Visual Abstract

Numerical nonlinear analysis of RC beams with un-strengthened and CFRP-strengthened opening drilled under service loads within shear zones

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ABSTRACT. The current research deals with the Reinforced Concrete (RC) beams numerical modeling using the finite element analysis software ANSYS-standard and the proposed strengthening procedure if it is urgently required to drill rectangular or circular in shape openings within their shear zones under different applied service loads levels. The shear zone is selected to investigate drilling the opening within since it is a critical zone to reduce the structural section effective area against shear. The RC beams are analyzed under two concentrated loads till failure before and after applying the proposed opening strengthening technique by means of the Carbon Fibers Reinforced Polymer sheets (CFRP). The main aim of this research is simulating the real practice situation conditions where the RC beam is subjected to the service loads, supported temporary by means of hydraulic jacks, the opening is drilled and then the strengthening is performed after which the jacking supports are released. The used finite element modeling (FEM) is verified using one of the available experimental studies of FRP-strengthened simply supported beams with and without openings which found in the literature before achieving the investigation. The results of proposed numerical nonlinear modeling are introduced. Many aspects of structural analysis such as the initial cracking loads, load deflection curves, cracking patterns and failure loads and modes for the reference (solid without opening) and un-strengthened & strengthened opening main control & services loaded RC beams are introduced and analyzed in details. CFRP opening strengthening improved the beams structural behavior as a whole.

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The service loads up to about 40% of the ultimate design strength relatively have an unnoticeable influence on the strengthened opening RC beams bearing capacities regardless the opening shape. Some important conclusions and recommendations for designer and executive engineers are stated.

KEYWORDS. Service loads, Reinforced, Concrete, Beams, Openings, Strengthening, Shear, CFRP, Finite element, Nonlinear analysis, ANSYS.

INTRODUCTION

Generally, for many decades ago, presence of the openings and holes in modern buildings is necessary for essential services utility pipes and ducts such as electricity, air conditioning and telecommunications instrumentations, water supply, sewage, plumbing, ventilations, lighting, and many other service networks. In the flat slabs constructions, these pipes and ducts are hanged between the reinforced concrete slab and the false ceiling which is fabricated usually to improve the aesthetic side. Consequently, the reduction in the floor height is considered as the main benefit. In traditional solid slabs constructions, the utility pipes and ducts are sometimes needed to be passed through openings in the web of certain beams which also reduces the floor height but it is aesthetically unaccepted. Existence of opening in reinforced concrete beams leads to premature initial cracking, after which excessive cracks widening and propagation occurred accompanied with stress concentrations around the opening corners and in the vicinity regions of its edges. Consequently, ultimate strength, stiffness, shear strength, initial cracking capacity and local recorded deflection and generated stress are strictly expected to be negatively influenced. This almost initiated failure semblances leading to the total RC beams collapsing.

Strengthening of the beams with openings is primarily depending on whether the building services are pre-planned or post-planned. In the case of the pre-planned openings, the sizes and locations of the openings are known in advance during the design stage. So, the sufficient strength and serviceability of beams with opening can saved before construction. On the other hand, the case of the post-planning means drilling the openings in the newly constructed building structures elements where the problem may arise to pass the utility pipes and ducts. So, the executive engineers should drill the openings without ignoring the structural safety and serviceability of the appropriate structural element. At that time, one of the probable diagnosis actions is the strengthening externally around the opening by means of an external reinforcing material such as Fiber Reinforced Polymer (FRP).

The structural response of RC beam due to an opening existence is affected by the openings structural characteristics depending on its geometrical shape, area aspect ratio and position. Upon service requirements, opening may be rectangular, circular, square, triangular, trapezoidal, diamond and even irregular shapes. Rectangular and circular openings are common used. The circular opening is structurally preferred more than the rectangular one where relatively high stresses concentration generated around the rectangular opening of unlike sharp corners. The different shapes of openings were experimentally studied first by Prentzas [1] as shown in Fig. 1 after which the structural classification depending on un-strengthened openings position under different loading types and the proposed strengthening methods were studied by several researchers [2-16].

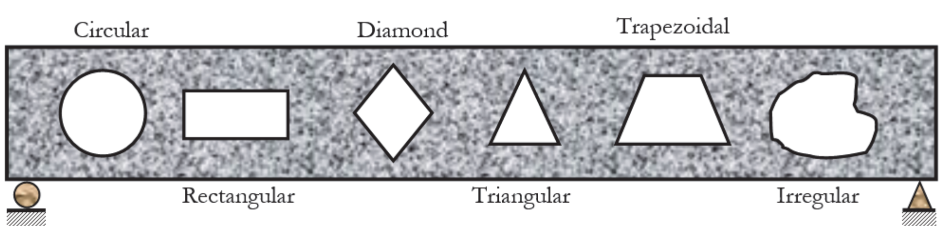


Figure 1: The probable shapes of the opening [1].

The openings are classified according to their geometric shape, size and location. Concerning the shape, many different shapes of openings such as rectangular, circular, square, triangular, trapezoidal, rhombus, and other irregular shapes as mentioned before. However, the most common shapes adopted are rectangular, circular and square due to their ease of application. Utilization of rectangular and square openings leads to the great stress concentrations at their sharp corners which causes some cracks unlike circular opening. These cracks are aesthetically unacceptable and lead to early failure since they are quicker than the cases with circular openings. Hence, it is proposed that the best shape of the opening is the circular since it allows for better load carrying capacity of the beam and the cracks are well distributed around the opening with minimized stress concentrations due to the absence of sharp edges. Regarding the size, openings are categorized into either small or large size openings. The opening size in many cases is defined as small or large based on its depth or diameter as a ratio of the total depth of the beam. Others define the opening according to its effect on the structural behavior of the beam [17]. The position of the opening along the effective span of the beam is classified into, openings in the shear zone or in the flexure zone. The modes of failure differ where the beams with openings in the shear zone fail by diagonal tension cracks across the opening while the beams with openings in the flexure zone fail in flexure beneath the opening. Hassan et al. [18] achieved one of the researches that concerning with the strengthened rectangular opening in RC beams shear zone where they analyzed numerically a group of strengthened reinforced concrete beams with openings strengthened by FRP sheets. The opening centerline was at different distances away from the support. The openings are strengthened by CFRP (carbon fibers) and GFRP (glass fibers) sheets using four strengthening schemes around the opening. The FRP strengthening schemes differ from each other in the number of layers, and orientation. The numerical results indicated that the ultimate loading capacity is slightly influenced by the distance measured from the opening centerline to the nearest support and the number of the layers of FRP. The results exhibited a significant influence due to the orientation and the direction of the FRP strengthening. Accordingly, the results of the GFRP and the CFRP were close and similar according to the authors who recommended that the use of GFRP is much preferable due to the reduced cost relative to the CFRP. Another example is the work of Shehab el-din et al. [19] where they investigated experimentally the effect of strengthening of reinforced concrete shallow T-beams with large openings at the critical shear region using CFRP (carbon fibers) and BFRP (basalt fibers) sheets. They achieved an experimental program contains nine shallow T-beams having large web openings and all tested beams are identical in reinforcements and dimensions. The nine beams included two control beams one was a solid beam without any opening and the other one with un-strengthened large opening at the critical shear zone. The other seven beams possessed a large opening in the beams web each strengthened differently. Six beams were with large web openings strengthened with six different schemes using CFRP sheets to determine the best strengthening scheme and the last beam possessed the large opening strengthened using BFRP sheets. The conclusions included that the strengthening by CFRP and BFRP sheets gained a great increase in the strength and the ductility behavior of beam. Also, the strengthening method had an effective role in attaining a good result. The provision of shallow RC beams with large openings in shear region caused a great reduction in the ultimate load capacity, stiffness, initial cracking and ductility. The best strengthening scheme was the complete wrapping around the opening and additional horizontal strips above, below and on the sides of the opening. Using of U-CFRP configuration without entire wrapping caused de-bonding of FRP sheets. The strengthening by CFRP was more effective in increasing the ultimate load capacity than BFRP because of the high elastic modulus and tensile strength. Numerous layers or extensive strips of BFRP may reinstate the full beam capacity. Also, Mansour [20] investigated numerically the shear strengthening of continuous RC beams having web openings with FRP layers. His numerical investigation was carried out using the validated numerical FEM to investigate the impacts of crucial parameters such as opening location, opening area and strengthening configuration on the shear behavior of continuous RC beams strengthened with FRP layers. Three different locations were suggested along the length of the RC beam to construct the web openings. On the other hand, the effects of five different opening areas on the load capacity and failure patterns were investigated. Moreover, two different FRP strengthening configurations were considered either strengthening above and below the opening or strengthening the whole RC beam height. He concluded that the reduction in the load capacities ranged from 7.3 to 66.1% compared to the solid beam. The ultimate loads of specimens strengthened over the whole RC beam height exhibited the highest values among analyzed specimens.

On the other hand, some others investigated the matter of opening existence and strengthening in both shear and flexure zones [21, 22]. Nair et al. [21] studied the opening strengthening by GFRP and CFRP sheets using various schemes. The beams models were fabricated so that one had a rectangular opening in the shear zone while the other one had the opening in the flexure zone. More six models were externally strengthened using CFRP and GFRP sheets. Four different schemes of the strengthening were applied. The first scheme considered the full external FRP wrapping around the boundary of the opening. The second scheme was performed by applying an internal wrapping of the FRP sheets inside the opening while the applying of both internal and external wrapping around and inside the opening was considered as the third scheme. The applying of the external wrapping around the opening using a double layer of FRP was the fourth one. The numerical analysis of the previous beams shows that the strengthening of the opening with CFRP is more efficient than GFRP sheets. Both external and internal CFRP strengthening around and inside the opening was the best scheme which increased and improved the ultimate load bearing capacity of the RC beam. El-Sisi et al. [22] studied RC beams with un-strengthened and CFRP strengthened opening under static and dynamic loads. They concluded that it is not recommended to drill any opening at the shear zone because the strength loss reached 57% even with the strengthening, especially for blast resisting structure. In addition, the strength was only recovered from approximately 11.95 to 32.46%. Other researchers studied the matter of opening existence and the strengthening with the fiber reinforced polymer (FRP) in flexure only such as Almusallam et al. [23].

Recently, since the majority of the available well-known and normally used analysis software programs by numerous researchers don't give the detailed fracture results due to the continuum simulation of materials such as concrete, some researchers such as De Maioet. al. [24] mainly spotlighted on the aspect of the cracking behavior of the RC structures by adopting an effectiveness numerical fracture and an embedded truss models to simulate the crack propagation, crack width, crack spacing and concrete-rebar interaction respectively. Also, Ferrian et al. [25] studied the cohesive approach by investigating the details of cracking and de-bonding phenomena arising in the concrete phase using two equations describing the stress requirement and the energy balance respectively. This approach can be used to simulate the FRP sheets gluing agent to the appropriate concrete surface.

Finally, studying the urgently post-planning opening existence in the RC beams shear zone and CFRP strengthening under various service loads levels on the structural behavior as a whole using the well known finite element analysis software ANSYS [26] assuming full interacted CFRP sheets to the appropriate concrete face is the main key in the current research. Hereinafter, the investigation program is presented, numerical modeling validation and evaluation are investigated and consequently the gained results are stated and analyzed to deduce the more important conclusions and recommendations.

INVESTIGATION PROGRAM

First of all, the current research investigated the openings drilled within the RC beams shear zone and the CFRP strengthening technique influence on the whole structural behavior enhancement of certain models cases. The shear zone wide is the distance between the applied concentrated load and the nearest support.

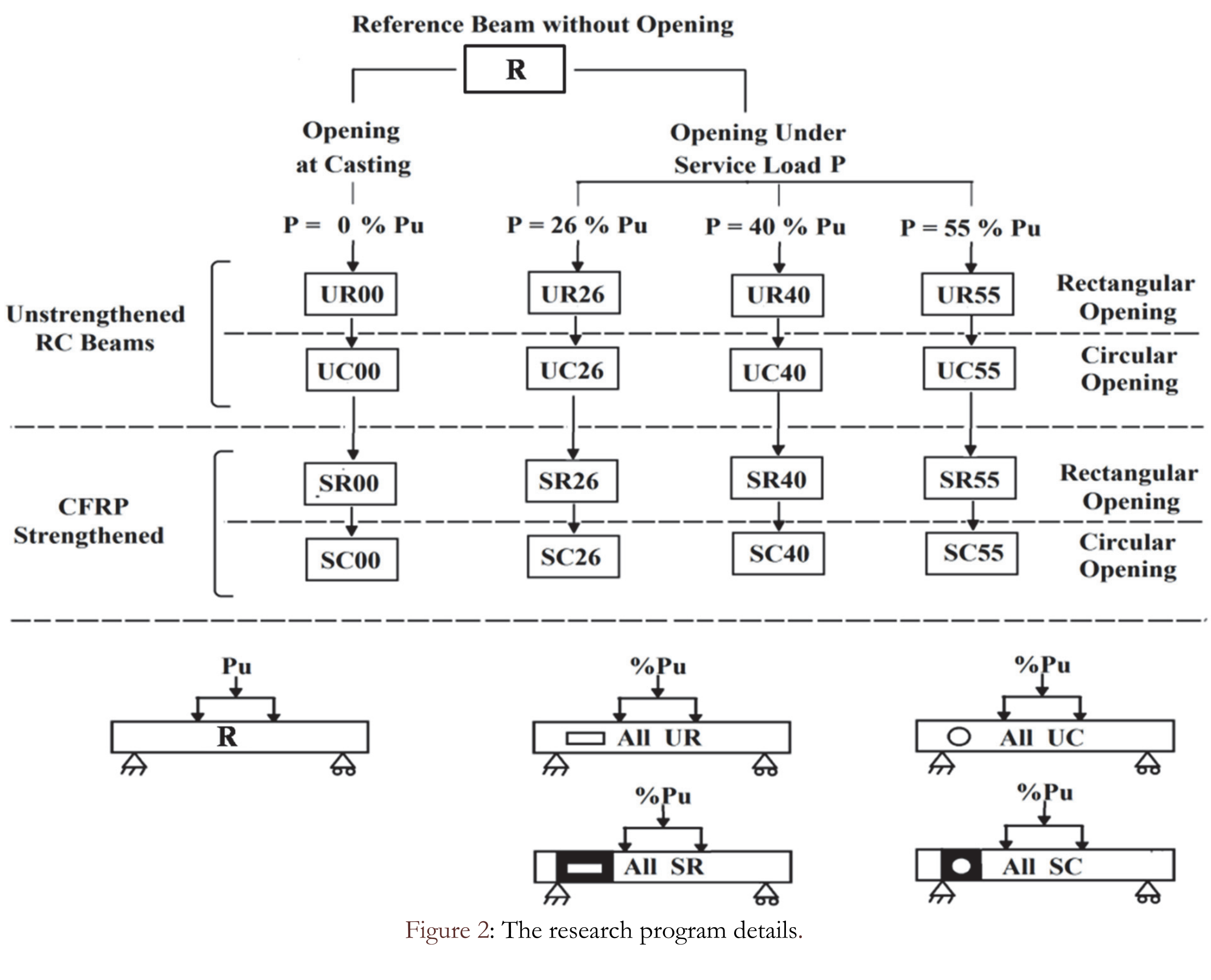


Figure 2: The research program details.

