Validated Finite Element Modeling of Lightweight Concrete Floors Stiffened and Strengthened with FRP

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Received: 22 May 2023 | Revised: 9 June 2023 | Accepted: 16 June 2023

 DOI: https://doi.org/10.48084/etasr.6055

ABSTRACT

The main challenge in designing Light-Weight Concrete (LWC) is to adapt most of the design, production, and execution rules from normal-weight concrete. Carbon Fiber-Reinforced Polymer (CFRP) composites provide strength and stiffness to the composite system. This study investigated the stiffness of an LWC flat slab with CFRP when subjected to human-induced vibration. This was determined by finding the natural frequency of the slab and comparing it with the acceleration limit ratio (human perception of vibration) of 0.5% g. In most cases, vibration characteristics are examined using commercial software based on Finite Element Analysis (FEA) methods that are powerful tools, but the user needs to understand the underlying assumptions and methods implemented, especially for reinforced concrete floor systems where inherent attributes, such as cracking, play an important role in the determination of vibration characteristics. This study used Abaqus CAE. The main idea of this study was that such software cannot detect the behavior of cracks in structures over the years and the effect on frequencies, as stiffness depends on the modulus of elasticity and not on the moment of inertia. Therefore, the natural frequency equation has a component that constantly accounts for the level of cracking on concrete slabs. This component was theoretically determined with detailed calculations that are not provided in the Design Guide for Vibrations of Reinforced Concrete Floor Systems. Then, the constant that accounts for the level of cracking k_1 was multiplied by the modulus of elasticity E and substituted in the latter's place in Abaqus to ensure the right behavior of the slab with and without CFRP. This study also investigated the properties of CFRP and how to represent it in the Abaqus. The numerical results showed good agreement with FEA and the acceptance criteria for walking excitations increased when using CFRP on a floor system.

Keywords-light weight concrete; validation; FRP; floor stiffness

I. INTRODUCTION

Since the 1990s, engineers and researchers have been aware of the advantages of combining concrete with FRP materials, as concrete aids in compressive resistance and stability and Fiber Reinforced Polymers (FRPs) provide tensile resistance [1]. This study considered a slab with and without FRP stiffening. A preliminary Finite Element Model (FEM) was prepared for the slab considered but with normal-weight concrete in the Design Guide for Vibrations of Reinforced Concrete Floor Systems [2]. This model was validated by comparing its results with the semi-empirical results of [2]. Subsequently, the validated model was adopted to show how Light-Weight Concrete (LWC) and CFRP affect the vibrational aspects of the slab. There are many advantages to having low density, as it helps reduce dead load, increases building progress, and reduces transportation and handling costs [3]. Also, by reducing the cross sections of the components, there is an increase in the demand for LWCs in many modern architectural constructions [4]. Figure 1 shows the flat plate reinforced concrete system considered in this study. For validation purposes, the indicated dimensions were the metric equivalent of the corresponding customary United States dimensions in [2].

The Design Guide for Vibrations of Reinforced Concrete Floor Systems was written with two objectives. It aimed to support structural engineers in selecting a suitable reinforced concrete floor system subjected to human-induced vibration and to present simplified approaches on estimating the vibration characteristics of reinforced concrete floor systems that can be used to evaluate whether the anticipated vibration is acceptable or not, by computing the natural frequency and comparing it with the ratio aspects.

