary, but probably also the most common method of teaching concepts, principles, and problem-solving methods in school. Stigler and Hiebert (1999) describe such an approach as the typical way of teaching mathematics in U.S. schools, and the highly authoritative volume *How People Learn* urges teachers to provide "…many examples in which the same concept is at work." (Bransford, Brown, & Cocking, 2000, p 20)

Looking at cases that are the same in one respect but differ in others to determine what they have in common is called induction. According to Fodor (1980), this is the only idea that exists to explain how novel meanings (concepts) are learned, and it simply does not work, for the reasons already cited. It follows then that there is no explanation of how we learn, find, create, or appropriate new meanings. Hence, by default, Fodor concludes that meanings (concepts) are innate. In our view, however, even if the concept (meaning) of "dog" were innate, you would never be able to separate that meaning from the meaning of "animal" if you had never encountered any animals other than dogs. Regardless of whether dogs were then called *dogs* or *animals*, the meaning of "dog" would be exactly the same as the meaning of "animal." Hence, you still would not have acquired the meaning of "dog." Nor of "animal" for that matter. The meaning of dog has to be learned, and this happens by coming across dogs, as well as other animals.

Similarly, if we lived in an entirely green world, then we would be unable to notice the greenness of everything. Hence, whether or not concepts (meanings) are innate, we must encounter alternatives to them if we are to be able to notice and grasp these concepts. Awareness of a particular number presupposes awareness of other numbers (or at least one other number), and awareness of a particular color presupposes awareness of other colors (or at least one other color). You cannot possibly understand what Chinese is simply by listening to different people speaking Chinese if you have never heard another language, and you cannot possibly understand what virtue is by inspecting different examples of the same degree of virtue. Nor can you understand what a "linear equation" is by looking only at linear equations.

You cannot arrive at a novel meaning through *induction*, but you can through *contrast*. In induction, the focused meaning, i.e., the one that you are trying to help another to make his or her own (e.g. "Chinese") is kept invariant, while the other features of the same entity (e.g. words) vary. In *contrast*, it is just the other way around. The focused meaning (e.g. language) varies, while other features (e.g. words) are invariant. Instead of saying different words in the same language (Chinese), you say the same word in different languages (one of which is Chinese).

Inductive learning is a frequent research topic, not the least in the field of machine learning (e.g. Michalski, 1983). As the conjecture being put to the test in our study is about how novel meanings are acquired, and as it states that they are not acquired through induction, we will leave those studies aside here.

1.2 Earlier attempts to put the conjecture to the test

Most work on Variation Theory has been carried out in the form of Learning Studies, the inspiration for which is the Japanese Lesson Study, which came to wider attention through the publication of Stigler and Hiebert's (1999) best-selling book *The Teaching Gap*. In this type of study, a group of teachers teaching a particular subject at a particular level together choose an object of learning (something to be learned) that is vitally important for students' continued learning and that has earlier been found to present difficulties for them. The teachers plan a lesson together, and one of them carries it out - usually in his or her own class - while the others observe. Afterwards, the group analyzes and discusses what happened in the classroom. The Learning Study is a hybrid form of Lesson Study and Design Experiment. It is a theory-based research undertaking whose important components include exploration of students' ways of making sense of the object of learning before and after the lesson(s). A Learning Study usually comprises three cycles, each building on the conclusions of the previous. Finally, a Learning Study is documented, frequently in publishable form. While Lesson study is primarily an arrangement for in-service training of the participating teachers, Learning study is primarily teachers' research, the results of which are supposed to be widely shared with other teachers. The Variation Theory of Learning has so far been the theoretical point of departure for the studies carried out. The model was originally developed right after the turn of the millennium in Hong Kong, and subsequently spread to other countries, notably to Sweden. Our estimate is that nearly 1000 such studies have been carried out by now (cf. Lo, 2009).

The main (quantitative) results of the studies published to date can be summarized as follows.

- In nearly all of the studies, students' results were better after the lesson(s) than before (Lo, Pong, & Chik, 2005). (Although this may appear self-evident, it is not. Unfortunately, there are many school lessons in which students learn nothing, or at least not what the teacher had hoped they would.)
- Students with weaker learning prerequisites usually learn the most. Hence, not only does the average rise, but the spread diminishes (Lo et al., 2005).
- In cases in which what the students had learned was observed not only immediately after the lesson but also on a later occasion, the results were often found to be better at the later time (thus indicating a content-specific "learning to learn" effect) (Holmqvist, Gustavsson & Wernberg, 2008).
- Results on national achievement tests increased for classes that had participated in several Learning Studies, an effect that in all likelihood was mediated by changes in teachers' regular ways of teaching (Maanula, 2011).
- When the same object of learning is dealt with in a Learning Study and in a Lesson Study by groups of equally well-qualified teachers, the quality of learning turns out to be strikingly higher in the former (Marton & Pang, 2006, 2008; Pang, 2010; Pang & Marton, 2003, 2005, 2007).
- When the three cycles of a Learning Study are compared, the results from the third are usually better than those from the second, and those from the second are usually better than those from the first (Lo, 2009).