This research spotlights on the rectangular in shape opening drilled under various service loads levels to investigate its undesired effects on the weakening of the corresponding beam whole structural behavior for the sake of recommending best precautions. The circular in shape openings are only considered for the sake of comparison. To achieve this goal, a numerical finite element non-linear analysis is carried out on seventeen simply supported RC beams loaded under two point static loads. All beams models are identical in dimensions, reinforcement details, material properties and loading setup. All RC beams are modeled as structurally simply supported with hinged and roller supports. The roller supports are modeled by constraining nodes vertical translation only in Y-direction while the hinged supports are modeled by constraining the nodes translations in all directions (X, Y and Z). The first beam model is solid with no opening and is considered as the reference one denoted by (R). Two other control models each with un-strengthened opening assumed to be drilled during the construction as pre-planned cases; i.e. drilled under no service loads. One of them is assumed to be with a rectangular opening and the other with circular one. The control models with rectangular and circular openings are denoted by UR00 and UC00 respectively. The openings are maintained to be of the same area regardless their shapes. Other three un-strengthened models each with rectangular opening denoted by UR26, UR40 and UR55 are modeled where the openings are assumed to be drilled during the application of the service loads which denoted by P as a percentage of the ultimate or theoretical failure criteria load which denoted by Pu of the reference solid beam without opening. These percentages of the service loads P are varying and assumed to be equal to about 26%, 40% and 55%Pu which are the interpretations of the models denotation numbers 26, 40 and 55 respectively. All models have ended denoted numbers as these percentage values. Another three un-strengthened models denoted by UC26, UC40, and UC55 are considered each with a circular opening. Also, the control strengthened opening beams models are two. The first one with a rectangular opening and is denoted by SR00 and the other with a circular one and is denoted by SC00. Additionally, other six CFRP strengthened models are considered. Three with rectangular openings are denoted by SR26, SR40 and SR55 and the others with circular openings are denoted by SC26, SC40 and SC55. The strengthening of beams models with opening is achieved by sheets around the periphery of the opening. It is numerically modeled according to the conditions of the well known software ANSYS standards [26]. The models features details are shown before in Fig. 2. Mainly, the research is investigating the effect of drilling a rectangular or a circular opening within the services loaded RC beams shear zones and achieving CFRP strengthening using nonlinear finite element analysis. Unidirectional CFRP sheets with 75mm width and 0.13mm thickness are used as an external strengthening technique. The used gluing agent to mount the CFRP sheets on position is an adhesive epoxy layer of 1mm maximum thickness.

INVESTIGATED MODELS DIMENSIONS AND REINFORCEMENT DETAILS

The dimensions of all seventeen RC beams in current research are identical. The numerically modeled RC beams are of 2300mm total length, with an effective span of 2000mm, depth of 250mm and width of 100mm. The main steel reinforcement consists of two top reinforcement longitudinal bars of ϕ10mm and four bottom longitudinal bars of ϕ10mm in diameter. The shear reinforcement is ϕ8mm diameter stirrups at 150mm apart as shown before in Figs. 3-a and 3-b. Rectangular openings are of 200mm width and 100mm height while the equivalent area circular openings are of 159.8mm diameter as shown in Figs. 3-c and 3-e. Both of them possess an area of about 20000mm^2. Both the circular and rectangular openings are of vertical central axes coincide to the shear zone central vertical axis since it is expected to be a practical case (as previously stated by the authors [15] and many others) at the beam section mid-depth. A concrete cover of 25mm for all beams is maintained. CFRP sheets extend 150mm beyond the right and left of the opening vertical boundaries whatever rectangular or circular and extend to the model extreme top and bottom face edges as shown in Figs. 3-d and 3-f.

MODELS MATERIAL PROPERTIES

Hereafter, the properties of the reinforced concrete beams models materials are detailed and the used meshing details of the nonlinear finite element are presented. The materials are concrete, steel reinforcements, carbon fiber sheets CFRP and epoxy resin as a gluing agent which is used to mount CFRP sheets in their appropriate positions.

The details of the homogeneous structural SOLID185 element [26] which is used to model the loading and supporting steel plates are represented in Fig. 4-a.

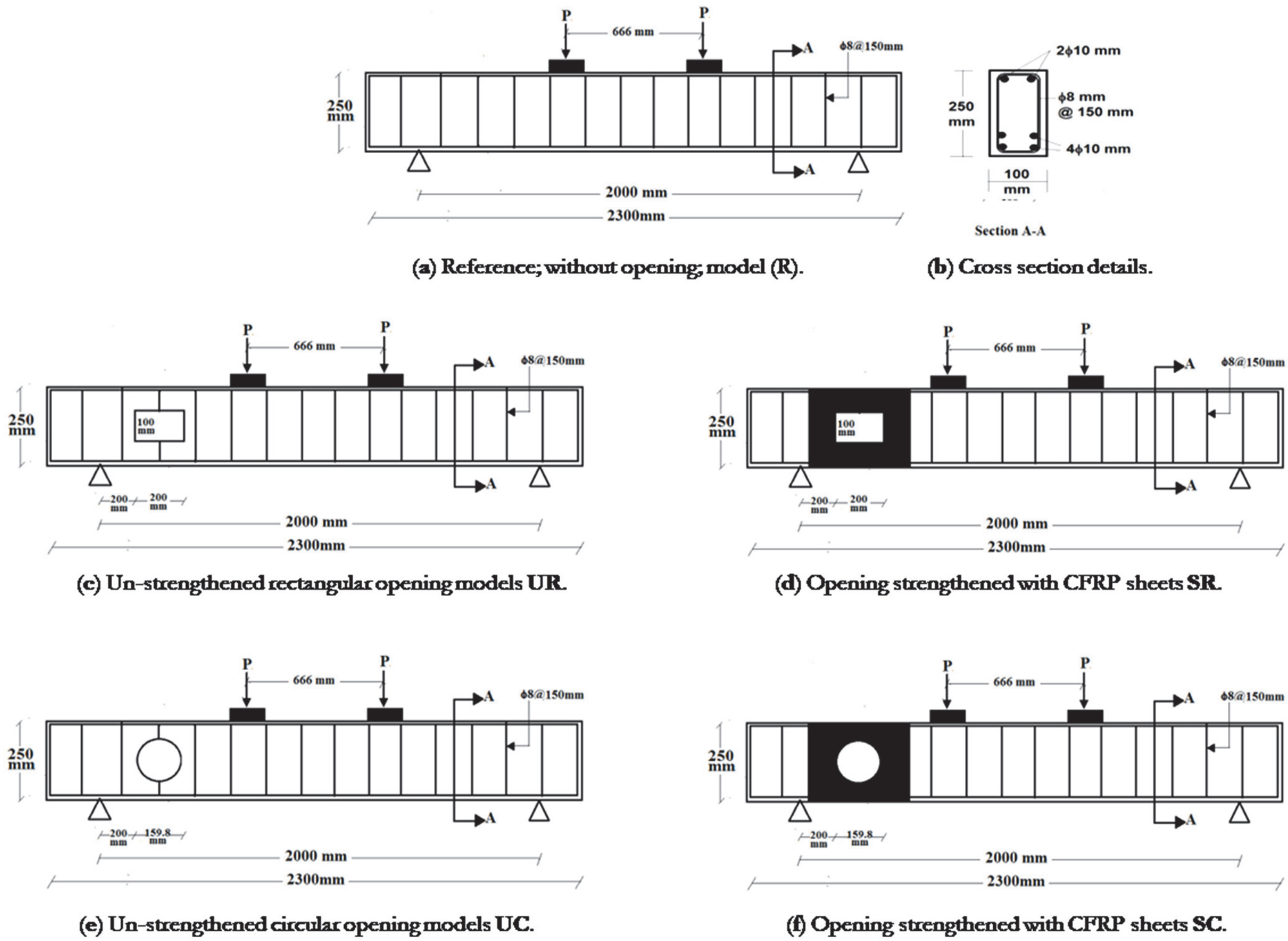


Figure 3: RC beam models dimensions, reinforcements, loading arrangement and the strengthening technique details

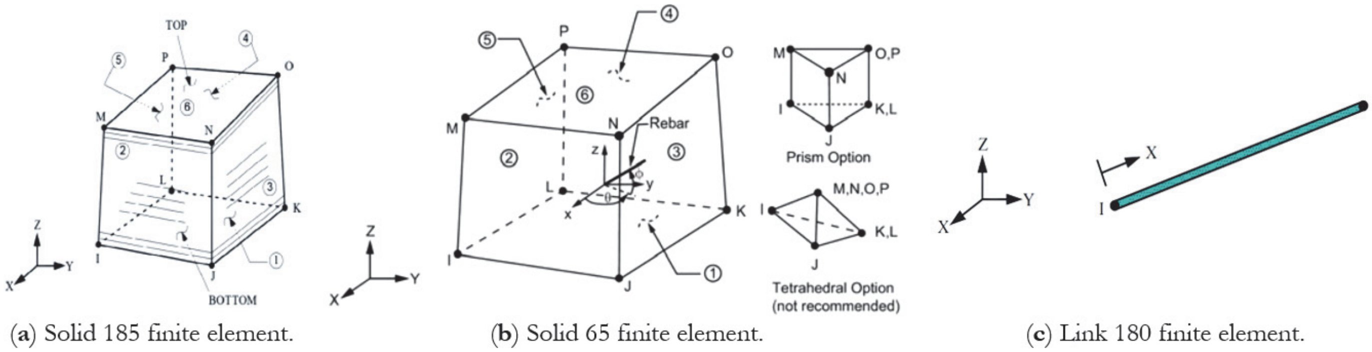


Figure 4: Types of the used finite elements [26].

Concrete

The considered concrete properties and failure criteria for all beams models in the current research such as the elastic modulus of concrete E_c, Poisson's ratio μ_c, ultimate compressive strength f_c, ultimate tensile strength which is typically ranges from 8% to 15% of the compressive strength [27] and is considered as $f_t \approx 0.1f_c$. The multi-linear isotropic simplified compressive uniaxial stress-strain curve characteristics are tabulated in Tab. 1.

Concrete is considered as a quasi-brittle material where the stress decreases gradually after the peak stress and the properties of concrete in compression and tension are different from each other.

The shear transfer coefficients βt and βc are for open and closed cracks respectively. They denote the reduction factors of the concrete shear strength for the subsequent loads inducing the shear sliding across the crack face [26]. These

coefficients values are vary from (0.0 to 1.0). The extreme low value of 0.0 represents a smooth crack (complete loss of shear transfer) while the extreme unity value represents a rough crack (no loss of shear transfer). In current research, the shear transfer coefficient values of βt and βc are considered to be 0.3 and 0.8 respectively since these values usually achieve a good convergence of the solution [15].
The ANSYS standard SOLID65 finite element is used for concrete modeling as shown in Fig. 4-b. The SOLID65 element contains eight nodal points located at a distance slightly less than the element length where the displaying cracking and crushing is at the eight nodal points or displaying the average value between them at the element center.

Concrete linear properties	Linear isotropic		
	Description		Property
	Modulus of elasticity (E_c)*		27828.04MPa
	Poisson's ratio (μ_{xy}) [27]		0.2
	Multi-linear isotropic compressive uniaxial stress-strain curve		
	Point No.	Strain (ε)	Stress (f); MPa**
	0	0.00000	0.00
	1	0.00025	6.90
	2	0.00038	10.38
	3	0.00040	10.90
	4	0.00060	15.96
	5	0.00080	20.58
	6	0.00100	24.68
	7	0.00200	36.85
	8	0.00300	40.00
	9	0.00350	39.50
Concrete non-linear properties	Description		Property
	Opened crack shear transfer coefficient (βt)		0.3
	Closed crack shear transfer coefficient (βc)		0.8
	Uniaxial crushing strength (f_c)*** = f_{cu}		40MPa
	Uniaxial cracking strength (f_t)**** ≈ 0.1 f_c		4MPa

Table 1: Linear and non linear concrete properties [26].

* E_c=4400 $(f_c)^{½}$ N/mm^2[28]. ** f= $E_c\varepsilon/[1+(\varepsilon/\varepsilon_o)^2]and\varepsilon_o$=2 f_c/E_c[29]. *** f_c= $E_c\varepsilon_o$. **** f_t= $E_c\varepsilon_{cr}$[30].

Steel reinforcement

Steel reinforcements are assumed to be of bilinear isotropic elastic-perfectly plastic material in tension and compression. The bilinear plasticity model can implement Von Mises yield surface related to the kinematic hardening and the plastic flow that are available in ANSYS [26]. The linear characteristics which used to identify the structural properties of the reinforced concrete beams steel reinforcing bars are modulus of elasticity E_s= 210000MPa and Poisson's ratio μ_s= 0.3 while the nonlinear yield and ultimate stress (f_y & f_u); of the main reinforcements and stirrups reinforcing bars are of 400 and 600MPa and 240 and 350MPa respectively. Longitudinal and transverse steel reinforcements are modeled by LINK180 finite element as shown in Fig. 4-c. The element is a uniaxial tension-compression element with three degrees of freedom at each node (i.e. translations in the nodal x, y, and z directions) and has the capability of the plastic deformation [26].

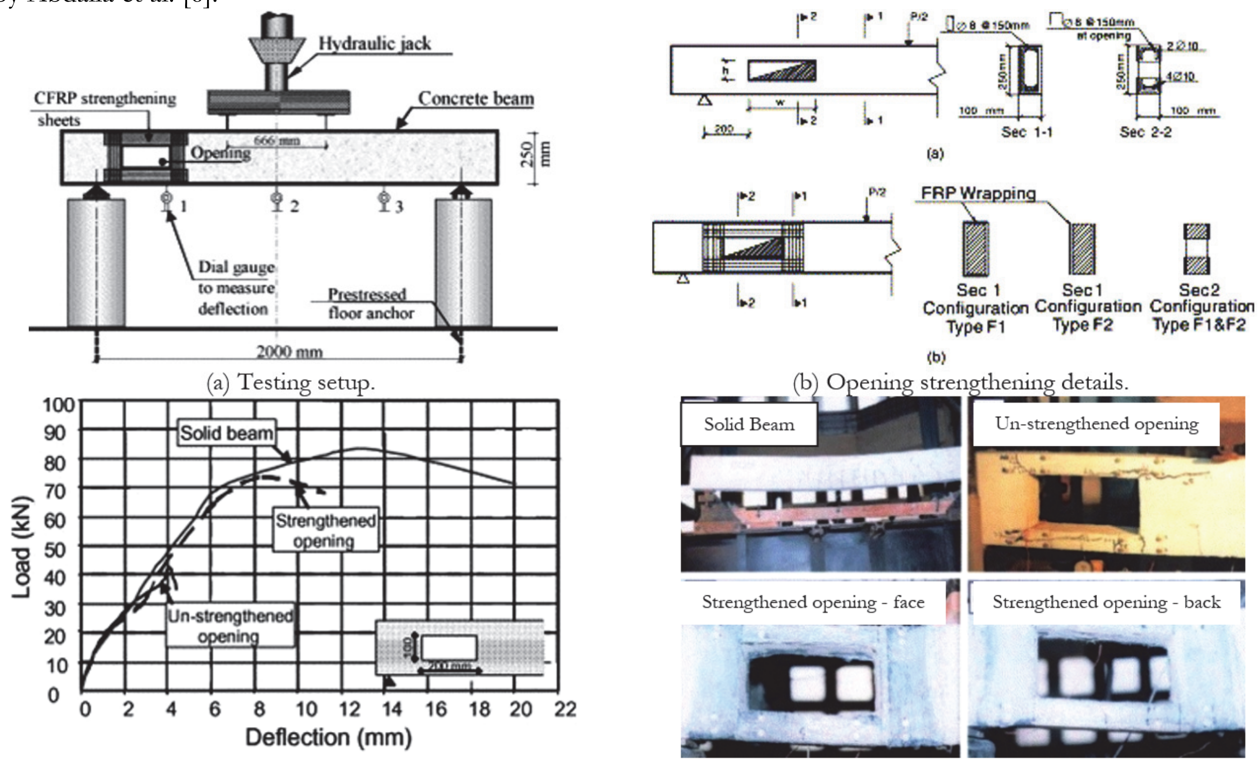
Carbon fiber sheets CFRP and epoxy as an adhesion agent

The epoxy resin available in the Egyptian market which is used to glue the CFRP sheets to the external concrete surface has an ultimate tensile strength of 30MPa, a modulus of elasticity of 4500MPa, a Poisson's ratio of 0.2 and a maximum elongation at break of 0.9% [31]. An assumed layer thickness of only one millimeter is used to glue CFRP sheets to the concrete surface in the appropriate positions around the opening. The epoxy is assumed to be a nonlinear material. The

CFRP sheets are classified into two types as unidirectional and bidirectional according to their orientation. The unidirectional CFRP sheets are used in current study since they behaves as a typically linear elastic material up to failure stage and does not display any yielding behavior as does the conventional reinforcing steel. The used CFRP sheets have a thickness of 0.13mm, a density of 1.82g/cm^3, an elongation at break of 1.5%, and an ultimate tensile strength of 3500MPa. The parameters used to identify the properties of CFRP sheets in ANSYS standard are the modulus of elasticity in the x direction (E_x = 230000MPa), the Poisson's ratio in the y-z plane (μ_{yz} = 0.3), the Poisson's ratios in xy and xz-planes (μ_{xy} = μ_{xz} = 0.22), the shear modulus in the xy and xz planes (G_{xy}=G_{xz}=11790MPa) and the shear modulus in the yz plane (G_{yz} = 6880MPa) [10]. Referring to Figs. 4-a and 4- b, the current research considers the layered structural solid SOLID185 and theSOLID65 elements as the finite elements types used to model the CFRP sheets and the epoxy resin layer respectively.

