

Focussed on characterization of crack tip fields

# Fretting in complete contacts - the use of Williams' solution

D. A. Hills, R. Flicek University of Oxford, UK david.bills@eng.ox.ac.uk, robert.flicek@eng.ox.ac.uk

**ABSTRACT.** Williams' solution may be used to characterize the corners of 'complete' fretting contacts. Here we look at the practicalities of conducting fretting tests using a range of different kinds of apparatus and the kind of results they can reveal of practical relevance to establishing a fretting database.

**KEYWORDS.** Fretting, complete-contacts; Williams' solution.

## INTRODUCTION

t the Second IJFatigue & FFEMS Joint Workshop held in 2013, in Malaga (Spain), we developed quite extensively a framework for understanding the fretting fatigue of complete contacts, using Williams' solution to characterize the local behaviour [1]. This can sometimes be quite a complicated process, as the stress intensity contributions from mode I and mode II loading may change sign, giving rise to local separation, and there are the competing non-linear local effects of slip (giving rise to fretting damage) and plasticity (giving rise to crack nucleation) [2,3]. For background material, these two papers should be consulted. They give, first, detailed consideration to length scales; all notch asymptotes subject to combined-mode loading have an *internal* length scale, even if the notch is semi-infinite extent. This is the quantity we have designated  $d_0$ , and is defined as

$$d_0 = \left| \frac{K_{II}}{K_I} \right|^{\frac{1}{\lambda_I - \lambda_{II}}} \tag{1}$$

where  $K_{I}$ ,  $K_{II}$  are the mode I, mode II generalized stress intensity factors respectively, and  $\lambda_I$ ,  $\lambda_{II}$  are the corresponding Williams' exponents [1]. Observation points nearer to the contact corner than  $d_0$  are mode I dominated, whilst those further away are either mode II dominated or may be subject to the influence of higher order terms in the series (in which case the field is not 'small scale' in character). There are two non-linear local effects, both forms of slip. First, there is the process zone within which non-linear effects give rise to the accumulation of damage, which is wholly analogous to the process zone at the root of a sharp V-notch. Secondly, there is the possibility of frictional slip (fretting itself), which clearly causes at least some local damage including surface finish degradation. If the process zone envelopes the frictional slip extent, we would say that the contact corner is 'notch-like' (and this remark applies *a fortiori* if the contact corner is actually adhered). In contrast, if the slip zone envelopes the process zone, we would argue that fretting damage is likely to be very important in controlling the strength of the contact corner.

In this paper, we want to take a more practical look at what these observations tell us about the properties of various fretting tests which are widely used, and what the range of test parameters should be so as to obtain the maximum useful output from the tests. It should be recalled that the notch-root generalized stress intensity factors act as a 'filter', and that, whatever the actual loading in the experiment, the locally experienced stresses can 'see' only the  $K_{lb}K_{ll}$  fields. Also, different types of tests probe different regions of  $K_{lb}$ ,  $K_{ll}$  space. So, in general terms, normal loading drives the state of



stress into the  $(K_I < 0, K_{II} > 0)$  quadrant, implying, as intuitively expected, firm closure of the contact corner, whereas the application of a shear force or tension drives the state of stress into either the  $(K_I > 0, K_{II} > 0)$  or  $(K_I < 0, K_{II} < 0)$  quadrants. These imply some degree of local opening [2, 3].

## CHOICE OF TEST

If of the early 'complete' fretting tests conducted at Oxford were carried out on a modified version of the group's general purpose fretting-fatigue apparatus, originally designed to conduct 'incomplete' (mainly Hertzian) fretting tests, which is shown in Fig. 1. One of the features which was thought to be very desirable was to use pad holders that could swivel, so intimate contact was made between the pad ends and the dogbone specimen faces. At the same time, the geometry was carefully arranged so as to attempt to ensure that the applied shear force was imposed strictly through the plane of the interface to ensure that there was no moment present tending to 'tip' the pads. Very large numbers of tests were carried out (over one hundred) on super-CMV steel, but the spread of results obtained was disappointing [4].



Figure 1: Fretting-fatigue test rig at the University of Oxford for studying both incomplete and complete contacts [2].