John Elliot, one of the founders of the "action research" movement in education, has evaluated two large-scale Learning Study projects carried out in Hong Kong. He concluded:

"The evaluation gathered convincing evidence of the positive impact of the process on teachers' and students' learning .... Learning Study is focused on realizing new kinds of pedagogical roles. From the evidence gathered in this evaluation it has enormous potential in this respect". (Elliott, 2004)

It seems, in other words, that the Learning Study approach has been something of a success story. What about our conjecture? Has it been supported in Learning Study research?

In our Learning Studies, every lesson was initially planned to be consistent with Variation Theory, and hence consistent with our conjecture. Differences between cycles were related to differences between different interpretations of the same ideas. Although this approach may be a good way to improve lessons, it is not really suitable for testing a theoretical conjecture. Accordingly, we carried out a few studies using comparison groups, controlling for the assumed generally positive effects of the co-operative Lesson Study model. Two groups of teachers, randomly selected for the two conditions (i.e., a Learning Study and Lesson Study condition), agreed on a particular object of learning. Together, they explored their students' understanding of that object, and planned a lesson on the basis of what they found and on their previous experience of teaching the same object of learning. One of the teachers then carried out the lesson, while the others observed. After the lesson, the group again explored students' understanding of the object of learning, and the lesson was analyzed in light of the results.

A researcher was present as a resource person during both the discussions and lessons. The only difference between the two conditions was that in the Learning Study group, the researcher introduced Variation Theory, which he did not do in the Lesson Study group. Although he participated in the discussions in both groups, he tried to act in a reactive rather than active (initiating) manner. The focus of the studies was a comparison of students' results under the two conditions in relation to a comparison of the patterns of variation and invariance brought about in those conditions (Marton & Pang, 2006, 2008; Pang & Marton, 2003, 2005, 2007). Although the results showed dramatic differences to the advantage of the Learning Study (and hence the theory on which it is based, as these patterns were controlled by the teachers - and by the students, of course - these comparisons had to be *post hoc*. To sharpen the comparison of patterns of variation and invariance, the researcher must be able to ascertain exactly what patterns are being compared. In quasi-experimental comparisons, such as that described here, there are usually no consecutive cycles. Even if a researcher tries to be as blind to the two conditions as possible, we can hardly claim that he or she has succeeded completely. In our case, the "theory group" may have had an advantage beyond that originating in the theory itself. Furthermore, the comparisons were made between the conditions in terms of the patterns of variation and invariance observed by the researcher, which means that they were *post hoc*, as noted, and hence the matter of empirical support for Variation Theory is not entirely straightforward.

1.3 There are no teaching experiments

A fair number of studies have been published in recent years in which the outcomes of learning have been found to be systematically related to the patterns of variation and invariance inherent in the conditions of learning. The lived object of learning (learning outcome) in these studies has generally been found to be related to the enacted object of learning (teaching and classroom interaction) in ways entirely consistent with our conjecture. The outcomes of learning, and differences therein, can be made sense of in terms of the patterns of variation and invariance or the differences in these patterns that are inherent in the conditions of learning (see, for instance, Fraser, Allison, Coombes, Case, & Linder, 2006; Fraser & Linder, 2009; Linder, Fraser & Pang, 2006; Marton & Pang, 2006, 2008; Pang, Linder & Fraser, 2006; Pang & Lo, 2012; Pang & Marton, 2003, 2005, 2013).

If lessons are to provide stronger evidence, then they must be defined in advance, and their effects on learning must also be predicted in advance. Kullberg (2010) carried out an interesting study in which she instructed teachers to teach particular objects of learning in terms of the critical features identified and patterns of variation and invariance employed in previous successful studies. The teachers were familiar with Variation Theory, according to which critical features and patterns of variation and invariance are powerful tools for communicating ways of handling a certain object of learning. Even when Kullberg's (2010) results supported her expectations, however, there were several cases in which the enacted pattern of variation and invariance differed from that expected. Although in some cases, the teacher had failed to open up dimensions

of variation to make it possible for the students to discern certain critical features, in others, the students opened up dimensions of variation that they were not supposed to under their specific condition, but that were critical for learning. In such cases, the class was meant to serve as a control, and the unpredicted changes may have strengthened or weakened the results.

1.4 A critical experiment

The only possible way to ensure that what is being compared is what we want it to be seems to be to build a pattern of variation and invariance into pedagogical tools: texts, tasks, examples, illustrations, problems, and the like. Variation and invariance - as far as the conditions of learning are concerned - can then be defined in terms of the relationships between the constituent parts of the pedagogical tools that are used. A study of this kind was carried out by Ki and Marton (2003). They investigated how non-native speakers of Cantonese could be helped to learn to attend to both the tonal and segmental (the sound but not the tone) aspects of Cantonese words simultaneously to identify their meanings. Cantonese is a tonal language in which the distinctions between six tones are of vital importance. The difficulty that speakers of non-tonal languages have when they try to learn it is not so much their inability to distinguish between two juxtaposed tones (Stagray & Downs, 1993) as their inability to link variation in pitch at the word level to variation in word meanings. Variation in pitch exists in all languages, but its significance in non-tonal languages is at the sentence- rather than word-level. Learning to pay attention to differences in pitch at the word level as a cue to differences in word meanings requires reorganization of the attentional field. Ki and Marton (2003) employed a set of nine words grouped in two ways. In the first, they were grouped to constitute three triplets, each characterized by one tone (the same within each triplet, but differing from the other two). In the second, three segments were grouped to constitute three triplets, each characterized by one segment and three different tones (see Figure 1).