Numerical non-linear modeling technique verification

The validation of the currently adopted RC beams and their proposed strengthening technique finite element modeling details according to the ANSYS software standard [26] is now detected to decide its reliability to perform the current planned research program guided by the available previous achieved experimental work results by Abdalla et al. [6].



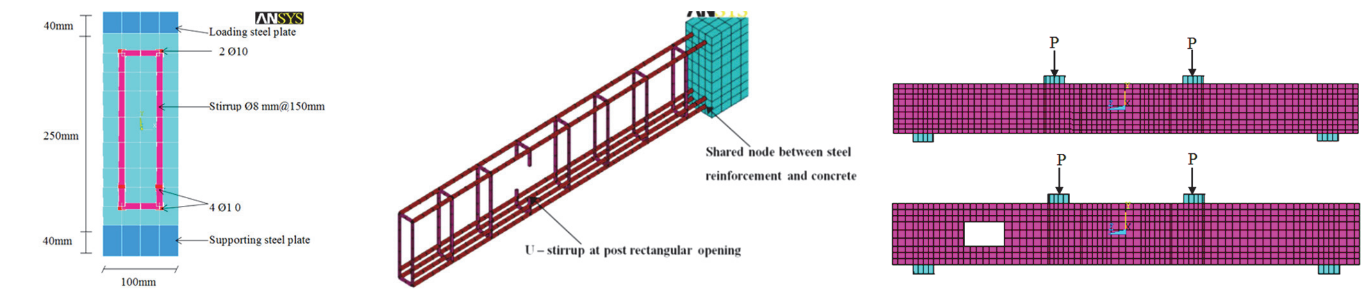
(a) Testing setup. (b) Opening strengthening details.

(c) Load – deflection curves. (d) Failed experimental models.

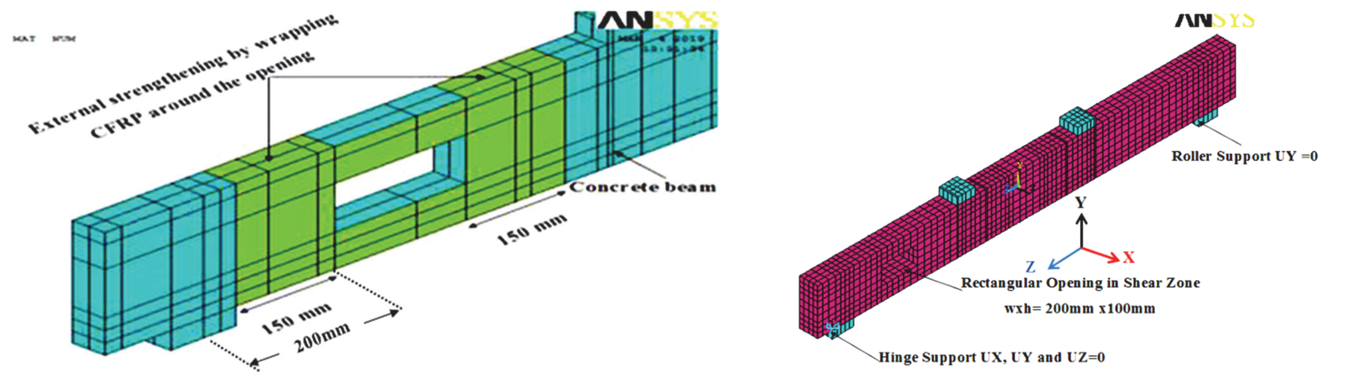
Figure 5: The previous achieved experimental work details which considered as the numerical verification reference [6].

First of all, the nonlinearity in ANSYS software is according to the nonlinear structural behavior which is simulated by a number of factors and it can be categorized into "material nonlinearity" and "geometric nonlinearity". The nonlinear material behavior is caused by the change in the stiffness during the different loading stages while the large deformations caused the change of the geometric configurations of the structure which leads to the nonlinear response of the structure. The "Newton-Raphson" approach is employed by ANSYS for solving the nonlinear problems. In this approach, the applied total load is subdivided into a group of load increments known as load steps. The load increments are applied over a number of the load sub steps. The load applied to the model should be increased gradually to avoid non convergence of the solution. The stiffness matrix of the model is modified to reflect the nonlinear changes in the stiffness of the structure at the end of each load increment before performing the next load increment [26]. The structural solution requires identifying: the type of the analysis (which is a structural type in the current intended study) and the analysis technique (small static displacement). The Newton-Raphson equilibrium iterations are applied in ANSYS for updating the stiffness

of models. The ANSYS software also utilizes an option to predict and control the load step sizes which known as "the automatic time stepping". Based on the physics of the models and the history of the previous researches in this field, it is concluded that the automatic time stepping will increase the load increment until it selects a maximum step to ensure a smooth convergence. On the other hand, in the case where the convergence behavior is abrupt, the automatic time stepping will divide the load increment until it equals the selected minimum load size. It is logically known that whenever the mesh is finer the results are relatively more accurate. In the current study, quick trials proved that meshing of size 25×25mm or finer approximately gave the same results. So, for the sake of time saver, meshing process is done so as to have cells maximum size of 25×25mm to get acceptable accuracy results.
Fig. 5 represents in brief the previous achieved experimental work details [6] which considered as a numerical modeling verification reference while Fig. 6 shows the finite element modeling details according to ANSYS [26]. Once the proposed numerical modeling technique is verified (as stated later) it is used to achieve the current research program.



(a) Cross section modeling details. (b) Reinforcements modeling details. (c) Numerical modeling mesh layout.

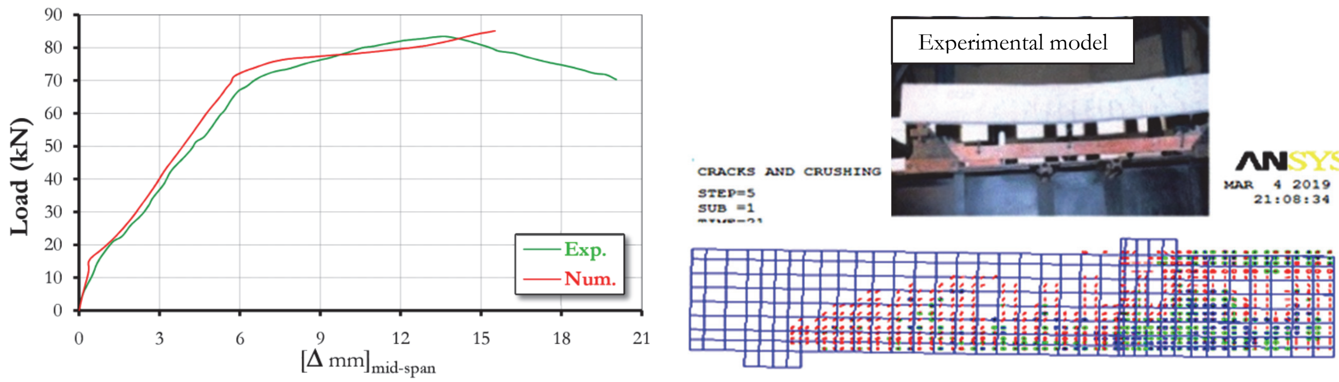


(d) Opening strengthening by means of CFRP sheets modeling details. (e) Loading points and supports details.

Figure 6: Finite element modeling details according to the ANSYS standard [26].

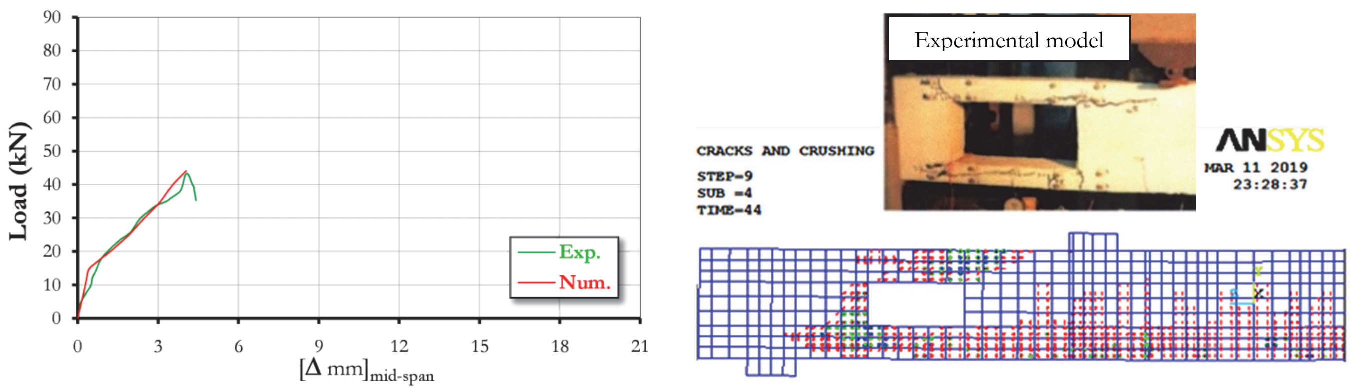
Figs. 7, 8 and 9 are representing a brief and limited comparative study to declare the proposed numerical modeling efficiency respecting the available previous experimental results [6] to judge its reliability to achieve the current research program. The ANSYS software usually shows the crack in the form of a colored outline circle in the crack plane, and the crushing in the form of an outline octahedron. Also, the outline circle has X shape through opening and closing the crack. Each integration point is capable of cracking in up to three different planes orthogonal to the principal axes where the first, the second and the third cracks displayed in the shape of a red, a green and a blue outlined circles respectively.
Regarding the solid beam model without opening, Fig. 7-a shows that the numerical maximum deflection at the model mid span is recorded to be 15.5mm which is higher than13.99mm that recorded experimentally. This proved an over estimated result by about 10.79%. The ultimate failure load recorded numerically is 2.41% more than that recorded experimentally. The initial cracking load Pcr of the proposed numerical modeling is recorded to be 26.67% less than that experimentally recorded since the detection of the fine cracks by the eye of the abstract or by the traditional inspecting devices is extremely difficult. So, relatively higher values are experimentally recorded. Both numerical and experimental mode of failure is flexure as shown in Fig. 7-b.
For the un-strengthened rectangular opening model shown in Fig. 8-a; the results convergence is noticed up to 79% of the experimental ultimate capacity at which slight differences are recorded. The ultimate failure load recorded numerically is 2.3% more than that recorded experimentally. The numerical maximum deflection at mid span at the ultimate failure load is 4.1mm which is identical to that recorded experimentally. The Pcr recorded numerically is 7.69% less than that recorded

experimentally. Both numerical and experimental mode of failure is shear due to excessive local shear deformation within the opening region especially at opening lower left and upper right corners as shown in Fig. 8-b.
For the strengthened rectangular opening model shown in Fig. 9-a; the results convergence is noticed up to Pcr of 17.72kN which is 11.4% less than the experimental Pcr which records 20kN. Regarding the ultimate failure load, the numerically recorded value is 2.7% more than that recorded experimentally. The numerical recorded deflection at the mid span is 9.8mm which is higher than 8.35mm that recorded experimentally. This proved an over determination of about 17.37%. Due to CFRP sheets existence, the cracking patterns is not clear in the experimental images as shown in Fig. 9-b.



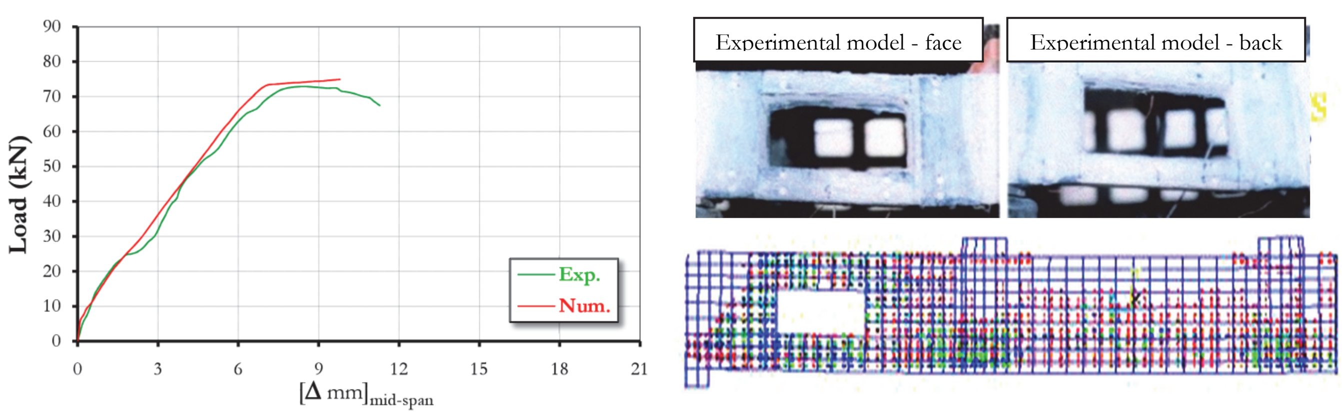
(a) Load-mid span deflection curve. (b) Cracking pattern.

Figure 7: Experimental *versus* proposed numerical modeling technique results for the control solid RC beam without opening.