One difficulty was that the usual increase in the coefficient of friction experienced during the first few cycles of loading typically increased the friction coefficient from about 0.4 to 0.7. In the case of Hertzian contacts, this simply has the result of reducing the width of the slip zones, and this happens in a smooth manner. But, in the case of sharp-edged contacts, it can make the difference between edge slip and a fully adhered contact, so the steady state problem is little more than a notch incorporating an interface. The interface, including asperities, is still likely to be a source of weakness, so the fatigue strength of the pair will be lower than that of a monolithic component incorporating a notch. A further feature of the contact pair is that, because all surfaces are machined prior to assembly, the contact corner is truly sharp, whereas, in the case of a notch, an internal corner is present which, no matter how careful the machining process, will always have some finite radius.

A principal advantage of the pad-on-dogbone arrangement is that it is possible to control three separate external loads – the normal force, P, which is normally held constant; the shear force, Q; and the bulk stress, T, which are both controlled by servo-hydraulic actuators as shown in Fig. 2. All three components of load excite both mode I and mode II loading, and the first really thorough derivation of the contact corner stress intensity factors was carried out by Flicek [5], with the following results

![](_page_2_Picture_0.jpeg)

	$K_I^A a^{\lambda_I - 1}$		-0.1537	0.1223	-0.0609	0.0751	$\left( P / a \right)$		
	$K_{II}^{A}a^{\lambda_{I}-1}$	}=	0.1280	0.1610 -0.1109	0.1562 $Q_1 / a$	$Q_1 / a$			
Ì	$K_I^B a^{\lambda_I - 1}$		-0.1534	-0.2116	0.2731	0.0752	$\left[ Q_2 / a \right]$	(2)	(2)
	$K_{II}^{B}a^{\lambda_{I}-1}$			0.4236	-0.4737	-0.1564	$\left[ T/a \right]$		

where  $K_{I^A}$ ,  $K_{II^A}$  correspond to the left corner in Fig. 2, and  $K_{I^B}$ ,  $K_{II^B}$  correspond to the right corner. By careful choice of the mix of normal, bulk, and shear loads, a wide range of  $K_I$ ,  $K_{II}$  space may be probed by this experiment.

![](_page_2_Figure_4.jpeg)

Figure 2: Idealised diagram of the pad-on-dogbone test rig [5].

#### **CANTILEVER SPECIMEN**

wholly different kind of very simple test was conducted by Juoksukangas et. al. [6]. This employs a simple cantilever strip, clamped between blocks, subject to a cyclically imposed tip deflexion. Juoksukangas et. al. obtained a very good set of results using this apparatus, and a smooth 'S-N' type curve was plotted. In their original paper, these authors used the notional stress at the root of the cantilever implied by elementary bending theory. It is, however, a simple step to obtain the calibration for the cantilever-root stress intensity factors. This is an important step because it means that the quantity being employed now represents the true state of stress at the critical point (here the cantilever root edges). Thus, what is being revealed is a true material property, which may applied to any other geometry where there is an internal right angle, with possible attendant interface slip, at the critical point. A limitation of the technique is that there are only two loads – the clamping force, which is kept constant, and the cantilever tip deflexion, which provides the oscillatory component of load. Also, it is assumed that the static load has only a secondary effect on the fatigue strength, and it is the oscillatory load which controls the contact strength. We obtained calibrations for the contact corner stress intensities in a discussion paper [7], and these were used to re-plot the experimental results found on a plot of  $\angle K_I$  versus  $N_I$  (number of cycles to macroscopic crack initiation), which is reproduced in Fig. 3. This figure may be thought of as displaying a definitive material property curve, with a fatigue limit,  $K_I^d$ , which might be adopted to solve any fretting-fatigue strength problem involving sharp-edged contacts.