	segmental$_1$	**segmental$_2$**	**segmental$_3$**
tone$_1$	word$_{11}$	word$_{12}$	word$_{13}$
tone$_2$	word$_{21}$	word$_{22}$	word$_{23}$
tone$_3$	word$_{31}$	word$_{32}$	word$_{33}$

	segmental$_1$	**segmental$_2$**	**segmental$_3$**
tone$_1$	word$_{11}$	word$_{12}$	word$_{13}$
tone$_2$	word$_{21}$	word$_{22}$	word$_{23}$
tone$_3$	word$_{31}$	word$_{32}$	word$_{33}$

Figure 1. Three triplets characterized by one segment and three different tones (if read by column) and by one tone and three different segments (if read by rows).

The participants' task was to learn to identify the meaning of the word they heard by selecting its English equivalent. If we consider each triplet as a sub-task, then to be able to come up with the meaning of

each word, a participant must be able to differentiate between the three words. If the three words in the sub-task have a tone in common, then the participant must learn to distinguish between the three different segments and link them to the three different meanings. If, instead, the three words in the sub-task have a segment in common, then the participant must learn to distinguish between the three different tones and link them to the three different meanings. Hence, when the segments vary, you learn segments, and when the tones vary, you learn tones.

The two ways of grouping the words can be seen as a comparison between two patterns of variation and invariance, that is, as induction and contrast from the point of view of tones. If we believe that language learners learn tones (i.e., differentiate between them) best if we offer them different examples of the same tone, then we group words into triplets, within which each has the same tone but a different segment, and ask learners to compare them. If we believe instead - as our conjecture suggests - that meaning (in this case, "the meaning of tones") derives from variation, then we group the words into triplets, within which the tones differ but the segment is the same, and ask learners to compare them. In Ki and Marton's (2003) study, the participants clearly learned to distinguish words more effectively by means of tones in the condition in which the tones were varied during the lesson and the segment remained the same, than in the condition in which the tone was invariant and the segments varied. The study thus demonstrated that learning is more effective under the contrast condition than under the induction condition, as predicted by our main conjecture (see also Guo & Pang, 2011). This was the first critical experiment in which it was put to the test.

1.5 Another way of putting the conjecture to the test

Above, we have argued - in agreement with Fodor (1978, 1980) - that induction is the most common means of trying to help others to acquire novel meanings, but it is certainly not the only one. In our own studies of the teaching and learning of Economics (Pang & Marton, 2003, 2005; Marton & Pang, 2006, 2008), we found that teachers frequently used neither induction, nor contrast. They differed from the teachers using Variation Theory by not only varying the focused aspect but also varying the unfocused aspect. The teachers *not* using Variation Theory actually used *more* variation than the teachers using Variation Theory.

The comparison between induction and contrast mentioned above and being the first critical test of the conjecture, can be illustrated in the following form:

induction		contrast	
focused aspect	unfocused aspect	focused aspect	unfocused aspect
i	v	v	i

In relation to the tone learning experiment described in the previous section, induction means that the participants learn one tone at a time in three different runs. In each run the tone is the same in every task, while the segments (the unfocused aspect) vary. In the case of contrast, there are three runs too, but in each run the segment is the same in every task, while the tone (the focused aspect) varies.

This is one way of putting the conjecture to the test. Contrast is aligned to the theory, induction is not. But what if the object of learning is Cantonese words (and not only tones)? Then we have two focused aspects (tonal and segmental). According to the theory, they should vary one at a time. But there is a third aspect, not new for the learners, hence unfocused. This is the meaning aspect of the words represented by pictures and English words in the experiment. It is not independent from the other two aspects: When one or both vary, the meaning varies too. The second way of putting the conjecture to the test is to compare the case when the two focused aspects vary one at a time, followed by both varying simultaneously (to bring the different aspects of the words together), with the case of having the focused aspects varying simultaneously

from the beginning. The former pattern of variation and invariance is consistent with the conjecture, the latter is not.

This is exactly the comparison that Ki, Ahlberg and Marton (2006) carried out (see Figure 2), demonstrating that the participants in the condition that was consistent with the main conjecture learned better than those in the condition that was not. Moreover, the conjecture was built into the pedagogical tools they used in the study, a computer-administered program that afforded variation, invariance, and feedback to the participants. In the first experiment, one of the aspects, tone, was considered focused (what is to be learned) and the other, segment, was considered unfocused. In this second critical experiment in which the conjecture was put to the test, both aspects were considered focused (they had to be learned).

tone	segment	meaning
v	i	v
i	v	v
v	v	v

tone	segment	meaning
v	v	v
v	v	v
v	v	v

Figure 2. Comparing patterns of variation and invariance consistent with (left) and not consistent with (right) the conjecture.