(**a**) Load- mid span deflection curve. (**b**) Cracking pattern.

Figure 8: Experimental *versus* proposed numerical modeling technique results for the RC beam with un-strengthened rectangular opening.



(**a**) Load-mid span deflection curve. (**b**) Cracking pattern.

Figure 9: Experimental *versus* proposed numerical modeling technique results for the RC beam with strengthened rectangular opening.

In general, the numerically recorded maximum central deflection and the ultimate failure capacity are of 9.37% and 2.47% in average more than that recorded experimentally while the numerical Pcr is 15.25% less.
Finally, referring to the previous limited comparative analysis of the validation Figs. 7, 8 and 9, it can be concluded that the proposed numerical modeling details are reliable enough to considering it in achieving the intended research program which aimed to mainly investigate the effect of the assumed practical applied service loads levels (which is simulated numerically as percentages of the solid model without opening ultimate design capacity) on the structural behavior of simply supported RC beams tested under two concentrated loads till failure and having post-openings strengthened by means of CFRP sheets within their shear zones as described before. Hereafter, the gained results of the current research program which is described in Fig. 2 are presented and analyzed.

RESULTS ANALYSIS AND DISCUSSION

The summarized results of all the studied models are tabulated in Tab. 2 for the sake of quick inspection reference preview. The discussed structural behavior general aspects of the analysis are limited only to the mid-span deflection, the initial cracking loads, the cracking patterns, the failure modes and the failure loads. All RC beams models are tested under two points of concentrated loads till failure. The results of the seventeen RC beams models are discussed hereinafter.

Beam model state		Considered Aspect of Analysis	Opening drilling at service loads as a percentage (%) of ultimate capacity of reference beam (**P_u**)							
			0 % P_u (0.0 KN)		**≈ 26 % P_u** (22 KN)		**≈ 40 % P_u** (34 KN)		**≈ 55 % P_u** (46.75 KN)	
Opening shape	Strengthening state	Measured parameter	Result	as % of (**R**)	Result	as % of **U?00***	Result	as % of **U?00***	Result	as % of **U?00***
No opening (reference beam **R**)	----------	Failure load (KN)	***85.0***	*100*	-------	---------	--------	---------	--------	---------
		Deflection (mm)	***15.5***	*100*	-------	---------	--------	---------	--------	---------
		Failure mode	Flexure		----------		----------		----------	
Rectangular opening	Un-strengthened	Beam ID	**UR00**		**UR26**		**UR40**		**UR55**	
		Failure load (KN)	44.0	51.80	43.0	97.7	40.7	92.5	40.0	90.9
		Deflection (mm)	4.10	26.5	4.0	97.6	3.85	93.9	4.79	116.8
		Failure mode	Shear		Shear		Shear		Shear	
	Strengthened	Beam ID	**SR00**		**SR26**		**SR40**		**SR55**	
		Failure load (KN)	75.0	88.20	74.0	98.7	72.6	96.8	62.0	82.7
		Deflection (mm)	9.80	63.2	9.35	95.4	8.95	91.3	5.83	59.5
		Failure mode	Flexure		Flexure		Flexure		Shear	
Circular opening	Un-strengthened	Beam ID	**UC00**		**UC26**		**UC40**		**UC55**	
		Failure load (KN)	55.53	65.3	53.4	96.1	48.2	86.8	47.6	85.7
		Deflection (mm)	6.40	41.3	5.51	86.0	5.48	85.6	5.21	81.4
		Failure mode	Shear		Shear		Shear		Shear	
	Strengthened	Beam ID	**SC00**		**SC26**		**SC40**		**SC55**	
		Failure load (KN)	76.0	89.4	74.6	98.2	74.0	97.4	69.0	90.8
		Deflection (mm)	10.90	70.3	9.75	89.4	9.64	88.4	6.81	62.5
		Failure mode	Flexure		Flexure		Flexure		Shear	

Table 2: The results summery of the investigated RC beam models according to the research program.
* Result as a percentage of concerned control model with un-strengthened (rectangular or circular) opening result at no service loads.

Deflection

The applied loading is incremented gradually and the deflections are recorded then the corresponding load-deflection curves are plotted. Load versus mid-span defection curves for un-strengthened and strengthened beams with rectangular and circular openings drilled under different service loads levels as percentages of ultimate failure load Pu of reference model without opening (R) are presented in the following figures.

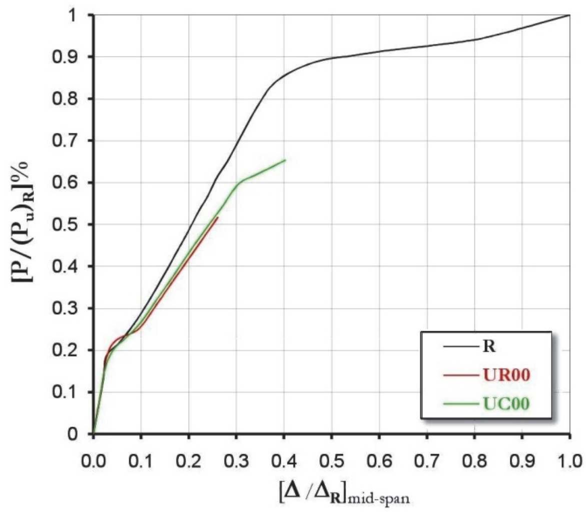


Figure 10: Influence of the openings drilling under no service loads on the beams models deflection compared to the (R) beam model.

The appropriate load deflection relation for the control un-strengthened rectangular (UR00) and circular (UC00) openings models in addition to the reference beam without any openings (R) are shown in Fig. 10. Drilling a rectangular opening of equivalent area as a circular one under no service loads within the shear zone seriously reduced the maximum mid-span deflection of the beam at failure to be only of 26% for (UR00) and 41% for (UC00) of that for the beam (R). In other words, this action reduced the maximum recorded mid-span deflection by about 74% and 59% for (UR00) and (UC00) respectively as can be deduced from Fig. 10. These results enhanced the undesired structural brittle behavior or the abrupt collapsing at failure although the opening drilling is assumed to be achieved under no service loads. Therefore, it is of serious importance to take care of the service loads application levels when it is required to drill the openings within the RC beams shear zones which has forced the authors to prepare the current research program to meet the practical conditions as possible as could. Strengthening the openings in the models (SR00 and SC00) seriously increases the mid-span deflection at failure to be up to 2.4 and 1.7 times that recorded for the un-strengthened opening models (UR00) and (UR00) respectively as can be deduced from Fig. 11. However, openings CFRP strengthening enhances the models desired structural ductile behavior.

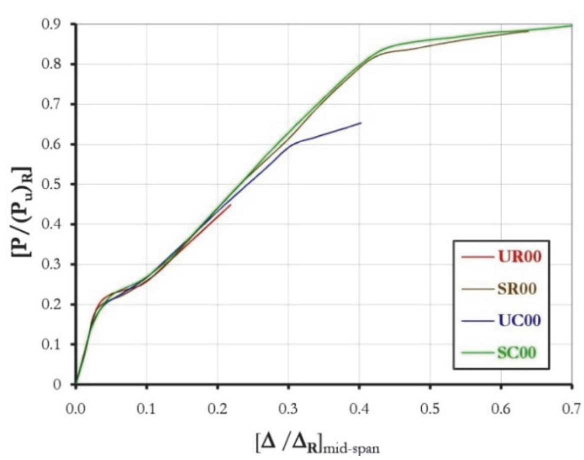


Figure 11: Influence of the openings drilling and strengthening under no service loads on the main control beams models mid-span deflection.

The influence of the different service loads levels up to 55%Pu on the behavior of the beams with un-strengthened rectangular and circular openings compared to the reference solid beam with no opening from mid-span deflection viewpoint is shown in Figs. 12 and 13 respectively.

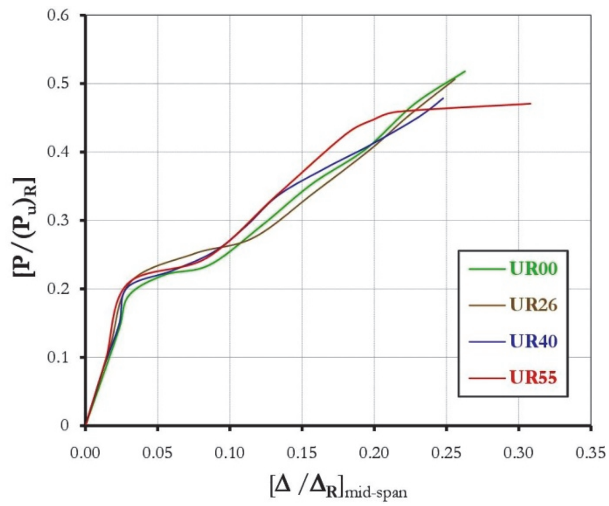


Figure 12: Deflection behavior of the beams with un-strengthened rectangular openings drilled under the different assumed service loads levels up to 55%Pu.

It can be stated that the service loads levels on the beam model with rectangular opening up to 26%, 40% and 55%Pu produce a mid-span deflection at failure equal to 0.98, 0.94 and 1.17 times that of the model UR00 under no service loads respectively as can be noticed from Fig. 12. For the sake of comparative analysis and as can be deduced from Fig. 13, the previous mentioned deflection proportional factors are reduced to be the same and equal to only 0.86 times that of UC00 for UC26 and UC40 models. Unlike them, the model UC55 records only 0.81 times that of UC00. These results prove more negative influence on the structural behavior from the deflection view point of the service loads levels up to 40%Pu of the models with circular opening than those have rectangular ones. High recorded deflection for model UR55 at failure is occurred due to the excessive local shear deformation within the opening region which is an expected result of the higher applied service loads when a rectangular opening is drilled.

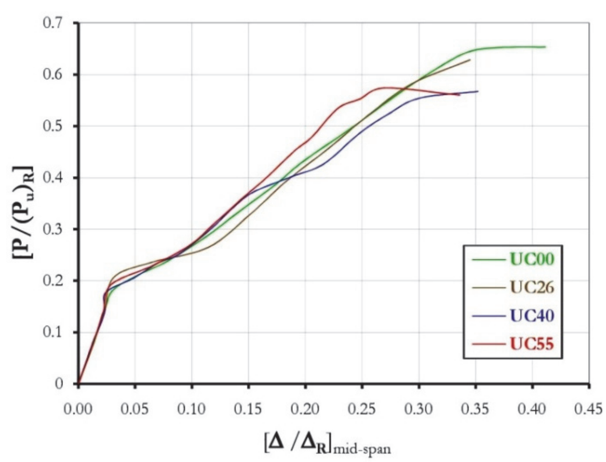


Figure 13: Deflection behavior of the beams with un-strengthened circular openings drilled under the different assumed service loads levels up to 55%Pu.

Figs. 14 and 15 declare the opening CFRP strengthening influence on the structural behavior from deflection viewpoint for beam models with rectangular and circular opening respectively. The openings CFRP strengthening under service loads level up to 40%Pu improves the desired ductile structural behavior by about 2.33 and 1.76 times those of un-strengthened ones from deflection viewpoint for rectangular and circular openings models respectively. The previous improvement factors are reduced gradually to be only of 1.21 and 1.31 respectively when the service loads level increased to be 55%Pu. These results emphasized the expected serious undesired influence from the deformational point of view of the higher service loads levels on the opening CFRP strengthening efficiency.

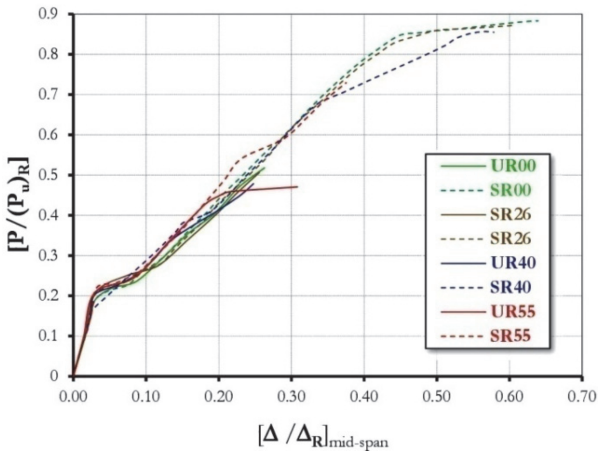


Figure 14: Influence of the rectangular opening drilling and strengthening under the different assumed service loads on the RC beams recorded mid-span deflection.

Additionally, it can be deduced from Figs. 14 and 15 that the openings strengthening under a service load level up to about 40%Pu increases the desired ductile structural behavior of the rectangular openings models from 0.725 times the un-strengthened circular ones in average to be 0.945 times. This improvement factor is reduced gradually from 0.93 to be of only 0.86 times as the service loads level increased up to about 55%Pu. These results emphasize again the previous mentioned undesired influence of the higher service loads levels on the strengthening of the rectangular opening compared to the circular ones.

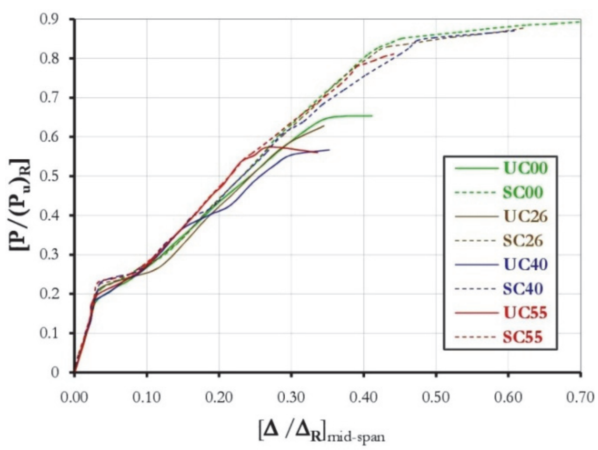


Figure 15: Influence of the circular opening drilling and strengthening under the different assumed service loads on the RC beams recorded mid-span deflection.

Fig. 16 summarizes the proportional deflection to the deflection of the reference beam results for all models at failure. The previous stated analysis is emphasized by the inspection of this bar chart contents.
Generally, it is clear that the opening CFRP strengthening is effective in improving the structural behavior of all tested models from the deflection viewpoint regardless the service loads level. It is noticeable that drilling and strengthening a rectangular opening within the shear zone of the beam at a service loads level of 55%Pu (SR55) records a noticeable low mid span deflection value of about of 0.63 times in average that drilled and strengthened at earlier service loads levels. This is believed to be owing to the excessive deflection that occurred locally within the opening region before application of the opening strengthening (UR55) which led the central deflection to be about of 1.19 times in average that recorded under a service loads level up to only 40%Pu which somehow forced the strengthened rectangular opening RC beam

models to failed earlier which enhancing the undesired abrupt failure as can be deduced from Fig. 16. It is worth to state that the previous mentioned factors are of 0.67 and 0.9 for the corresponding circular opening cases.
The numerically full collapsing state of the model UR55 under 55%Pu service loads is discussed later in Fig. 25.