![](_page_2_Figure_8.jpeg)

![](_page_2_Figure_9.jpeg)

![](_page_3_Picture_1.jpeg)

## BRIDGE TYPE SPECIMENS

ong before our understanding of partial slip contact problems had developed to the point it has now, and before clear separation of the nucleation and propagation phases of crack growth were considered, materials scientists were conducting fretting tests in which 'bridges' were clamped to either rotating-bending or simple tension dogbone specimens. The oscillating values of surface strain meant that shear forces were developed beneath the feet of the bridges, and this usually produced some slip, and hence fretting damage. The principal advantages of the arrangement were: (i) the fretting components were relatively easy to make, and (ii) particularly in the case of rotating-bending, the assembly was maintained in balance, so it could be operated at high speed, and the number of cycles could be clocked up very quickly. On the other hand, although the tests are easy and quick to conduct, their interpretation is correspondingly very difficult. There have been a number of attempts to analyse the bridge-foot contact problem, notably by Hattori and Nakazawa using a boundary element method [8,9]. Recently, Noraphaiphipaksa et. al. [10] have made a very good analysis of bridge-style contact pads clamped to a dogbone specimen subject to oscillatory tension, which is shown in Fig. 4.

![](_page_3_Figure_4.jpeg)

Figure 4: Idealised diagram of one quarter of the bridge-style test rig [11].

This test rig has the advantages of: (i) two-way symmetry, and (ii) all deformation occurs in the plane of analysis, which therefore is strictly two-dimensional in nature. The nature of the contact is such that the application of a bulk tension produces a shear force on each 'foot' of the bridge. This enables a single actuator servo-hydraulic test machine to be used to conduct fretting-fatigue tests. Noraphaiphipaksa et al.'s analysis is extremely revealing and shows that at the loads which were typically employed, sufficient twisting of the feet occurred for there to be significant receding of the contact as the outside of the foot separated, which significantly complicates interpretation of the results. This is because during part of the load cycle the contact of the foot is 'complete' with an attendant singular contact pressure, and during part of the loading cycle the contact has shrunk and is incomplete in nature, so there will certainly be at least some partial slip occurring. We re-analysed Noraphaiphipaksa et al.'s results in a discussion paper [11] and fitted generalized stress intensity factors to the contact edge solutions. The calibrations found are given here

$$\begin{cases} K_{I}^{\mathcal{A}} a^{\lambda_{I}-1} \\ K_{II}^{\mathcal{B}} a^{\lambda_{I}-1} \\ K_{II}^{\mathcal{B}} a^{\lambda_{I}-1} \end{cases} = \begin{bmatrix} -0.1693 & -0.1669 \\ 0.3656 & 0.5092 \\ -0.5617 & 0.6163 \\ -0.3034 & -0.2770 \end{bmatrix} \begin{cases} p_{0} \\ \sigma_{0} \end{cases}$$
(3)

where  $K_{I}^{A}$ ,  $K_{II}^{A}$  correspond to the left corner in Fig. 3, and  $K_{I}^{B}$ ,  $K_{II}^{B}$  correspond to the right corner. We are then in a position to show very easily the conditions under which the feet remain in intimate (complete) contact and when they recede (and become incomplete), and this information is shown in Fig. 5. As will be clear, the situation is quite different for the 'inside' and the 'outside' of the feet.

It is clear that in the tests which Mutoh carried out, a very complicated load cycle existed, and the loads would have to be 'backed off' considerably in order for his investigation to probe strictly 'complete' contacts. Equally, the philosophy developed here shows that it is perfectly possible to devise forms of the 'bridge' which do maintain intimate contact up to higher loads, and this would seem to be a viable way to go, but with a more restrictive probing of  $K_{l_i}K_{l_i}$  space than is possible using the two-actuator and single-pad arrangement.

![](_page_4_Figure_2.jpeg)

Figure 5: Diagram of  $K_{II}$  versus  $K_{I}$  showing the implications of Williams' solution for contact behaviour [11].

#### SHRINK FIT PROBLEMS

recent set of experiments intended to investigate the fretting-fatigue strength of shrink-fit joints is being developed by Bertini et. al. [12]. These tests are still at an early stage of refinement but show promise.

## SUMMARY

ur ambition is to produce a viable laboratory technique which reveals the fretting-fatigue strength of materials in a laboratory test, and which may be applied to a wide range of practical problems. At the outset, a distinction should be made between incomplete contacts and complete contacts. For the former of these, the relevant asymptotic forms take a particular form (square-root bounded), and the tremendous amount of data generated in many laboratories throughout the world has been very satisfactorily correlated [13]. In contrast, complete contacts are much more challenging to characterise, and an important starting point is to think of them not as contacts between two components brought together, but as a monolithic entity which may be split, even separated, along small parts of the contact, especially the edges. The domain in which the analysis is carried out is therefore very different, and the problems are notch-like. Indeed, in some cases, when the edge is adhered, they are virtually indistinguishable from a sharp notch. Nevertheless, the presence of slip will have some influence on the propensity of the contact edge to start a crack. We do not attempt to speculate what this might precisely be, but instead rely on a laboratory experiment to determine the contact strength. Here, we have reviewed the relatively modest amount of data available, together with its sources.