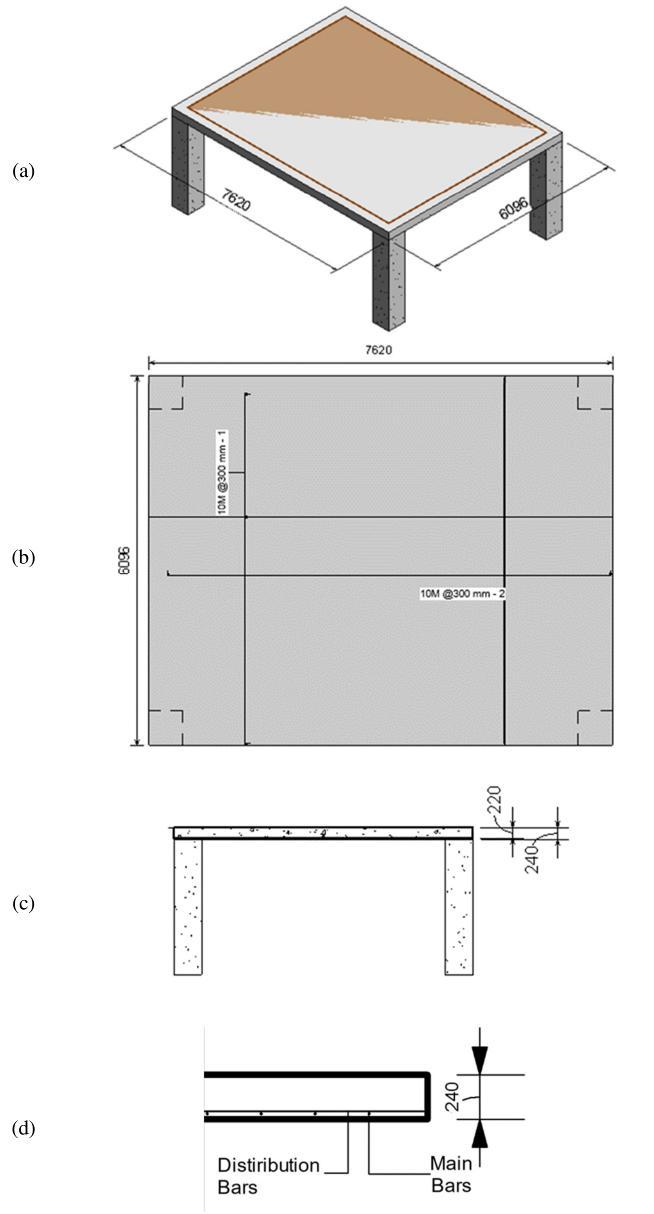

Fig. 1. Flat plate system details: (a) 3D view, (b) top view, (c) longitudinal section, (d) cross section.

The peak acceleration as a fraction of the acceleration of gravity a_p/g is less than or equal to the acceleration limit a_0/g for the appropriate occupancy, as shown in (1) and (2), respectively. The natural frequency is the quantity of the floor system's response to the sources that can cause vibration and is related to how occupants will perceive such vibrations. Many methods and resources are available to determine this property.

$$f_i = \frac{k_2\lambda_i^2}{2\pi l_1^2}\left[\frac{k_1 E_c h^3}{12\gamma(1-v^2)}\right]^{1/2} \quad (1)$$

$$\frac{a_p}{g} = \frac{65e^{(-0.35fn)}}{\beta w} \leq \frac{a_\circ}{g} \quad (2)$$

Service bending moments were taken from Table V from the Guide of normal weight concrete [2].

A. *Finite Element Analysis*

FEM is useful in providing the overall performance of structurally reinforced concrete. It also provides a reasonable prediction of the loading capacity and crack formation, but realistic fracture conditions, such as crack patterns, are difficult to obtain [5]. The three-dimensional Finite Element Analysis (FEA) of the flat plate system can be achieved using Abaqus [6], as it is one of the leading finite element packages, notwithstanding the quality of the pre and postprocessing capabilities. Abaqus consists of Abaqus Standard and Abaqus Explicit for dynamic analysis. Abaqus CAE (Complete Abaqus Environment) is used for data input for FEA and allows monitoring and viewing of the results through an input file prepared with a text editor.

B. *Walking-Induced Vibration of Flat Plate System*

There are important requirements in designing building structures related to people's sensitivity to a vibration that is the comfort of building occupants and safe building conditions. The dynamic response of a building loaded with dynamic loads requires finding the response. One of the responses is that human activities such as walking or jumping can cause strength much greater than their body weight [7].