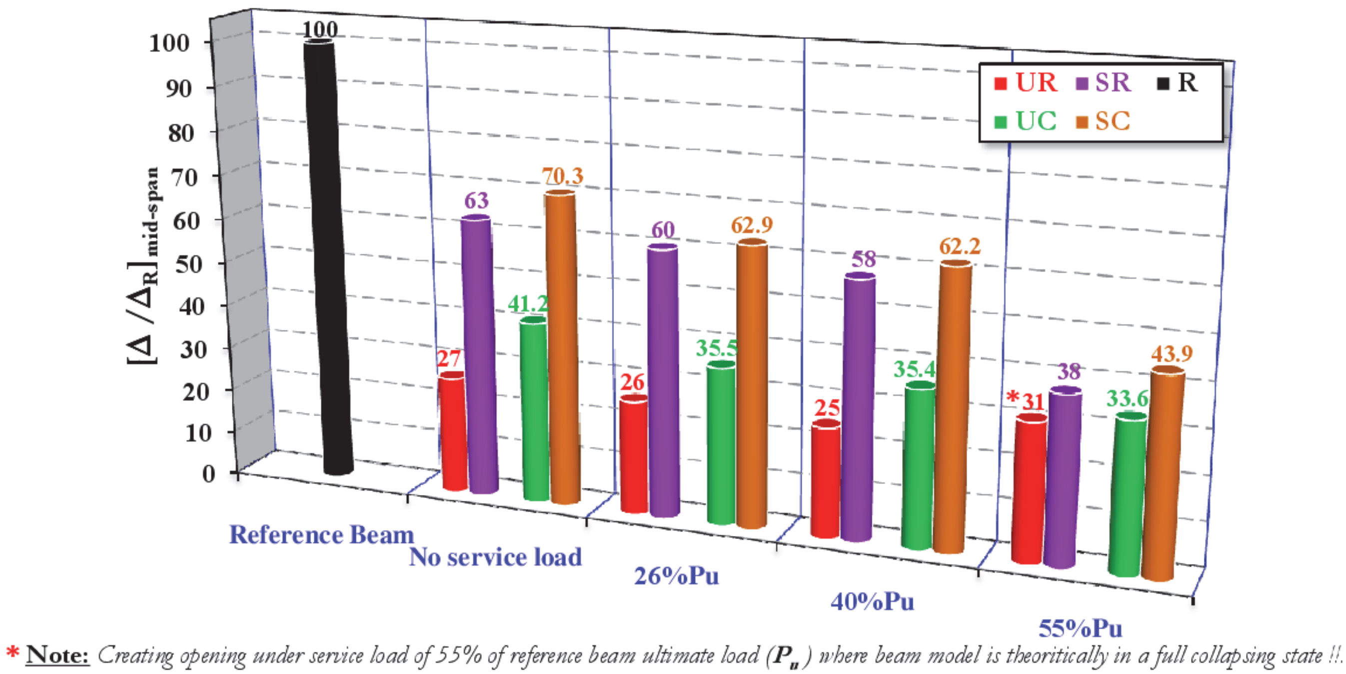
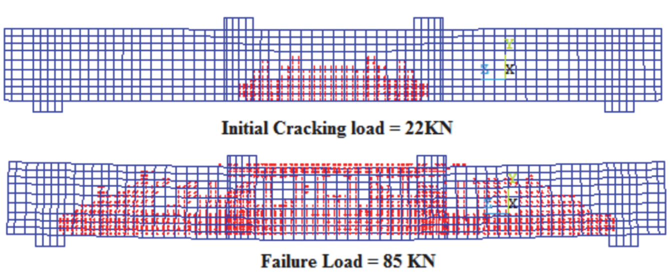
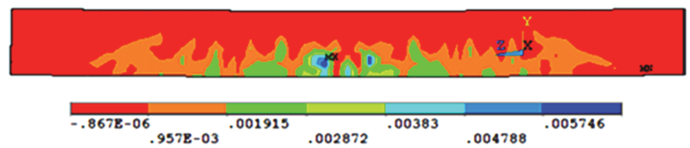


Figure 16: Influence of the opening drilling and strengthening under the different assumed service loads levels on the RC beams models mid-span deflection at failure.

Cracking patterns and maximum generated strain contour areas in concrete
It is worth to remind the reader that the ANSYS software usually shows the crack in the form of a colored outline circle in the crack plane, and the crushing in the form of an outline octahedron. Also, the outline circle has X shape through opening and closing the crack. Each integration point is capable of cracking in up to three different planes orthogonal to the principal axes where the first, the second and the third cracks displayed in the shape of a red, a green and a blue outlined circles respectively.



(a) Patterns of initial cracking and cracking at failure.

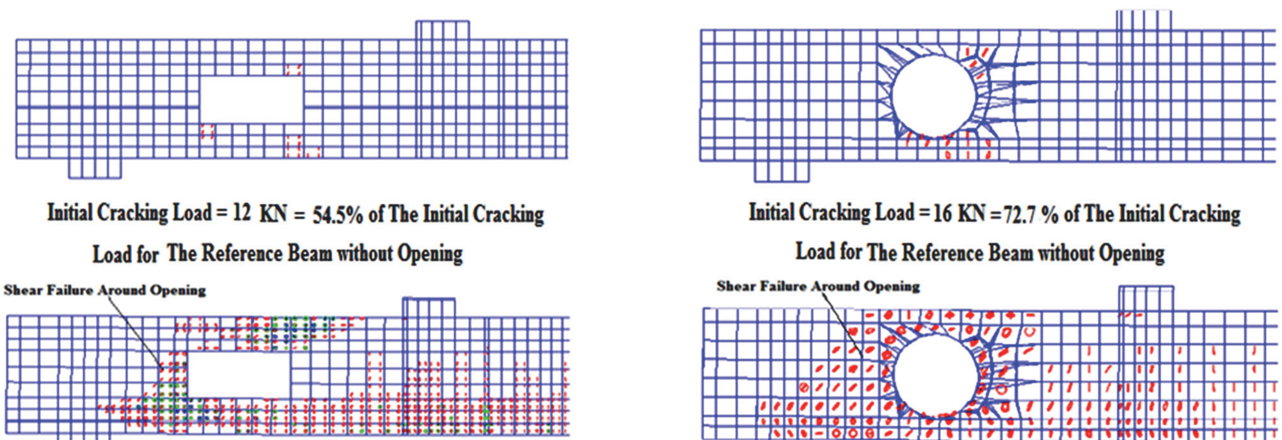


(b) Concrete maximum generated strain contour areas at failure.

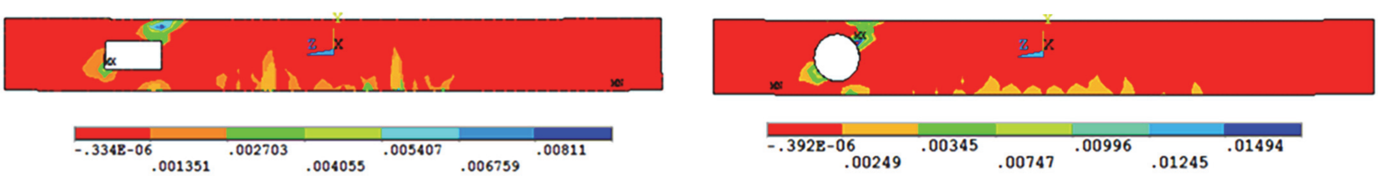
Figure 17: Structural status details of the reference beam (R).

The cracking pattern of the reference beam (R) is shown in Fig. 17-a. The cracks propagated in a manner similar as the flexural mode of failure. The initial flexural cracks occurred at a load of 26%Pu at the mid-span bottom fibers of the tension zone. The cracks propagated and widened in the middle third of beam model till they reach near the extreme top fibers at which they gradually reduce the area of the compression zone after which a tensile failure occurred simultaneously with the yielding of the reinforcing steel bars which is known as under reinforced section concrete failure.
As the loading continued the cracks extend beyond the middle third to the outer thirds and form what so called "tooth like" pattern at failure stage. The failure mode of the reference beam is flexural at which the recorded strain exceeded the predefined failure limit as shown inFig.17-b. By inspecting the values at the strain contour scale bars it is realized that the

concrete tensile strain have reached the extreme values and is found to be about of 0.006 within the beam mid span region just upper of the main tension steel reinforcements due to the steel-concrete full bond assumption.
The cracks pattern of the control un-strengthened beams with opening drilled under no service loads (UR00 and UC00) are shown in Fig.18-a. The cracks propagation views through the control un-strengthened beams with the rectangular and circular opening at Pcr and at the failure are presented. By loading the rectangular opening beam, the first cracks appeared are the shear cracks (diagonal to the opening edges) at a load of 19%Pu while it is equal to 24%Pu for the circular opening case. These shear cracks congested as the loading level increased and the diagonal shear cracks jammed around the rectangular opening corners due to the expected generated stress concentrations. In the case of circular opening the diagonal shear cracks spread uniformly in the opening periphery. These diagonal shear cracks in both the rectangular and circular opening beams are followed by minor flexural cracks that appeared at the mid span. As the loading level increases the diagonal tension (shear) cracks spread intensively around the rectangular opening corners and the circular opening periphery until the strains exceeded the predefined ultimate strain of concrete at which the concrete fails as shown in Fig.18-b. This is owing to the opening un-strengthened yet state and the absence of the opening special reinforcements where the openings are assumed to be post-planned openings made. Accordingly, the concrete suffered alone all the developed shear stresses.



(a) Patterns of initial cracking and cracking at failure.

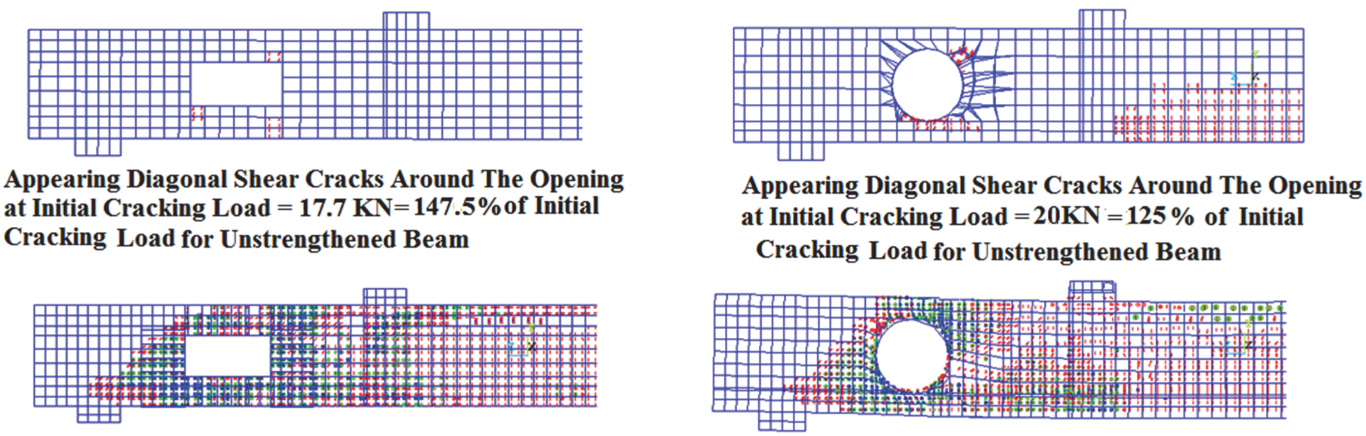


(b) Concrete maximum generated strain contour areas at failure.

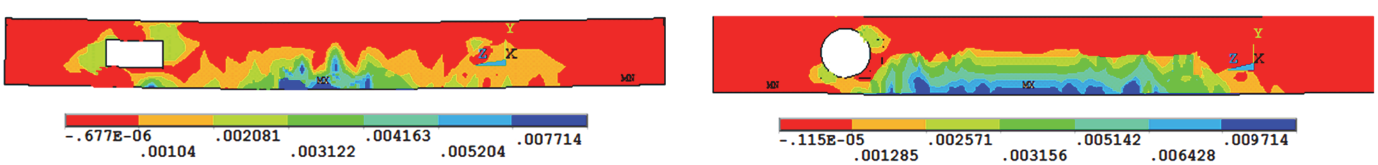
Figure 18: Structural status details of the un-strengthened control beams models UR00 and UC00 drilled under no service loads.

By comparing the cracks pattern of the control beam with circular opening (UC00) and that with the rectangular one (UR00), it is declared that the (UC00) model structurally performs better than the (UR00) one as initially expected. This is believed to be owing to less stresses concentration around the circular opening which allows for higher applied loads and higher deflection values before failure unlike the rectangular opening case where it is failed at a lower capacity with relatively less recorded deflection values.
The cracking patterns of the control beams with CFRP strengthened opening drilled under no service loads (SR00 and SC00) are shown in Fig.19-a. Regarding the (SR00) model, the first appeared crack is a shear crack (diagonal to the opening edges) observed at a load of 21%Pu which is 1.11 times the Pcr of the beam model UR00. In the other hand, for the case of (SC00) model, the first appeared crack is the shear crack (around the opening boundaries) at a load of 25%Pu which is only 1.04 times the Pcr of the model UC00. Additionally, it is accompanied with minor flexural cracks initiated at the mid span. It is eligible to state that the Pcr in the case of model SC00 is 1.19 times that of model SR00. The initial shear cracks appeared as loading increased and the diagonal shear cracks begin to grow around the opening lower left and upper right corners after which the minor flexural cracks appeared within the mid span region at a load value of 27%Pu. On the other hand for the model SC00, also the shear cracks increased as loading increased and the diagonal shear cracks

generated around the periphery of the circular opening and followed by extra flexural cracks within the mid span region at the same load level as SR00 model. As the loading is gradually incrementing, the diagonal tension (shear) cracks spread around the corners of the model SR00 while regarding the SC00 model, the diagonal shear cracks increased around the periphery of the circular opening followed by a growth of the flexural cracks at the model mid span region more rapidly than the case of the model SR00. Subsequently, the flexural cracks widen and propagated within the middle third region of the beam after which the beam going to failed in shear at which CFRP sheets begin to resist diagonal tension stresses in both strengthened beams models SR00 and SC00. Additionally, the shear cracks widened around the periphery of the circular and rectangular opening corners as the loading level increased followed by growth of the flexural cracks at the mid span region. Flexure cracks propagate up towards the extreme compression top fibers of the model and gradually reducing the compression zone area depth causing a crushing failure of the extreme top surface of the beam which occurred at failure loads of 88% and 89%Pu for models SR00 and SC00 respectively as can be deduced from Fig.19-b where the maximum generated strain contour areas at failure are presented. Simultaneous yielding of the steel reinforcement is observed for both cases. Referring to the Fig. 18-b, it can be deduced from Fig. 19-b after inspecting values at the strain contour color scale bars that the strengthening of the circular opening reduced the generated concrete strain by about 35% while this percentage is found to be only about of only 5% for the case of the rectangular opening which proved negligible effect of the strengthening of the rectangular opening on generated stress in concrete within the opening corners periphery regions. Also, due to the opening strengthening, the color scale dark blue area that has high generated strain and consequently stress at the opening upper right corner disappeared and instead it is found within the beam mid span third as originally expected.



(a) Patterns of initial cracking and cracking at failure .



(b) Concrete maximum generated strain contour areas at failure.