Discerning an aspect amounts to separating it from other aspects. Two aspects can be distinguished from each other if one varies and the other is invariant. Furthermore, if there are two focused aspects that learners are expected to learn to discern, then they should be varied one at a time, rather than simultaneously. If we want these learners to relate the two aspects, then we should vary them simultaneously, but only after they have been discerned. In the second experiment carried out by Ki, Ahlberg and Marton (2006), there was a third aspect, meaning, that was assumed to be recognized by the learners (they were expected to make sense of the pictures representing the meanings). This aspect is a function of the other two aspects and cannot be kept invariant when any of the other aspects vary; nor does it interfere with the experience of variation in the other aspects, of which it is a function. The first critical experiment showed that letting the focused aspect (that which is to be learned) vary, while keeping the unfocused aspect (that which has already been learned) invariant yields better learning than keeping the focused aspect invariant and letting the unfocused aspect vary. The conjecture was thus supported. In the second critical experiment it was shown that in the case of two focused aspects varying one at a time, and then varying both simultaneously, yields better learning than letting both focused aspects vary from the beginning, even when an unfocused aspect, which is a function of the two focused aspects varies at the same time. The conjecture was supported again. It was put to the test in a third critical experiment in a study reported in the next section. In this case, in addition to the two focused aspects (demand and supply) and the unfocused aspect being a function of the two (price), there was an additional unfocused aspect involved (good) independent of the two focused aspects which - according to the theory - was supposed to remain invariant. Again, two patterns of variation and invariance - one consistent with, and one not consistent with our main conjecture - were compared in terms of their effect on learning. And the conjecture was supported once again. This is the empirical contribution of the present paper.

2. The study

2.1 Understanding pricing

The point of departure for this study was an earlier study in which 10-year-old children were taught to discern price as a function of demand and supply (Lo, Lo-Fu, Chik & Pang, 2005). That study, in turn, built on an earlier study of qualitatively different ways of understanding price and pricing (Dahlgren, 1978). In both of these studies, with minor differences, it was found that most children - and many adults - see price as a function of the attributes of goods. For instance, if something is expensive, then it is because it is big, beautiful, tastes good, etc. Price is thus seen as an attribute of the good in question, and linked to its other attributes, not as a function of market conditions (notably demand and supply), as economics tells us that it is. Some see price as a function of demand only, and others as a function only of supply. For others still, price is a function of both demand and supply, or rather of the relationship between the two, which is roughly in accordance with the canonical conceptualization of price in classical, liberal economics. We use the expression "learning to see something in a certain way" as synonymous with "making a novel meaning your own" or "appropriating a meaning." All three refer to the capability to discern certain aspects of a phenomenon and focus on them simultaneously.

What then are "ways of seeing something"? They are categories of description used to depict the various appearances of something or the different ways in which it is experienced (or its different meanings). The research specialization of phenomenography (Marton, 1981; Marton & Booth, 1997; Marton & Pang, 2008) is the study of categories of description depicting appearances, experiences, and meanings. It posits that if a learner exhibits a certain way of seeing something, then this does not imply that he or she *has* that way of seeing (as a mental representation, for instance). What then does it imply? It implies that he or she is seeing - or has seen - a particular phenomenon in a particular way under particular circumstances. Further, the fact that he or she is so seeing implies that he or she *is able - or has been able* - to see that particular phenomenon in that particular way under the given particular circumstances. Accordingly, what we might wish to explore is the extent to which the same person can see the same phenomenon in the same way under different circumstances. If he or she can, then this could be interpreted as demonstrating that he or she has separated the particular way of seeing this particular phenomenon from the particular circumstances. Becoming an "expert" frequently amounts to being able to see particular phenomena in particular ways under widely varying circumstances (cf. Chi, Feltovich, & Glaser, 1981; Goodwin, 1994; Marton & Booth, 1997, p. x; Sandberg, 1994).

Hence, phenomenography does not tell you what individuals' ways of seeing something *are*. It tells you *how* their ways of seeing something *vary* (between people under the same circumstances and/or within people under different circumstances). The different categories of description together constitute *the outcome space* (of how the particular phenomenon might be experienced). As previously mentioned, studies have established four categories of description that together constitute *the outcome space* of the experience of price.

2.2 Making it possible to learn to see price in a more powerful way

Are the different ways of seeing price equally powerful? We do not believe that we can always - or even most of the time - find a universal ordering of how valid and powerful different ways of seeing the same thing are. In a planned economy, and according to Marxist economics, for example, price is *not* a function of demand and supply. However, we can delimit a set of contexts and settle for ordering the different options within that set. We could thus argue that it is better to enable learners to see something in an additional way that we believe to be powerful - in certain contexts, that is - than not doing so.