C. *Upgrading the Flexural Capacity of Slabs*

The main cause of damage is that the flexural capacity cannot meet the demand. Fiber-reinforced plastics (sheets or laminates) are externally attached to the slab at the top or bottom surface (positive or negative bending moment demand). Such a scheme was used to upgrade the seismic capacity of the slabs of a four-story R/C building built in 1933, subjected to the 1995 earthquake in Kozani, Greece [8]. Vibration measurement has been proven to be promising in Structural Health Monitoring (SHM). This scheme is used to study the behavior of structures, such as vibrations and damage, by evaluating the natural frequency and other dynamic parameters [7] using the properties of the model of Table VI after converting the units to N/mm and assuming the length of the column to be 3 m, as shown in Figure 2.

II. CARBON FIBER REINFORCEMENT POLYMER

A. *Overview*

Carbon fibers, also called graphite fibers, are lightweight and strong fibers with excellent chemical resistance. They dominate the aerospace market and are considered an orthotropic material [9]. CFRP composites are extremely stiff and brittle and susceptible to galvanic corrosion. Their use in field applications has grown since the late 1980s due to their low cost and importance in strengthening improperly or insufficiently designed structural elements, especially in seismic areas [10]. The use of CFRP strengthening techniques improves the stiffness and flexural strength of RC members and reduces crack spacing and crack width [11].

B. *Orthotropic Material*

An orthotropic material has three planes of symmetry, as shown in Figure 2, which coincide with the coordinate planes.

A unidirectional fiber-reinforced composite can be considered orthotropic. One plane of symmetry is perpendicular to the fiber direction, and the other two can be any pair of planes parallel to the fiber direction and orthogonal among themselves. Only nine constants are required to describe an orthotropic material: E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, v_{12}, v_{13}, and v_{23} [9].

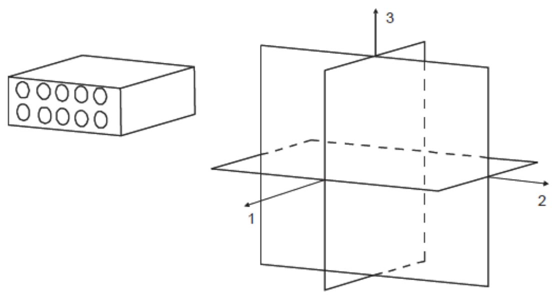

Fig. 2. Orthotropic material.

C. Mechanical Properties

The modulus of elasticity value of carbon fibers is approximately 240 GPa [8]. Based on modulus and strength, carbon fibers can be divided into [8]:

- Ultra-High-Modulus (UHM) for > 450 GPa
- High-Modulus (HM) between 350-450 GPa
- Intermediate-Modulus (IM) between 200-350 GPa
- Super High-Tensile (SHT) for tensile strength > 4.5 Gpa

In the mechanics of materials, both fibers and matrix are assumed to be isotropic. Isotropic material stiffness is completely represented by two properties: modulus of elasticity and Poisson's ratio. Using micromechanics, the combination of two isotropic materials, fiber and matrix, is represented as an equivalent, homogeneous, and anisotropic material [9]. The stiffness of the equivalent material is represented by five elastic properties: E_1 is the modulus of elasticity in the fiber direction, E_2 is the modulus of elasticity in the direction transverse to the fibers, G_{12} is the in-plane shear modulus, G_{23} is the out-of-plane shear modulus, and v_{12} is the in-plane Poisson's ratio.

TABLE I. MATERIAL PROPERTIES OF THE LAMINA [13].

E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	v_{12}	v_{13}	v_{23}
131.9	9.51	9.43	5.27	7.03	3.39	0.326	0.341	0.485

In general, all types of FRP exhibit high tensile strength and low density and are non-corrosive [12]. CFRP composites are very durable, have excellent fatigue properties, and can withstand most environmental conditions. Many challenges prevent the additional growth of the FRP market. Some of these challenges are the brittle failure of FRP-strengthened RC structures due to sudden failure modes such as FRP rupture or debonding, deterioration of the mechanical properties due to harsh environmental conditions such as wet-dry cycles and freeze-thaw conditions, reduction in strength due to the effects of improper installation procedures, and lack of agreement among debonding behavior and bond length models [10].

III. VALUES IN ABAQUS

A. Part Module

Three main parts were defined for the concrete, longitudinal rebars, and CFRP: solid extrusion, wire planar, and shell planer, respectively.