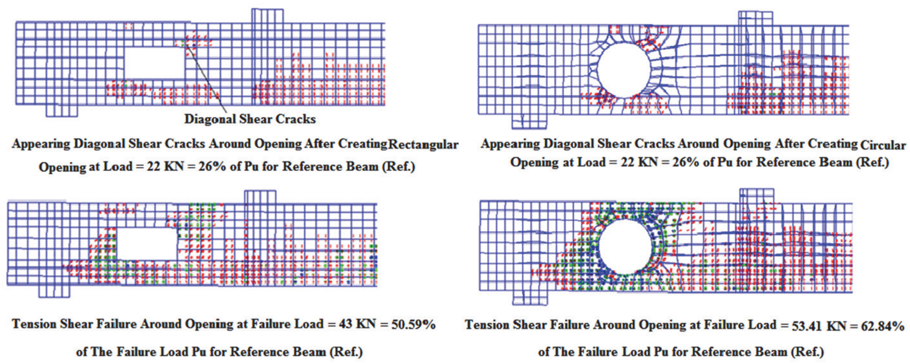
Figure 19: Structural status details for RC beams with strengthened rectangular and circular openings models SR00 and SC00 drilled under no service loads.

For the sake of comparison, the crack patterns for all un-strengthened models under different service loads levels are collected in the same figure, Fig. 20. The cracking patterns for the un-strengthened models with rectangular and circular openings (UR26 and UC26) drilled under a service loads level of 26%Pu of the reference solid model (R) are shown in Fig.20-a.

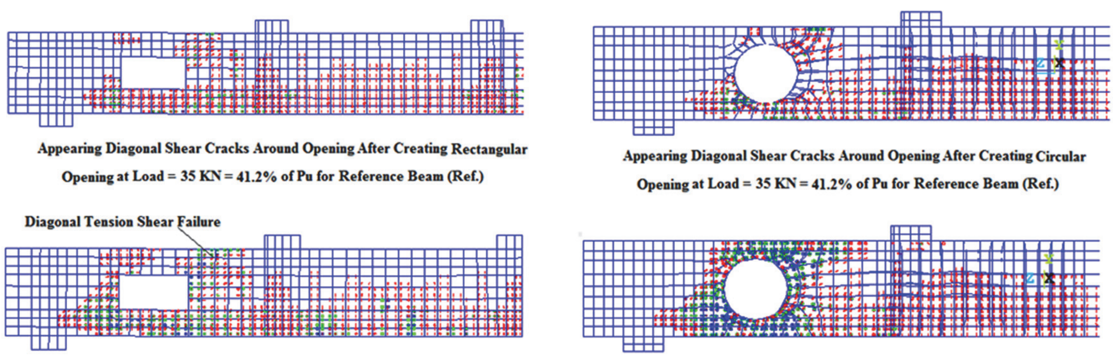
Referring to the Fig. 17-a, the flexural cracks initiated at the mid span regions within the tension zone of the reference beam which are the first cracks kind that appeared at a load of 26%Pu. After drilling a rectangular opening at the same service loads level (26%Pu of the solid model R), the diagonal or shear cracks appeared immediately around the opening corners due to the reduction of the model cross section effective area at the opening position which forced the failure mode to be changed from flexure to shear as a direct result of drilling the opening within the shear zone as will be discussed later. The diagonal shear cracks are propagated and congested around the opening till failure at a much lower loading level due to the reduction of the shear strength of the beam cross section which reduced the effective depth along the opening wide distance.

The cracking patterns and propagation for the un-strengthened beams with rectangular and circular opening drilled under other different service loads levels of 40% and 55%Pu of the reference beam R are similar to the cracks propagation characteristics of the beams models UR26 and UC26 as can be noticed from Figs. 20-b and 20-c except that the initial cracks occurred at relatively lower applied loading levels.

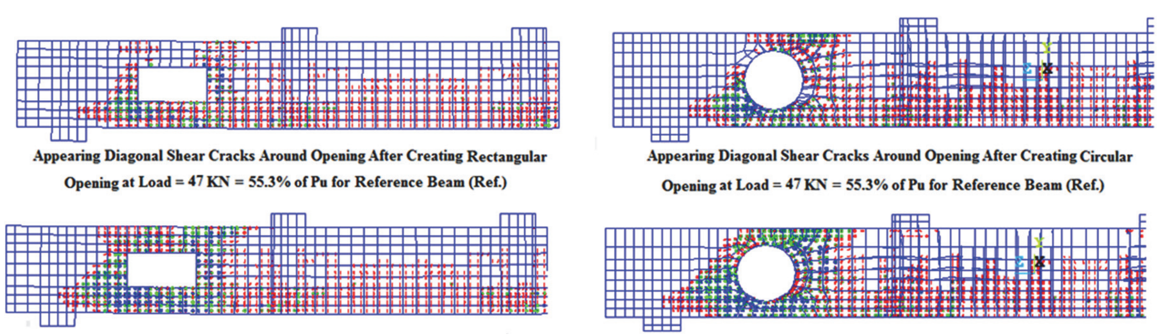
No other noticeable details concerned the crack patterns are observed while the slight differences in the generated strain are discussed later in Fig. 22.



(a) Patterns of initial cracking and cracking at failure for U26 models.



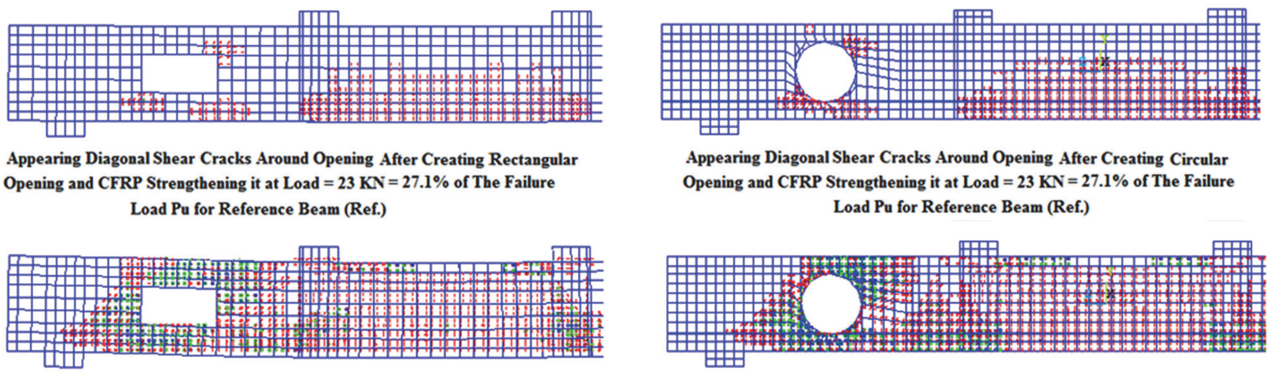
(b) Patterns of initial cracking and cracking at failure for U40 models.



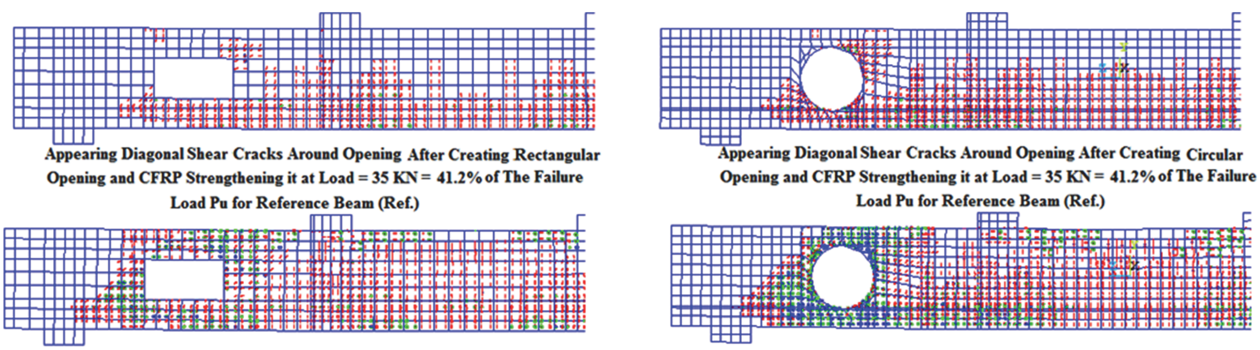
(c) Patterns of initial cracking and cracking at failure for U55 models.

Figure 20: Cracking status for the un-strengthened rectangular and circular openings drilled under service loads levels up to 55%Pu.

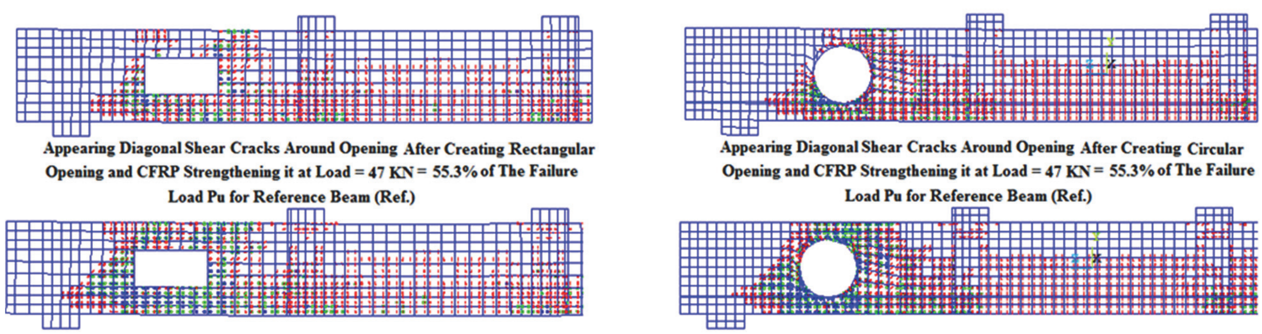
For the sake of comparison, the cracking patterns for all strengthened models under different service loads levels are collected in the same figure, Fig. 21. The cracks propagation for the strengthened rectangular and circular opening models SR26 and SC26 at initial cracking and failure load values are presented in Fig.21-a. It is worth to remind that the first crack is of a flexural kind and it is appeared at a loading level of 26%Pu at the models mid-span before drilling the rectangular or circular opening and strengthening them by CFRP sheets. After drilling the openings and performing the strengthening, the shear cracks appeared around the opening at a loading value of about 1.05 times that of the reference solid model Pcr simultaneously with flexural cracks appearance at the beam span middle third region. As the applied loading increased, the diagonal tension (shear) cracks initiated and propagated around the rectangular opening corners of the model SR26 and within the vicinity to the periphery of the circular opening of the model SC26 after which the flexural cracks within the span middle third of the beams SR26 and SC26 are intended to become wider and wider. Consequently, they are seemed to be fail in flexure at which CFRP sheets resisted the diagonal tension and the shear stresses generated within the openings regions of the strengthened models. The flexure cracks propagated towards up the extreme compression fibers of the beam resulting in reducing of the compression zone area depth which is followed by the crushing of the extreme top surface of the beam which occurs at the maximum failure load of 87% and 88%Pu for the beam models SR26 and SC26 respectively simultaneous with yielding of the reinforcement at the span middle third for both. The crack patterns in the case of circular opening is the same as the rectangular, but the cracks around the circular opening are distributed around the periphery of the circular opening unlike the rectangular where the cracks are concentrated at the opening corners.



(a) Patterns of initial cracking and cracking at failure S26 models.



(b) Patterns of initial cracking and cracking at failure for S40 models.



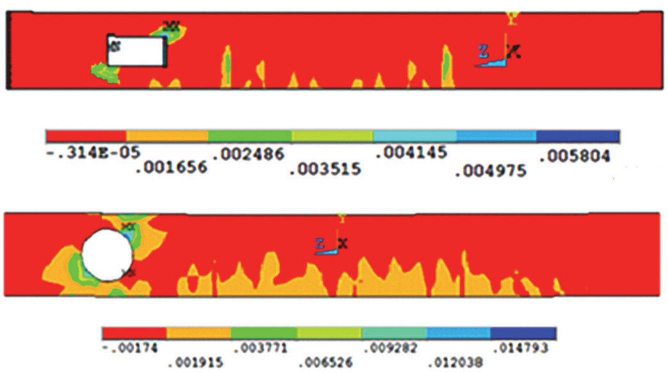
(c) Patterns of initial cracking and cracking at failure for S55 models.

Figure 21: Cracking status for the strengthened rectangular and circular openings drilled under service loads levels up to 55%Pu.

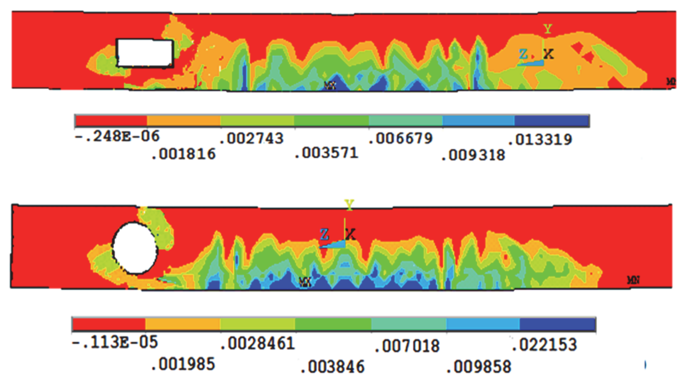
The cracking patterns and propagation features of the strengthened models with rectangular and circular opening (SR40 and SC40) are similar to the cracks propagation of the models SR26 and SC26 as can be noticed from Fig. 21-b except that they occurred at relatively little lower applied loading levels of 85% and 87%Pu respectively. Also, the cracks initiation and propagation for both strengthened rectangular and circular opening models (SR55 and SC55) are shown in Fig.21-c.

Where the opening has not yet been drilled and strengthened by CFRP sheets, the first cracks that developed at applied loading of 26%Pu are flexural cracks which appeared within the beam span middle third in the tension zone. Increasing the applied loading on the reference solid model up to 55%Pu causing wide spreading of the cracks which surpass the beam span middle third to the boundaries of the beam span exterior thirds regions at a load of 55%Pu. After drilling the rectangular and circular openings and strengthening them at a service loads level of 55%Pu (SR55 and SC55), diagonal shear cracks immediately appeared around the openings due to their existence reduction effect on the RC concrete sections effective area. After strengthening the openings by CFRP sheets, models SC55 and SR55 re-sustain the applied loading till they fail at a loading level of 81% and 73%Pu respectively.

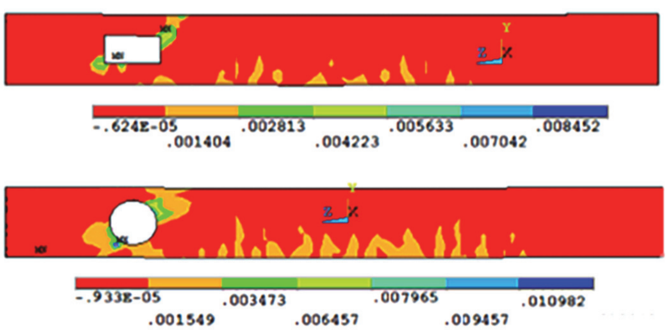
Finally, the openings strengthening with CFRP sheets has very desired influence on minimizing the propagation and widening of the cracks especially within the vicinity of the openings regions for all models regardless the various service loads levels. Unlike un-strengthened openings, the service loads levels relatively have a negligible influence on the cracks kind at failure regardless the drilled opening shape.



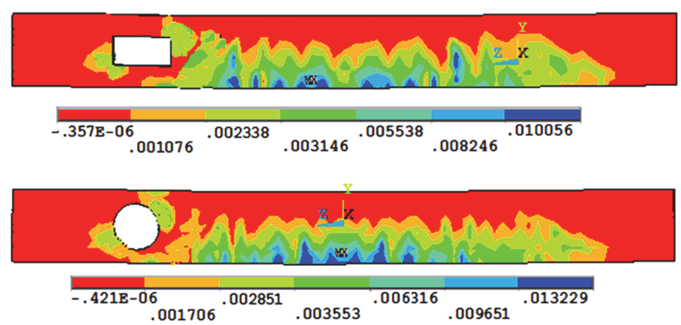
(a) Maximum generated strain at failure full view for U26 models.