Accordingly, we may try to help learners to see something in a new way, that is, in a way that they have previously been unable to. Although we can certainly try, we can never be certain of success. At best, we can ascertain that this new way of seeing *might* have been instilled, that is, that under the conditions

given, it is possible that the learners learned to discern certain critical features, which is exactly what Lo et al. (2005) did in five primary school classes (Grade 4) in Hong Kong in the context of a Learning Study. The aim of each lesson in this study was the same: to enable the students to see price as a function of demand and supply in novel situations. After the lesson, a novel question was used to probe their way of seeing price.

2.3 The enacted object of learning

A double-lesson was used to help the students to learn to discern demand and supply, and the relationship between the two, as determinants of price. During the lesson, the students formed groups and participated in an auction of four items (a mechanical dinosaur, a doll, a dinosaur card, and a stationery set). The auction was repeated several times, with variations. To encourage the students to focus on and discern the critical aspects of supply and demand separately, changes were made in supply (by varying the number of items available) while demand was kept invariant, and then changes were made in demand (by varying the purchasing power through changes in the auction money afforded the groups) while supply was kept invariant. After each auction, they were asked what would be a reasonable price for a new, limited-edition mechanical dinosaur if people had more money to spend. After the groups had written their answers on a worksheet, the teacher engaged the class in a discussion of the case of supply going down and demand going up.

Did the teachers who took part in this study achieve their goal? If so, to what extent did they do so? As can be seen from Table 1, their attempts were not especially successful, with the possible exception of class 4B (see the frequencies for category D, considered the canonical conception here).

Table 1

Distribution of conceptions in pre- and post-tests in Learning Study carried out by Lo et al. (2005)

	Class 4A		Class 4B		Class 4C		Class 4D		Class 4E	
Conceptions of price	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
A. Attributes of the good	6.1%	15.2%	7.7%	0.0%	0.0%	9.7%	17.9%	0.0%	6.5%	9.7%
B. Demand	39.4%	45.5%	64.0%	28.2%	77.5%	71.0%	50.0%	78.6%	51.5%	48.4%
C. Supply	0.0%	6.0%	2.6%	7.7%	3.2%	3.2%	10.7%	3.6%	19.4%	6.5%
D. Demand and supply	3.0%	9.1%	10.3%	61.5%	3.2%	12.9%	7.1%	14.2%	9.7%	22.6%
E. Other non-economic reasons	3.0%	0.0%	7.7%	2.6%	0.0%	0.0%	3.6%	0.0%	3.2%	0.0%
Unclassified	48.5%	24.2%	7.7%	0.0%	16.1%	3.2%	10.7%	3.6%	9.7%	12.8%

Rather than ask whether (and why or why not) seeing price in terms of demand and supply is too difficult for 10-year-old children, we are more eager to understand the striking difference in results between

class 4B and the other classes. (As is shown in Table 1, while the frequency of the target conception (D) increased after the lesson from 10.3 to 61.5% in class B, it increased from about 6 to about 15% in the other classes). Did something happen in this class that did not in the others? Or did something happen in all of the classes except 4B? Prompted by the same curiosity, Lo et al. (2005) did indeed come up with an interpretation for the discrepancy in their results: the necessary conditions for discerning a simultaneous variation in demand and supply were present only in class 4B, which was the only class in which the unfocused aspect (the good, i.e. the item for auction) was invariant throughout the entire sequence of variation and invariance in the focused aspects (demand and supply). Differences of this kind (the focused aspect varying and the unfocused aspect remaining invariant versus both aspects varying) have also been found in two other studies, and in both cases were linked to rather dramatic differences in what the participants had learned (i.e., according to the outcome measures) (Marton & Pang, 2006; Pang & Marton, 2003). The conjecture that we want to put to the test here has two component parts: what is expected to vary in sequence (the focused aspects) and what is expected to remain invariant (the unfocused aspect) throughout. In the study reported here, we wanted to compare two conditions: one consistent with the second component part (the unfocused aspect remaining invariant throughout) and one not consistent with it. Could we replicate the findings of the aforementioned study (Lo et al., 2005), which served as our point of departure, with the same difference built into pedagogical tools? Figure 3 shows the comparison carried out.

demand	supply	meaning	good
v	i	v	i
i	v	v	i
v	v	v	i

demand	supply	meaning	good
v	i	v	v
i	v	v	v
v	v	v	v

Figure 3. Comparing patterns of variation and invariance, consistent (left) and not consistent (right) with the conjecture.

2.4 Design of the study

To reduce the number of factors that could affect the outcome, we tried to build the pattern of variation and invariance (which we assumed to be necessary) into the task structure of the learning resources in such a way that the entire experiment would be an interaction between students and the auction game tool: the computer. Students were invited to attempt to achieve the object of learning by using two different computerized learning resources during an independent learning session that lasted approximately one and a half hours and was held in the multi-media learning center of the participating school. In line with Lo et al. (2005) study, in both learning resources, the economic principle to be dealt with was the determination of the market price through the interaction of supply and demand. An auction game was used to embody the variation in the dimensions of supply and demand. To test whether it is crucial to keep the auction item in question invariant, so as to enable students to focus on and discern the critical aspect of the interaction between supply and demand more readily and effectively, the two learning resources were identical in all respects but: one resource made use of the same product (i.e., boxes of candy) throughout the auction game, whereas the other featured different products within and across each round.