B. Property Module

Three sections were defined, where solid and homogeneous were defined and assigned to the concrete mass and two truss sections were defined and assigned to the longitudinal rebars and stirrups. For concrete, the values of Young's modulus and Poisson's ratio of 6713.8 and 0.2 were inserted, respectively. The E with the effect of crack was:

$$Ec = k_1 \times \rho c^{1.5} \times 0.043\sqrt{fc} = 0.54 \times 1440^{1.5} \times 0.043 \times \sqrt{28}$$
$$= 6713.8\ MPa$$

Table II shows the values inserted for CFRP.

TABLE II. VALUES INSERTED FOR CFRP

E_1	E_2	Nu_{12}	G_{12}	G_{13}	G_{23}
170000	9000	0.34	4800	4800	4500

IV. STEEL REINFORCEMENT OF SLAB

The maximum spacing of the main bar of the slab must be at least 300 mm or three times the effective depth, and the maximum spacing of the distribution bar of the slab must be at least 450 mm or five times the effective depth. Meanwhile:

$$Effective\ Depth = 150 - 20(cover) - \left(\frac{10}{2}\right)(rein.) = 125mm$$

Therefore, the spacing used was 300 mm in two ways.

V. DYNAMIC LOADS GENERATED BY HUMAN ACTIVITIES

The dynamic loads generated by human activities can be grouped into two types according to the person-structure interaction. The first is when there is a loss of contact with the structure, such as running and jumping, and the second is when there is no loss of contact with the structure, such as walking. A human being produces a dynamic load during walking or running that can be represented by a force that varies in time with components in three directions: vertical, lateral, and frontal. The vertical component is the most investigated because it presents greater amplitudes compared to the others. This dynamic force is produced by the acceleration and deceleration of the body mass. For walking, its frequency range has been stated to be between 1.6 and 2.4 Hz. Although there has not been the same comprehensive investigation for running or jumping activities, the typical frequency range for running is considered 2.0 to 3.5 Hz while for jumping it is 1.8 to 3.4 Hz [1]. People are exposed daily to vibrations on floor slabs or on footbridges caused by different sources of excitation. The evaluation of human sensitivity to these vibrations involves psychological and physical aspects. Human-induced vibrations can cause serviceability problems and discomfort to users. The main factors that influence human sensitivity are position, such as standing, sitting, or lying, and the type of activity that depends on age, gender, mood, vibration frequency, displacement amplitudes, damping, and finally the acceleration of dynamic excitation [1]. The natural frequency of two-way reinforced concrete floor systems can be obtained using some fundamental concepts in plate theory. The simplifying assumptions made in the following discussion enable the natural frequency to be determined straightforwardly and consistently for all two-way systems [2].

Assume that a reinforced concrete flat plate or a voided slab system can be modeled as a rectangular, isotropic plate where the primary vertical deflection is due to flexure. Since the slab is supported only by columns, it is free to deflect at any point except at the locations of the columns, where it is generally assumed that vertical displacements are negligible and that rotations are not restricted. Equation (1) can be used to determine the natural frequency f_i, which applies to rectangular plates with corner supports.

$$f_i = \frac{k_2\lambda_i^2}{2\pi l_1^2}\left[\frac{k_1 E_c h^3}{12\gamma(1-v^2)}\right]^{1/2} \quad (1)$$

where h is the total thickness of the flat plate or voided slab, γ is the mass per unit area of the plate, v is the Poisson's ratio, l_1 is the longest of the two center-to-center span lengths of the plate panel, and λ_i^2 is a function of the panel aspect ratio l_1/l_2. Table III shows the values of λ_i^2 for the fundamental mode of vibration. In [13-15] there are values of this parameter for other modes and panel aspect ratios.