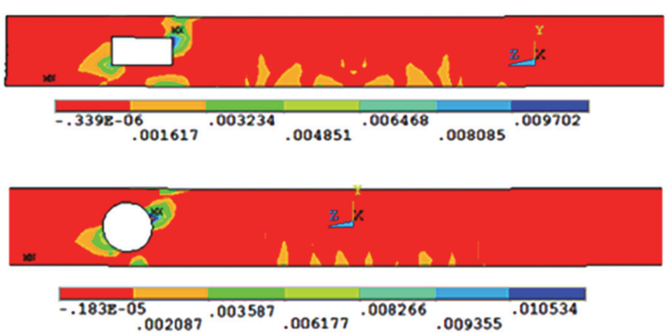
(d) Maximum generated strain at failure full view for S26 models.



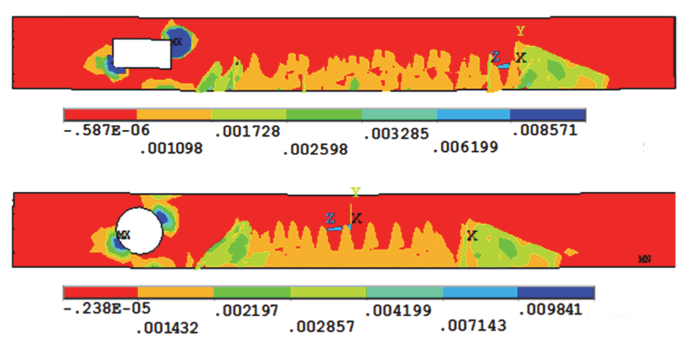
(b) Maximum generated strain at failure full view for U40 models



(e) Maximum generated strain at failure full view for S40 models.



(c) Concrete maximum generated strain contour areas for U55 models.



(f) Concrete maximum generated strain contour areas for S55 models.

Figure 22: Concrete strain contour areas at failure of the RC beams with un-strengthened and strengthened rectangular and circular openings drilled under service loads levels up to 55%Pu.

For the sake of comparison, the generated strain contour areas for all strengthened models under the different service loads levels at failure are collected in the same figure, Fig. 22. The strain dark blue color scale indicates the contour areas of the maximum generated tensile strain values. It is clear that the service loads level of 55%Pu weaken the whole beam capacity to sustain the applied loads due to the local high stress concentration at the strengthened openings diagonal chords ends in the direction parallel to the shear cracks as shown in Fig. 22-f. This leading to a relatively rapid local failure before the beam bottom fibers reached the maximum defined characteristic strain level unlike the cases of the lower service loads levels 26% and 40%Pu as shown in Figs. 22-d and 22-e respectively. The opening strengthening forced the dark and light blue colors to be appeared within the beam span middle third bottom fibers especially for low service loads levels of 26 and 40%Pu where these colors don't appear before except within the vicinity of the opening periphery for all models regardless the opening shape. This proved more desired structural behavior. Regardless the different service loads levels, Figs. 22-a , 22-b and 22-c declare that the opening existence without CFRP strengthening leaded to an instant rapid local failure within the opening regions before the models bottom fibers reached the maximum defined characteristic strain level.

It can be deduced after inspecting the values at the strain contour color scale bars that the strengthening of the rectangular opening under a service loads level of 55%Pu reduces the maximum generated concrete strain at failure by about 12% due to the relatively earlier occurrence of collapsing which caused before by the high service loads level application. The higher service loads causing an increase in the concrete strain by about 130% and 19% of that of the cases of low service loads levels 26% and 40%Pu respectively. This proved the serious effect of the higher service loads level on the maximum generated strain in the models with strengthened rectangular openings especially within the opening corners periphery regions.

The same deductions can be stated for the circular opening models where the previous mentioned corresponding comparative percentages are a reduction of about 7% and increasing of about 50% and 21% for models SC55, SC26 and SC40 respectively. This indicates the relatively more efficiency of the opening strengthening in improving the structural behavior for the models with rectangular openings than those with circular ones which is an initially expected deduction due to the original expected superiority of the circular opening under the same structural conditions. Also, due to the opening CFRP strengthening, the dark blue areas of high generated strain and consequently stress at openings upper right corners disappeared and are reappeared instead within the beam mid span third as expected except the models with openings drilled under a service loads of 55%Pu where the maximum stresses are also generated at the opening vicinity regions especially at upper right corners.

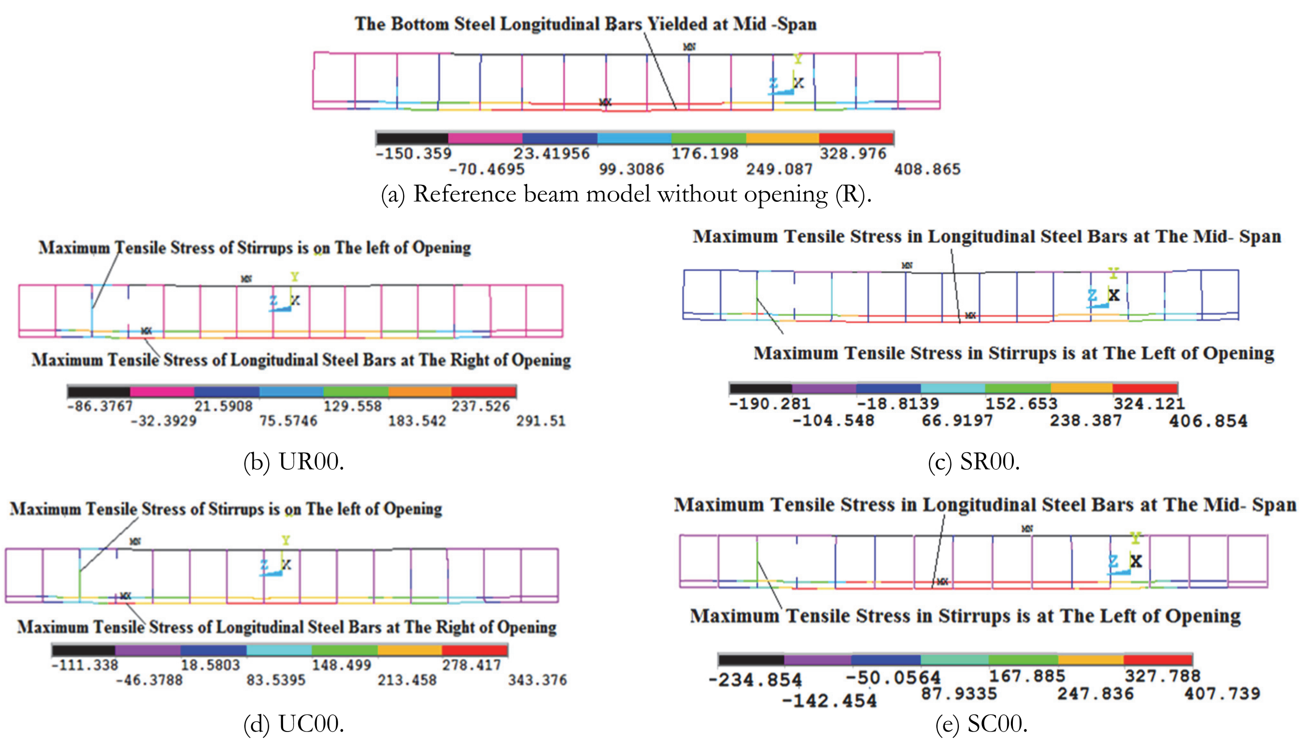


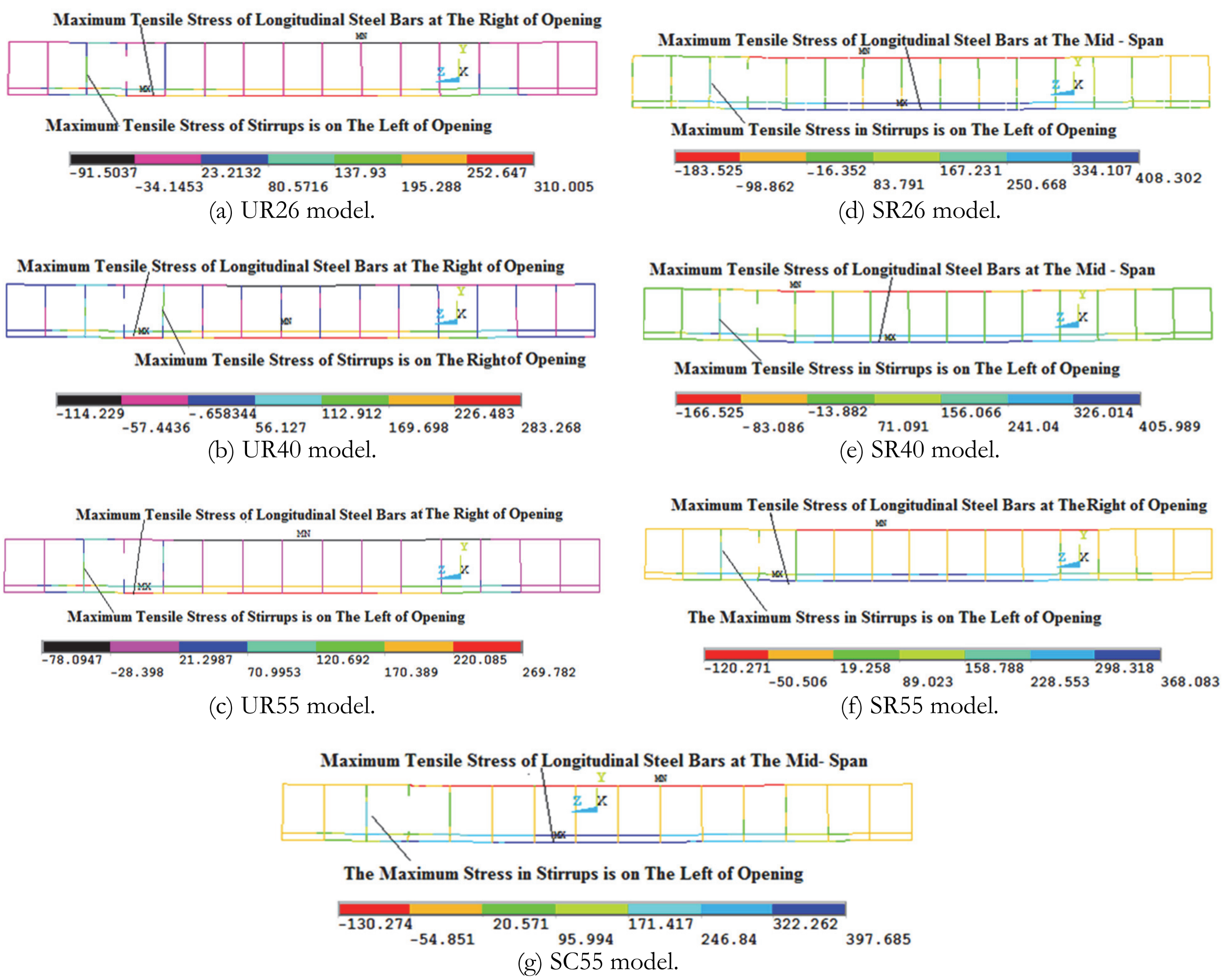
Figure 23: Strengthening effect on the generated stress (in MPa) in the reinforcements for the main control beams at failure.

Maximum generated strain in steel reinforcements

By inspecting the generated stress values on the bottom reinforcements at the stress contour color scale bar for the reference solid model (R) as shown in Fig. 23-a, it is clear that the stress slightly exceeds the value of 400MPa which is the indicator of the yielding of the steel reinforcement bars at failure and then the model exhibits a large plastic deflection. On the other hand, the stresses generated on the top reinforcement bars are negative hence indicating a compressive stresses which are much lower than the yield stresses. Also, by inspecting the generated stresses in the vertical stirrups it is realized that their stresses values are less than the yield stress value of 240MPa at failure.

Figs. 23-b and 23-d represent the generated stresses in the reinforcements for the models UR00 and UC00 where it is noticeable that the maximum generated stresses occurred in the tension reinforcement bars at the right of the opening but they do not reach the yield stress due to that both of them failed in shear due to the excessive local shear deformations within the opening region. For the model UC00, it is worth to state that the shear stress are better distributed around the circular opening where the concrete withstands shear stresses till the stirrups yielded in shear and recorded 108.8% of the yielding criteria just left to the opening towards the left support.

Figs. 23-c and 23-e declare that both tension and compression generated stresses in the steel reinforcements bars of the model SC00 are higher than those of the model SR00 case. Unlike un-strengthened opening models, the opening strengthening effect is positively obvious where the maximum generated stresses values are slightly over the characterized yielding limit at failure and are of 1.4 and 1.19 times that recorded for un-strengthened opening models and approximately are the same as that recorded for the solid model without opening for the rectangular and circular opening models respectively. Strengthening of the rectangular opening is more efficient in structural behavior improvement than the case of circular ones from the generated stress on the steel reinforcement bars viewpoint.



(a) UR26 model. (d) SR26 model.

(b) UR40 model. (e) SR40 model.

(c) UR55 model. (f) SR55 model.

(g) SC55 model.

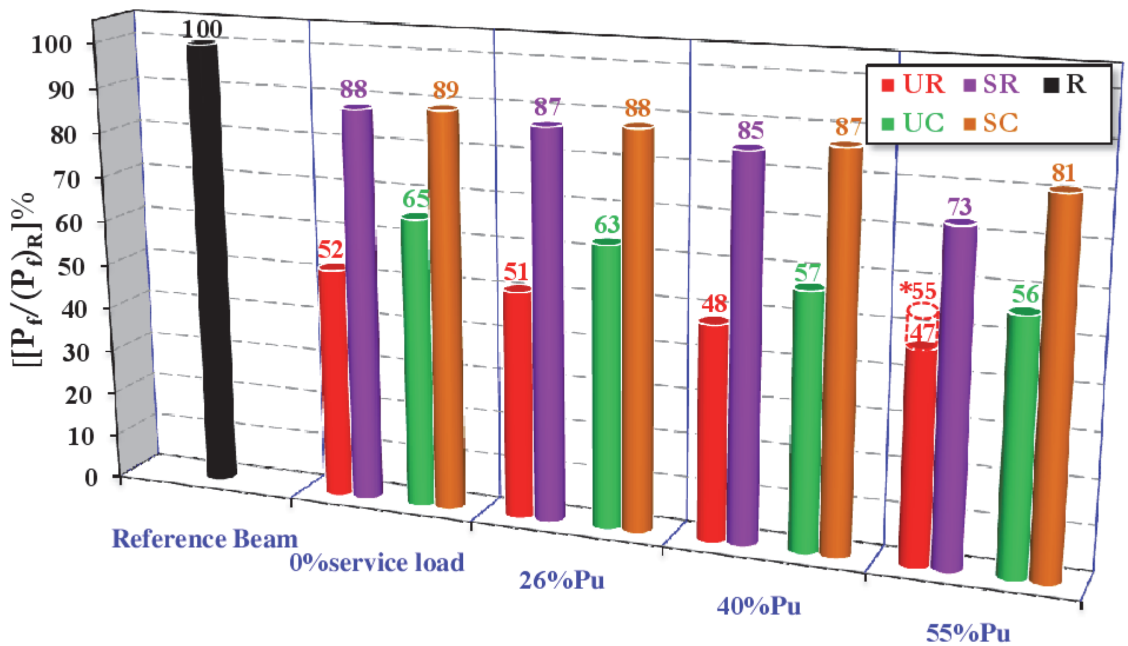
Figure 24: Influence of the opening drilling and strengthening under the different service loads levels up to 55%Pu on the generated stress (in MPa) in the reinforcements at failure.