Seventy-eight Grade 4 students from four classes of one school in Hong Kong participated in the study. Within each class, students were randomly divided into two groups, with each given one of the two learning resources. To minimize the teacher effect, learning took place in an autonomous manner, with the students involved playing the computerized auction game on their own, although the researcher gave a five-minute summary at the end of the session to remind the students of the key learning points. (Note that it was impossible for the researcher to know under which condition each student was working. The only difference between the two conditions was that students in the same multi-media learning center received one or the other of the two versions of the learning resource, with the distribution of the two completely randomized.)

To obtain students' existing understanding of the object of learning before they engaged with the learning resources and to form a baseline for comparison of the learning outcomes of the two groups, a pre-test was administered to all students. Then, immediately after the independent learning session, they were required to complete a post-test to allow evaluation of their mastery of the object of learning.

In both tests, the students were asked to consider a problem relating to a real-life scenario embodying the principle in question, i.e., the interaction of supply and demand in determining the market price of a good. They were also asked to elaborate upon the factors they had considered in setting that price. The questions in the pre- and post-tests were essentially identical, except that the product in question varied. Mirroring Lo et al. (2005) study, a hot dog and a box of biscuits were used. Students who were asked a question about the hot dog in the pre-test were asked about a box of biscuits in the post-test, and vice versa. The pre- and post-test questions were as follows: *Have you ever tried the hot dogs (biscuits) sold in the school shop? Do you know how much they cost? Maybe you know or you don't know. Anyway, just for your information, hot dogs are (a box of biscuits is) now sold at HK$5. Suppose that you are the new owner of the shop. What price would you set for a hot dog (box of biscuits)? Would you set the current price, or a different price? What would you consider when you set the price?*

The students' answers were analyzed and described in terms of the aforementioned set of four categories of understanding.

2.5 The learning resources

To build a relevance structure (Marton & Booth, 1997, p. 143) that would enable students to appropriate the object of learning, they were given the task of bidding on goods for an upcoming New Year's celebration through the computerized auction game. In the first round, students were introduced to the basic rules and operation of the game. Each student was given HK$400 in auction money and asked to bid for and thus try to obtain as many items as possible from the nine being auctioned, which were displayed on screen with their base prices shown. Each round of the auction came to an end after three minutes or once the student had used up all of his or her money, whichever came first. The average prices of the goods auctioned were then calculated and shown to the student so that he or she could associate possible changes in those prices with changes in the conditions of each round of the auction, such as the amount of auction money provided, the number of goods to be auctioned, or both.

As previously noted, the only difference between the two learning resources was that for the "different goods" group the nine items, which included different kinds of snacks such as potato chips, chocolate bars, biscuits, and so on, differed both within each round and between rounds, whereas for the "same goods" group the nine items were all the same, i.e., every item was a box of candy.

In the second round (see Figures 4 and 5), to bring students' focal awareness to bear upon the dimension of demand, demand was deliberately varied (by varying students' purchasing power by changing the amount of auction money they were given) while the supply of goods was kept invariant. Each student's auction money was cut by HK$200, thus diminishing their purchasing power and demand for goods. The supply of goods for auction, however, remained invariant, with the number of items kept at nine. Everything was identical for both learning resources except that the nine items for auction remained invariant in the "same goods" design (the same nine boxes of candy as in the first round, whereas the type of goods varied in the "different goods" design, changing from the nine kinds of snacks in the first round to nine kinds of soft drink in the second.)

Day 2				Group A
Record of the average price of items for auction				
	Day 1	Day 2	Day 3	Day 4
Amount of auction money	HK$400	HK$200		
Number of items auctioned				
Average price of items for auction				
Now the shop has closed. Are you happy with the items that you have obtained? Reflect on the auctions on Days 1 and 2 and complete the following task.				
Task: Compare today's average item price with yesterday's. What have you found?				

Figure 4. Same goods design (round 2).

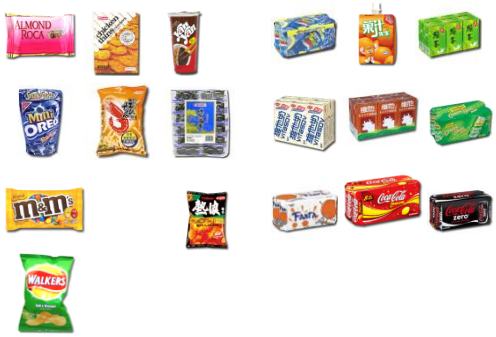
Day 2				Group B
Record of the average price of items for auction				
	Day 1	Day 2	Day 3	Day 4
Amount of auction money	HK$400	HK$200		
Number of items auctioned				
Average price of items for auction				
Now the shop has closed. Are you happy with the items that you have obtained? Reflect on the auctions on Days 1 and 2 and complete the following task.				
Task: Compare today's average item price with yesterday's. What have you found?				

Figure 5. Different goods design (round 2).