TABLE III. VALUES OF λ_i^2 FOR A CORNER-SUPPORTED, RECTANGULAR PLATE [2].

l_1/l_2	λ_i^2
1.0	7.12
1.5	8.92
2.0	9.29

The constant k_2 accounts for the effect of rigidity at the joint between the slab and the columns, which are cast monolithically in typical reinforced concrete structures. Column size has a direct impact on joint rigidity: the larger the column size, the greater the slab stiffness and fundamental frequency. Instead of a more exact analysis, the following are approximate values of k_2 that can be used in (1) and are based on column size c_1 [16]:

$$k_2 = \begin{cases} 1.9\ for\ c_1 \le\ 24\ in. \\ 2.1\ for\ c_1 >\ 24\ in \end{cases} \quad (3)$$

where the constant k_1 accounts for the level of cracking in the concrete slab and can be determined by dividing the effective moment of inertia I_e by the gross moment of inertia I_g. When k_1 is less than 1.0, the stiffness of the slab is less than the gross stiffness, and the natural frequency of the system is reduced [2]. For a flat plate, I_e for a panel is approximated by adding the average I_e of the column strip in one direction to the average I_e of the middle strip in the orthogonal direction. The average I_e for the column and middle strips is used to take into account positive and negative regions in the strips. The following equation is applicable for square panels that have both ends continuous [17]:

$$Average\ I_e =\ 0.7I_m + 0.15(I_{e1} + I_{e2}) \quad (4)$$

where I_m refers to I_e at the midspan section and I_{e1} and I_{e2} refer to I_e at the end sections of the panel [2]. For a rectangular panels system, I_e for a panel is approximated by taking the average of the average I_e values in the column strips (cs) and middle strips (ms) in both the x and y directions:

$$I_{\ e\ for\ the\ panel} = \frac{(I_{e\,cs\,x}+I_{e\,ms\,y})+(I_{e\,cs\,y}+I_{e\,ms\,x})}{2} \quad (5)$$

In (4), the effective moment of inertia I_e can be determined for sections with relatively low reinforcement ratios. The contribution of shrinkage restraint to cracking should also be considered in such systems. To account for this effect, the use of a two-thirds factor applied to M_{cr} was recommended in [18], which is equivalent to using a reduced modulus of rupture $f_r\ =\ 5.0\sqrt{f\ '_c}$, while in [19] it was proposed using $f_r\ =\ 4.0\sqrt{f\ '_c}$ to determine I_e.

$$I_{\ e}\ = \frac{I_{cr}}{1-\left(\frac{M_{cr}}{M_a}\right)^2\left[1-\frac{I_{cr}}{I_g}\right]} \le I_g \quad (6)$$

A reduced modulus of rupture $f_r\ =\ 4.5\lambda\sqrt{fc'}$ was adopted for vibration analysis in [2] for flat plate and voided slab systems.

A. Flat Plate System Walking Excitation

An existing flat floor system residential building will be converted into an office building. The vibration of the system was evaluated assuming normal occupancy of the office. Table IV shows the design data. It was also assumed that the office space will have some non-structural components [2]. Notice that w_c = 1440 kg/m^3 for LWC.

TABLE IV. DESIGN DATA [2]

Concrete	Compressive strength f'_c= 4,000 psi	f'_c= 28MPa
	Density w = 150 pcf	
Reinforcing steel	Yield strength f_y=60,000 psi	f_y = 412MPa
Loads	Superimposed dead load = 20 psf	
	Live load (design) = 65 psf (includes partition)	
	Live load (vibration) = 11 psf	
Spans	Typical bay: 25'–0"×20'-0	7620×6096mm
Members sizes	Slab thickness h_s= 9.5"	h_s = 240mm
	Columns 22"×22"	560×560mm

B. Determination of Concrete Properties

The modulus of elasticity is determined as:

$$E_c\ =\ w_c^{1.5}\ 33\sqrt{f'_c} \quad (7)$$

$$Dynamic\ E_c\ =\ 1.2E_c$$

$$Modular\ ratio\ n\ = \frac{E_s}{1.2E_c} =\ 29000000/2138400\ =\ 13.6$$

C. Determination of the Bending Moments at the Critical Sections for a Typical Interior Bay

Since the limitations of ACI 13.6.1 are satisfied, the direct design method can be used in the bending moment determination at the critical negative and positive sections in the column and middle strips along the span. The bending moments are determined by calculating the total static moment in each direction and then applying the appropriate coefficients given in ACI 13.6.3. Table V gives a summary of the service bending moments. It was assumed that the 25-foot span is in the north-south direction [2].