Fig. 24 represents the influence of the opening strengthening under the different studied service loads levels on the maximum generated stress at failure in the models reinforcements especially for the strengthened rectangular opening since the approximately typical values that are recorded for the circular opening cases except the case under a service loads of 55%Pu as shown in Fig. 24-g.
In general, it is noticeable that the maximum generated stresses occurred in the stirrups in between the opening left edge and the left support region. Also, the maximum stresses in the main tension bars are recorded at the right of the opening but they do not reach the characterized yielding stress due to that most of the un-strengthened opening models failed in shear within the opening region except the cases of the strengthened opening models that sustain service loads up to only 40%Pu where the main steel reinforcement bars at the middle third of the beam model span have maximum recorded stress values slightly over the characterized yielding limit.
Regarding the strengthening of the rectangular opening, the maximum recorded generated stress in the reinforcements at failure are of 1.32, 1.43and 1.37 times that of un-strengthened models for the service loads levels of 26%, 40% and 55%Pu respectively as can be deduced from Figs. 24-a, b, c, d, e & f. These results indicate the serious effect of the service loads level on the developed stresses in the reinforcements.
Increasing the service loads level up to 55%Pu reduces the maximum developed stresses in the reinforcement bars for un-strengthened and strengthened rectangular opening models to be 0.87 and 0.9 times that recorded for the case of 26%Pu respectively. These results indicated the noticeable effect of the high service loads levels on turning the failure from the preferring manner that occurring duo to the yielding of the main reinforcements to be occurred within the opening regions which is a non preferable abrupt manner. Also, more relatively noticeable desired effect of the opening strengthening is proved for the service loads levels load up to not more than 40% of the RC solid beam ultimate design capacity since the stresses values recorded in the main steel reinforcement bars are slightly over the characterized yielding limit at failure. At a service loads level of 55%Pu, the circular opening strengthening increased the maximum recorded stress in the main steel reinforcements to be 1.08 times that of the strengthened rectangular one regardless that the generated stress in the steel reinforcement bars in both of them did not yet reached the characterized yielding limit at failure. This result indicates a little more preferable structural behavior of choosing the circular opening option than rectangular one as can be deduced from Figs. 24-f and 24-g and as a well-known deduction.

Failure loads and modes

Fig. 25 declares that by drilling either the rectangular or the equivalent area circular opening within RC beams shear zone regardless the level of the service loads, a serious reduction in the maximum carrying capacities occurred. Consequently, an undesired changing in the failure mode from the flexural to shear is occurred. It is noticeable that drilling the rectangular opening under no service loads leads to about 48% reduction in the failure load respecting the result of the solid beam model with no opening. In the other hand, drilling equivalent circular area opening instead leads to bout only 35% reduction which indicating that the circular openings are preferable as previously stated by many other researchers. Concerning the strengthened circular and rectangular opening drilled under no service loads, minor reductions of only 11% and 12% are recorded respectively.
Respecting the (R) model result, the openings drilled while the beam models sustain service loads levels up to 26%Pu failed at 37% and 49% reduction in the ultimate capacity for the circular and rectangular openings respectively. In other words, it is realized that by drilling circular or rectangular opening under service loads level of 26%Pu, the ultimate failure load proved only slight further reductions of about 3.1% and 1.9% of the maximum load carrying capacity of the corresponding control beams models UC00 and UR00 respectively. Drilling a rectangular opening under 26%Pu changes the failure mode from a flexure to shear mode as previously stated in Tab. 2. Comparing the cases of drilling circular and rectangular openings under a service loads level of 40%Pu to those drilled under only 26%Pu resulting in maximum loading capacity reductions of 9.5% and 5.9% respectively. It is realized that drilling a rectangular opening under 40%Pu changes the failure mode from a flexure to shear mode typical as those drilled under a service loads level of 55%Pu as previously stated in Tab. 2.
The failure loads of the models with circular and rectangular opening (UC55 and UR55) are equal to 56% and 47% of that of the solid (R) model respectively. In other words, drilling the opening at a high service loads level such as 55%Pu has a serious effect on the failure of the beam models. This indicates that the circular opening proved relatively more satisfied structural behavior response than the rectangular one which is believed to be owing to the less stress concentrations than that generated diagonally at the ends of the rectangular opening chord in the direction of shear cracks propagation which are not occurred by the similar manner in the case of the circular opening models.
Also, regardless the service loads level, Fig. 25 declares that by drilling either the rectangular or the equivalent area circular opening within RC beams shear zones and strengthening them, a serious improvement in the maximum load carrying

capacity is achieved compared to the un-strengthened cases. Consequently, a desired changing in the failure mode from shear to the flexure is gained for low service loads level cases up to 40%Pu. In general, relative more strengthening efficiency is gained for the rectangular opening than circular opening cases due to the original preferable structural performance of the opening circular shape from stress concentrations point of view. Drilling the circular or rectangular opening under a service loads level up to 26%Pu and strengthening around them externally by means of CFRP sheets (SC26 and SR26) force the ultimate failure load to be 0.88 and 0.87 times the ultimate load of the solid model with no opening respectively. This proved a great efficiency of increasing the ultimate loading capacity of such RC beams with opening in addition to changing the failure mode from the undesired shear to the desired flexure as stated before in Tab. 2. The failure loads of the models SC26 and SR26 are improved by about 1.4 and 1.7 times those for un-strengthened beams UC26 and UR26 respectively. This proved the relatively more strengthening efficiency for rectangular opening model than the circular one. In other words, drilling the circular or rectangular opening within RC beam shear zone under a service loads level of 26%Pu caused reductions of the ultimate load capacity of 37% and 49% while by opening CFRP strengthening performance, these reductions percentages are satisfactory and reduced to be only of 12% and 13% respectively. Drilling the circular or rectangular opening under a service loads level up to 40%Pu and strengthening it externally by means of CFRP sheets force the ultimate failure load to be 0.87 and 0.85 times Pu of (R) model respectively. This indicated a great efficiency of increasing the ultimate load carrying capacity of the RC beams with opening in addition to changing the failure mode from the undesired shear to the desired flexure as stated before in Tab. 2. The failure loads of models SC40 and SR40 are improved by about 1.43 and 1.52 times those of un-strengthened ones UC40 and UR40 respectively. Also, this indicates the relatively more strengthening efficiency of the rectangular opening than the circular one. In other words, drilling the circular or rectangular opening within RC beams shear zone under a service loads level of 40%Pu caused reductions of the ultimate load carrying capacity by about 43% and 52% respectively while by the opening CFRP strengthening performance, these reductions percentages are satisfactory and reduced to be only 13% and 15% respectively.



* **Note:** *Creating opening under service load of 55% of reference beam ultimate load (P_u) where beam model is theoritically in a full collapsing state !!.*

Figure 25: Influence of the opening drilling and strengthening under various service loads on the models ultimate (failure) capacity.

Drilling the circular or rectangular opening under a service loads level up to 55%Pu and performing the strengthening by means of CFRP sheets force the ultimate failure load to be 0.81 and 0.73 times the ultimate failure capacity of the solid model without opening respectively. These results indicate an acceptable efficiency of increasing the ultimate load carrying capacity of the RC beams with opening under this high service loads level although the undesired shear failure mode of the un-strengthened opening case remain the mode of failure after the opening strengthening application as stated before in Tab. 2.

Fortunately, performing the circular or rectangular opening CFRP strengthening improved the ultimate failure capacity by about 1.44 and 1.55 times those of un-strengthened cases respectively although the numerically decided full collapsing state that noticed for the un-strengthened rectangular opening model UR55 under a service loads level of 55%Pu. In other words, drilling the circular or rectangular opening within RC beams shear zones under 55%Pu service loads caused reductions in the ultimate load carrying capacity of about 44% and 53% respectively while by performing the opening CFRP strengthening, these reduction percentages are reduced to be only 19% and 27% respectively which are too enough to be considered as a very exciting strengthening gained result and quite acceptable at this critical service loads state from the authors point of view.

Unfortunately, regarding the notification that previously stated at the end of the analysis of Fig. 16, beam model UR55 is numerically failed earlier in a shear mode at applied loading of only 47%Pu, i.e. under an applied load lower than 55%Pu. Also, the model with strengthened rectangular opening SR55 failed at shear under relatively lower load than those with strengthened circular opening SC55 which also failed in shear as stated before in Tab. 2. The undesired shear failure is believed to be owing to the partial frustration influence of the CFRP sheets opening strengthening on the stresses concentration occurrence especially at the rectangular opening corners. Regardless the notification of UR55 model earlier failure, drilling a rectangular or circular opening within the shear zones of RC beams under a service loads level of 55%Pu and strengthening them by means of CFRP sheets enhances the RC beams ultimate load carrying capacity and somewhat prevents the occurrence of the sudden or immediately undesired failure at the earlier stages of the service loads even though the failure mode is shear.

Generally, the ultimate failure bearing capacities are only of 49.5% and 60.3%Pu in average for the models cases with un-strengthened rectangular and circular opening respectively when the service loads level percentage increased to be up to 55%Pu of reference solid beam (R). These restoration percentages are increased to a somewhat enough satisfactory degree and are founded to be 83.5% and 86.3% in average for the strengthened rectangular and circular opening respectively. Again, it is worth to mention that strengthening the numerically decided failed un-strengthened rectangular opening beam model UR55 under a service loads level of 55%Pu increased its failure capacity to be 73%Pu as shown in Fig. 25 in spite of the opening CFRP strengthening is assumed to be applied where the RC beam is considered in the full collapsing state which indicates the serious gained retrofitting efficiency from the failure capacity point of view.

CONCLUSIONS

The current research paper deals with the RC beams numerical modeling and the CFRP strengthening technique if it is urgently needed to drill an opening within their shear zones under the service loads application. The results are peer analyzed and discussed as stated before. Then the main gained important conclusions and recommendations are stated as follows:-

1- Strengthening the openings within RC beams shear zone by means of CFRP sheets under no service loads is seriously increased the mid-span maximum deflection at failure unlike un-strengthened cases regardless the opening shape which enhances the desired or "what known as" ductile structural behavior of the RC beams.

2- The opening drilled without any strengthening action regardless the applied service loads level forces the corresponding RC beam towards an instant rapid local shear failure within the opening region before the RC beam bottom fibers reached the maximum defined characteristic strain and consequently maximum generated stress. So, the opening strengthening action is seriously recommended anyhow.

3- Un-strengthening and strengthening the drilled circular openings within the RC beam shear zone by means of CFRP sheets exhibited high load carrying capacities more than the beams with un-strengthening and strengthening rectangular openings regardless the applied service loads level.

4- The ultimate failure capacity are only of 0.55 and 0.68 times that of the solid RC beam in average for the beams with un-strengthened rectangular and circular opening respectively where the service loads level is assumed to be 55%Pu of the solid beam.

5- The CFRP strengthening of the circular or rectangular opening within RC beams shear zones under a service loads level up to only 40%Pu changes the failure mode from an undesired shear to the desired flexural failure mode accompanied with a satisfactory increasing of the beams ultimate load carrying capacity to be about 0.875 times in average that of the solid beam without opening.

6- Strengthening the openings by means of CFRP sheets has very desired influence on improving the failure capacities of all studied cases regardless the service loads level. Unlike un-strengthened openings, the service loads level up to

40%Pu relatively have unnoticeable influence on the bearing capacities of the RC beams with strengthened opening regardless the shape of the opening.

7- Retrofitting (even what numerically considered as collapsed beam) RC beams with rectangular opening within the shear zone under higher service loads levels may enhances their ultimate bearing capacities to be about three quarters (73%) and to be more than four fifths (83%) of the capacity of the solid RC beams without openings and the beams with strengthened rectangular opening under no service loads respectively which proved a serious importance of performing the opening strengthening anyhow as possible as could.

8- Finally, regardless its shape and strengthening state, it is not recommended to drill any opening at all within the shear zones of the RC beams suffering a service loads level more than 40% of the design ultimate loading capacity. From the practical requirements point of view, if it is no way to avoid drilling a rectangular opening within the shear zone of a simply supported RC beam under higher service loads conditions more than 40%Pu, the opening must be strengthened immediately by means of the CFRP sheets anyhow before releasing the temporary supports.

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NOMENCLATURE

β_c : Shear transfer coefficient value for closed crack; ranges from 0.50 to 0.90.
β_t : Shear transfer coefficient value for open crack; ranges from 0.05 to 0.50.
Δ : Deflection at the beam model central section (i.e.: at the beam mid-span).
Δ_R : Deflection at the reference beam model central section (i.e.: at the solid beam mid-span).
ε : Strain.
ε_o : Strain value at the ultimate compressive strength of concrete.
ε_{cr} : Maximum tensile strain of concrete (uniaxial cracking strain).
μ : Poisson's ratio.
E : Bending modulus of elasticity.
G : Shear modulus of elasticity.
f : Stress for any value of strain.
f_c : Ultimate compressive strength of concrete.
f_{cu} : Concrete characteristic ultimate cube compressive strength.
f_t : Concrete tensile strength (concrete uniaxial cracking strength).
P : Service loads level as a percentage of Pu.
P_{cr} : Initial cracking load.
P_f : Failure load.
$(P_f)_R$: Failure load of the reference beam model [$(P_f)_R$=Pu].
Pu : Ultimate capacity of the reference beam model.
R : Reference (solid) beam model without opening.
SC : Strengthened circular opening beam model.
SC00 : Strengthened circular opening beam model at no service load.
SC26 : Strengthened circular opening beam model at a service loads level of about 26%Pu.
SC40 : Strengthened circular opening beam model at a service loads level of about 40%Pu.
SC55 : Strengthened circular opening beam model at a service loads level of about 55%Pu.
SR : Strengthened rectangular opening beam model.
SR00 : Strengthened rectangular opening beam model at no service load.
SR26 : Strengthened rectangular opening beam model at a service loads level of about 26%Pu.
SR40 : Strengthened rectangular opening beam model at a service loads level of about 40%Pu.
SR55 : Strengthened rectangular opening beam model at a service loads level of about 55%Pu.
UC : Un-strengthened circular opening beam model.
UC00 : Un-strengthened circular opening beam model at no service load.
UC26 : Un-strengthened circular opening beam model at a service loads level of about 26%Pu.
UC40 : Un-strengthened circular opening beam model at a service loads level of about 40%Pu.
UC55 : Un-strengthened circular opening beam model at a service loads level of about 55%Pu.
UR : Un-strengthened rectangular opening beam model.
UR00 : Un-strengthened rectangular opening beam model at no service load.
UR26 : Un-strengthened rectangular opening beam model at a service loads level of about 26%Pu.
UR40 : Un-strengthened rectangular opening beam model at a service loads level of about 40%Pu.
UR55 : Un-strengthened rectangular opening beam model at a service loads level of about 55%Pu.