In the third round, to help students to shift their focal awareness to the dimension of supply, supply was deliberately varied while demand was kept invariant. The number of items for auction was reduced from nine to seven, whereas the amount of auction money remained the same (HK$200). However, the only - but critical - difference between the two learning resources was that all seven items in the "same goods" design remained boxes of candy, whereas the seven items used in the "different goods" design now differed from those in the two previous rounds, with participants being asked to consider different kinds of balls in this round.

Unlike the classroom study carried out by Lo et al. (2005), we introduced a fourth auction round in which variation was introduced in both the demand and supply of goods in a simultaneous manner. Our purpose was to help students to focus on the dimensions of both in determining the market price of a good. To this end, the auction money given to students was increased from HK$200 to HK$400, and the number of items to be auctioned was reduced from seven to six. This round thus involved a simultaneous variation in the supply of goods and variation in purchasing power (demand for the goods), the aim of which was to enable students to discern the critical aspects of experiencing price and pricing. As before, the only difference between the two learning resources was that the six items in the "same goods" design remained the same, whereas a new set of items (six different kinds of decorations) was introduced in the "different goods" design.

Lastly, similar to the procedure in the earlier study Lo et al. (2005), students were asked a question (for instructional purposes) about what would happen to the price if the supply were increased and purchasing power decreased. In the current study, they were invited to predict the direction of change in the market price, that is, whether the price would go up or down, if the amount of auction money was decreased from HK$400 to HK$100 while the number of items to be auctioned increased from six to 11.

As noted, the learning session concluded with a five-minute summary delivered by the researcher to remind students of the key learning points in the computerized learning resources. He simply read the following PowerPoint slides to the two groups of students at the same time.

1. (Slide 1) "Compare the auction game on Days One and Two. As the auction money given to you on Day Two was less than that on Day One, your income decreased. When your income decreased, your purchasing power also decreased. This made your demand for goods decrease. As the supply of goods on Day Two was the same as that on Day One, the average price of goods was lower."
2. (Slide 2) "Compare the auction game on Days Two and Three. As the auction money given to you on Day Three was the same as that on Day Two, your income and purchasing power remained unchanged. Your demand for goods also remained unchanged. As the cost of production, such as the prices of raw materials, electricity, and labor increased, the supply of goods decreased. As a result, the average price of goods on Day Three was higher than that on Day Two."
3. (Slide 3) "Compare the auction game on Days Three and Four. As the auction money given to you was more than that that on Day Three, your income and purchasing power increased, and your demand for goods also increased. At the same time, the increase in the cost of production made the supply of goods decrease. As demand increased and supply decreased, the average price of goods on Day Four was higher than that on Day Three."
4. (Slide 4) "The price of a good is determined by its supply and demand. The supply of a good is affected by its cost of production, such as the prices of raw materials, electricity, and labor, whereas the demand for a good is affected by people's income and purchasing power. When businesses set the price of a good, they need to consider the factors affecting supply and demand at the same time."

3. Results and findings

The results presented in Table 2 show that students who belonged to the group using the learning resource with the "same goods design" outperformed their counterparts using the learning resource with the "different goods design" in the post-test, in which statistically significant difference was observed between the two groups (χ^2 = 10.36, p = 0.03 (< 0.05); effect size = 0.32). (Note, in particular, the relative frequencies for the target understanding - Category D - in the post-tests for the two conditions.)

Table 2

Distribution of conceptions, pre- and post-test

Conception of price	"Same goods design" Group (40 students)				"Different goods design" Group (38 students)			
	Occurrence		Percentage		Occurrence		Percentage	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
A. Attributes of the good	10	1	25.0%	2.5%	4	2	10.5%	5.3%
B. Demand	14	11	35.0%	27.5%	14	15	36.8%	39.5%
C. Supply	5	9	12.5%	22.5%	2	5	5.3%	13.2%
D. Demand and supply	**5**	**16**	**12.5%**	**40.0%**	**5**	**6**	**13.2%**	**15.8%**
E. Other non-economic reasons	6	3	15.0%	7.5%	13	10	34.2%	26.3%
Total	40	40	100%	100%	38	38	100%	100%

χ^2 = 7.92 (df = 4) (p = 0.09, i.e., p > 0.05) – Pre-test

χ^2 = 10.36 (df = 4) (p = 0.03, i.e., p < 0.05) – Post-test

We can see in Table 2 that while the frequency of the target conception (D) increased after the lesson from 5 to 16 (of 40) under conditions consistent with the conjecture, it increased from 5 to 6 (of 40) under conditions not consistent with the conjecture.

4. Conclusions

Only in a restricted sense was this study a replication of Lo et al. (2005) investigation. We wanted to find out if invariance or variation in an unfocused aspect can really have such a strong impact on the learning of the focused aspects as was interpreted to be the case in the previous study. The question could be answered in the affirmative and the conjecture was thus supported.

It should be noted that in both the original and follow-up studies, variation was restricted. When the supply was invariant, demand went down (instead of going up in one case and down in another), and when demand was invariant, the supply went down (instead of going up in one case and down in another). In the

last round, only one of the four combinations of demand (up/down) and supply (up/down) was realized. We decided not to include all four combinations as it would have made the task too difficult for such young participants. In all of the circumstances considered, it is of course possible that the results would have differed had the students been exposed to more of the possible differences among the patterns of variation and invariance.