#### **ACKNOWLEDGEMENTS**

.C. Flicek would like to thank Rolls-Royce plc and the Technology Strategy Board for financial support under the programme SILOET-II. The authors wish to thank Rolls-Royce plc for granting permission to publish this work.

### REFERENCES

[1] Hills, D.A., Flicek, R.C., Dini, D., Sharp contact corners, fretting and cracks. Frattura ed Integritá Strutturale (Fracture and Structural Integrity), 25 (2013) 27-35. doi:10.3221/igf-esis.25.05

![](_page_5_Picture_1.jpeg)

- [2] Flicek, R., Hills, D.A., Dini, D., Progress in the application of notch asymptotics to the understanding of complete contacts subject to fretting fatigue. Fract. Eng. M., 36 (2013) 56-64. doi:10.1111/j.1460-2695.2012.01694.x
- [3] Flicek, R.C., Hills, D.A. Dini, D., Sharp edged contacts subject to fretting: A description of corner behaviour. Int. J. Fatigue, 71 (2015) 26-34. doi:10.1016/j.ijfatigue.2014.02.015
- [4] Mugadu, A.E.B., Studies in Fretting Fatigue of Complete Contacts. University of Oxford D.Phil. dissertation, Trinity Term 2002.
- [5] Flicek, R.C. Analysis of Complete Contacts Subject to Fatigue. University of Oxford D.Phil. dissertation, Michaelmas Term 2014.
- [6] Juoksukangas, J., Lehtovaara, A., Mäntylä, A., Development of a complete contact fretting test device. P. I. Mech. Eng. J-J. Eng., 227 (2013) 570-578. doi:10.1177/1350650112466162
- [7] Hills, D.A., Flicek, R.C., Dini, D., A discussion of: Development of a complete contact fretting test device by J Juoksukangas et al. P. I. Mech. Eng. J-J. Eng., 228 (2014) 123-126. doi:10.1177/1350650113500946
- [8] Hattori, T., Nakamura, M., Ishizuka, T., Fretting Fatigue Analysis of strength improvement models with grooving or knurling on a contact surface. In Attaia, M. H. and Waterhouse, R. B. (Eds.) Standardization of Fretting Fatigue Test Methods and Equipment, ASTM STP 1159. Race Street, Philadelphia. ASTM (1992) 101-114
- [9] Nakazawa, K., Sumita, M., Maruyama, N. Effect of contact pressure on fretting fatigue of high strength steel and titanium alloy. In Attaia, M. H. and Waterhouse, R. B. (eds) Standardization of Fretting Fatigue Test Methods and Equipment, ASTM STP 1159. Race Street, Philadelphia. ASTM (1992) 115-125
- [10] Noraphaiphipaksa, N., Kanchanomai, C., Mutoh, Y., Numerical and experimental investigations on fretting fatigue: Relative slip, crack path, and fatigue life. Eng. Fract. Mech., 112 (2013) 58-71. doi:10.1016/j.engfracmech.2013.10.007
- [11] Hills, D.A., Flicek, R.C., A discussion of "Numerical and experimental investigations on fretting fatigue: Relative slip, crack path, and fatigue life" by N. Noraphaiphipaksa, C. Kanchanomai, and Y. Mutoh. Eng. Fract. Mech., 113 (2015) 52-55. doi:10.1016/j.engfracmech.2014.10.024
- [12] Bertini, L., Beghini, M., Santus, C., Baryshnikov, A., Resonant test rigs for fatigue full scale testing of oil drill string connections. Int. J. Fatigue, 30 (2008) 978-988. doi:10.1016/j.ijfatigue.2007.08.013
- [13] Hills, D.A, Thaitirarot, A., Barber, J.R., Dini, D., Correlation of fretting fatigue experimental results using an asymptotic approach, Int J. Fatigue 43 (2012) 62-75. doi:10.1016/j.ijfatigue.2012.02.006.