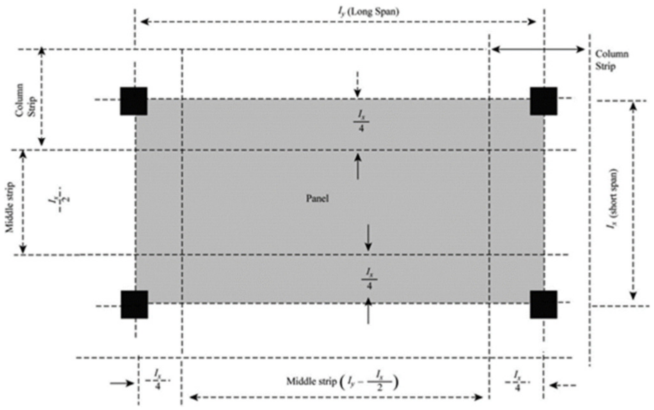

Fig. 3. Dimensions of c_s and m_s.

TABLE V. SERVICE BENDING MOMENTS (FT-KIPS) [2]

		North-South		East-West	
		Column Strip	Middle Strip	Column Strip	Middle Strip
Negative	D	91.2	29.8	70.1	22.9
	L	42.7	14.0	32.9	10.7
Positive	D	39.1	26.1	30.1	20.0
	L	18.3	12.2	14.1	9.4

D. Determination of the Required Flexural Reinforcement

Table VI shows the required flexural reinforcement. The provided area of steel is set equal to the minimum area of steel where required and the sections in all cases are tension controlled [2].

TABLE VI. REQUIRED FLEXURAL REINFORCEMENT (in^2) [2]

	North-South		East-West	
	Column Strip	Middle Strip	Column Strip	Middle Strip
Negative	5.39	2.05	4.15	3.08
Positive	2.31	2.05	2.05	3.08

E. Determination of the Effective Moments of Inertia I_e and the Crack Coefficient k_1.

The effective moment of inertia of the column and middle strips is determined using (7). For this floor system with rectangular panels, I_e for the panel is determined by (4), where the average of the average I_e values in the column strips (cs) and middle strips (ms) in both directions are used. Detailed calculations are provided below for the section containing the negative flexural reinforcement in a column strip in the north-south design strip. Calculations for the other sections are similar [2].

For a column strip width of 10 ft, the section properties y_t and I_g are 4.75 in and 8574 in^4, respectively. I_e can be determined by:

$$a_1 = \frac{column\ strip\ width}{nA_s} \tag{8}$$

$$kd = \frac{\sqrt{2da_1+1}-1}{a_1} \tag{9}$$

$$I_{cr} = \frac{(Column\ strip\ width)kd^3}{3} + nAs(d-kd)^2 \tag{10}$$

$$M_{cr} = \frac{f_r I_g}{y_t} \tag{11}$$

$$I_e = \frac{I_{cr}}{1-\left(\frac{M_{cr}}{M_a}\right)^2\left[1-\frac{I_{cr}}{I_g}\right]} \leq I_g = \frac{3043}{1-\left(\frac{32.1}{91.2+42.7}\right)^2\left[1-\frac{3043}{8574}\right]}$$
$$= 3160 \text{ in}^4 \tag{12}$$

Similarly, I_e = 2071 in.^{4} for the positive section in this column strip. Using (5), the average I_e for this column strip in the north-south direction is $I_{e|cs,N\text{-}S}$ = (0.7×2071) + 0.15 (2×3160) = 2398 in.^{4}. The average I_e for the middle strip in the north-south direction and the column and middle strips in the east-west direction are determined by (7) similarly: $I_{e|ms,N\text{-}S}$ = 3126 in.^{4}, $I_{e|cs,E\text{-}W}$ = 2546 in.^{4}, and $I_{e|ms,E\text{-}W}$ = 12861 in.^{4}. The effective moment of the panel is determined by (6) as $I_{e/panel}$ = 10466 in.^{4}. The gross moment of inertia of the panel is averaged in the same way as the effective moment of inertia:

$$I_{g|N-S} = I_{g|cs,N-S} + I_{g|ms,N-S} = 17148\ in.^4$$

$$I_{g|E-W} = I_{g|cs,E-W} + I_{g|ms,E-W} = 21435\ in.^4$$

$$I_{\ e/panel} = 19292\ in.^4$$

The constant k_1 accounts for the level of cracking in the slab and is determined by dividing the effective moment of inertia of the panel by the gross moment of inertia of the panel $k_1 = \frac{10466}{19292} = 0.54$.

F. Determination of the Natural Frequency.

The fundamental frequency for a flat plate floor system is estimated by (1). The quantities in this equation that have not been previously determined are as follows [2]:

- k_2 = 1.9, since the column is less than 24 in.
- λ_i^2 = 8.02 by linear interpolation from Table III for l_1/l_2 = 1.25.
- *Mass* = 1587.7 lb
- Γ = 3.175 slugs/ft^2

Therefore,

$$f_i = \frac{1.9 \times 8.02}{2\pi \times 25^2}\left[\frac{0.54 \times 307929600 \times (9.5/12)^3}{12 \times 3.175(1-0.2^2)}\right]^{1/2} = 5.83 \text{ Hz.}$$

For comparison purposes, an FEA was performed to determine f_i for this flat plate system. Using the nonlinear option which includes the effects due to cracking, f_i was found to be 8.7 Hz with a crack coefficient equal to 0.71. The crack coefficient was determined by dividing the maximum deflection obtained from the elastic analysis by the deflection from the inelastic analysis of the system. Thus, the simplified method yielded a conservative estimate for f_i, approximately a 10% difference [2].

VI. RESULTS AND DISCUSSION

According to (2), a floor system is satisfactory if the peak acceleration as a fraction of the acceleration of gravity a_p/g is less than or equal to the acceleration limit a_0/g for the appropriate occupancy [2]. For an office occupancy, the damping ratio β can be taken as 0.03 for offices that have some nonstructural components but no full-height partitions. Thus,

$\frac{a_p}{g} = \frac{65e^{(-0.35\times5.83)}}{0.03\times51124} = 0.0055$ for f_i calculations without CFRP, and $\frac{a_p}{g} = \frac{65e^{(-0.35\times6.18)}}{0.03\times51124} = 0.00487$ for f_i from Abaqus with CFRP.

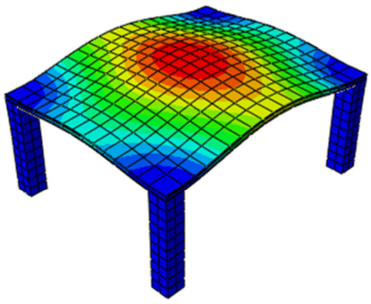

Fig. 4. f_i calculation with CFRP

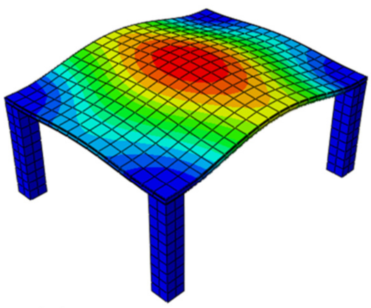

Fig. 5. f_i calculation without CFRP.

VII. CONCLUSION

After detailed calculations for the natural frequency according to [1], it was cleared that the natural frequency of the LWC flat plate without FRP is higher than the frequency applied due to human activities, while for walking excitations, a_p is approximately equal to 0.55% of g. This acceleration is more than the acceleration limit a_0, which is equal to 0.50% of g for office occupancies [2]. On the other hand, the natural frequency of an LWC flat plate with CFRP in FEM is 0.487%. Therefore, this flat plate system is satisfactory for walking excitations (human perception of vibration) only with CFRP, which is useful in increasing the stiffness and strength of concrete structures. It should be noticed that this LWC satisfied the strength limits even without FRP.

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