Quite a few of the students in the comparison group managed to learn to discern the critical features of pricing, even though the good was not invariant. In general, even if there is variation in several dimensions, learners may be able to block out all dimensions but one, that on which they happen to focus. There is an interesting twist concerning how this question appeared in the experiment, however. As noted, in the target group, a number of items of the same type of good were offered in each round at the same base price. In the comparison group, the same number of items as in the target group were offered at the same base price. Further, whereas the target group considered the same type of good both within and between rounds, the comparison group considered different goods in each round. The difference between the conditions was illusory, however: all the relevant factors (the number of items available, the amount of money participants had, and the base price of goods) were exactly the same. The only element that differed was the irrelevant labels placed on the goods. The only thing those in the comparison group had to do was to separate what was relevant for their decisions from what was not, and bracket the latter. If all of them had done so, then the conditions for the two groups would have been the same. As we can see from the differences in outcome, however, this was not the case. Our finding that the comparison group was affected by the irrelevant differences in the item labels implies that quite a few learners in that group failed to separate relevant information from irrelevant information, and therefore failed to see the former.

The main contribution of this study is the support it provides for our conjecture: if both the focused and unfocused aspects of the object of learning vary, then it is more difficult to discern the focused aspects and relate them to one another than if the unfocused aspect remains invariant while the focused aspects vary. We found this to be true in the current study, even though the unfocused aspect was completely redundant. However, the conjecture also addresses the question of how we can acquire new meanings (or how we can learn to see certain things in certain ways). As mentioned earlier, Fodor (1978, 1980), and others, claim that there is no answer to this question and, in fact, there cannot be any. Meanings are innate.

However, we argue that regardless of whether meanings (concepts) are innate, or of the sense in which they are (or are not) innate, we have to learn to discern them as aspects of the world around us, and for this to happen, there are necessary conditions. These necessary conditions are specific to particular meanings and to learners' particular experiential history. They can be formulated in terms of patterns of variation and invariance among instances that do and do not have that particular meaning. Our conjecture is thus very straightforward, as is the way in which it can be put to the test. We simply have to create the necessary conditions in one case and ensure that they are absent in another, as Pang and Marton (2003) did in their aforementioned study. Then, we can compare the two cases and determine whether, as expected, all participants in the first case learn the target meaning, whereas none of those in the second do. If these are indeed the results, then the conjecture is strengthened.

Obviously, this is not what happened. Even if we can demonstrate that contrast is more powerful than induction as far as the learning of new meanings is concerned, we cannot demonstrate that new meanings cannot be learned through induction. After all, some learners seem to learn in that condition too, and certainly not all learners will learn even if all possible steps are taken to make it possible for them to do so. The relationship between what is learned, on the one hand, and the conditions of learning, on the other, is stochastic rather than deterministic. But why is this so?

Returning to the experiment reported in this paper, beyond the fact that the target principle (price as a function of the relationship between demand and supply) was made explicit to both groups, there is a more general answer to the foregoing question. Our conjecture concerns the pattern of variation and invariance *as experienced by the learner*, whereas a pattern of variation and invariance that can be controlled by the researcher refers to the patterns *as seen by the researcher*. What might the relationship between the two look like? One condition of experiencing variation is that there is variation to be experienced. Making sure that

this condition is met is the first step toward making learning possible (which in our view is what teaching is all about). However, variation can also be experienced because of previous experiences. Experienced variation is thus not necessarily the experience of what is present in the learning situation as seen by the observer. On the other hand, even if there is variation, it is not necessarily experienced by all learners. In conclusion, when comparing two randomly selected groups, we would expect more learners to experience variation if it is present than if it is not.

However, experiencing variation not only concerns the variation to be experienced in a relevant dimension; it also presupposes invariance in other dimensions. In other words, variation can be experienced only against a background of invariance. In this sense, experienced variation is a function of invariance, and, as previously stated, experienced variation is also a function of variation. Our intention with the present study was to illustrate that learning (in the sense of the discernment of the necessary features of a phenomenon) is a function of experienced variation (by the learner), which is a function of both variation and invariance (as seen by the observer). This we did, with a focus on the latter (invariance).

We have thus shown that introducing redundant information (different goods) that is correlated with a variation in critical aspects (a change in demand and supply) significantly reduces the likelihood of learners being able to discern the critical features of the object of learning. A seemingly subtle difference between two conditions, both representing 90 minutes of pedagogical effort, is proved to play a key role in what the students managed to learn.

Keypoints

- This paper addresses one of the oldest unsolved mysteries of learning: How do we make novel meanings our own?
- The answer suggested is: By discerning, separating and bringing together the critical aspects of what we learn about.
- A critical aspect can be discerned and separated through the experience of variation in that aspect against the background of invariance in other respects.
- We have put our conjecture to the test by embedding it in a pedagogical tool.
- The conjecture was supported.

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