An Analytical Solution for the Dynamics of a Functionally Graded Plate resting on Viscoelastic Foundation

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ABSTRACT

This paper deals with the dynamic response of Functionally Graded Material (FGM) plates resting on a viscoelastic foundation under dynamic loads. The governing equations are derived by using Hamilton's principle using the classical plate theory and the higher-order shear deformation plate theory. Using state-space methods to find the closed-form solution of the dynamic response of functionally graded rectangular plates resting on a viscoelastic foundation. Numerical examples are given for displacement and stresses in the plates with various structural parameters and the effects of these parameters are discussed. The result of the numerical example shows a marked decrease in displacement and stresses as the coefficient of viscous damping is increased.

Keywords-dynamic; FGM plate; viscoelastic; viscoelastic foundation; analytical method

I. INTRODUCTION

Structural engineering problems such as in beams, frames, plates, and shells can be solved by experimental methods [1-3], analytical methods [4, 5], and numerical methods such as finite elements [6-11]. The dynamic problem of beams or plates on foundations has been investigated by many researchers [12-17]. When using analytical and semi-analytical methods, we depend on the boundary condition to choose the approximation function of the displacement fields. Usually, the Navier approach [18] and the Ritz method [19] are used for simple rectangular plates, and the Ritz method [20] and Fourier-Bessel series [21] for circular plates. The dynamic stability of the orthotropic plates subjected to an arbitrary dynamic load was studied with the Galerkin method in [22]. The dynamic response of plates [23] on an elastic foundation is subjected to moving loads using the strip method. An asymptotic theory was used in [24] to study the dynamic response of anisotropic inhomogeneous and laminated plates. The Galerkin method was used to deal with the analysis of the nonlinear dynamic response of a laminated composite plate subjected to blast loading in [25]. The wave propagation of the rectangular FGM plates [26] with clamped supports under impulse load using the dispersion relation and integral transforms. The dynamic behavior of fiber-reinforced plastic sandwich plates with PVC foam core was analytically studied in [27]. The dynamic stiffness matrix was constructed in [28] for an infinite or semi-finite Timoshenko beam on the viscoelastic foundation to the harmonic moving load. The stochastic finite element method was utilized for the calculation of the variability of the eigenvalue coefficient of the function of the graded beam.

In this paper, the governing equation of the functionally graded plates resting on viscoelastic foundation is solved by Hamilton's principle. The sinusoidal function and the Navier approach are used to find the dynamic solution of functionally graded simply supported rectangular plates subjected to step loading.

II. GOVERNING EQUATION OF THE FGM PLATE

We consider an FGM plate resting on a viscoelastic foundation. The geometry of the plate and the coordinate system is shown in Figure 1.

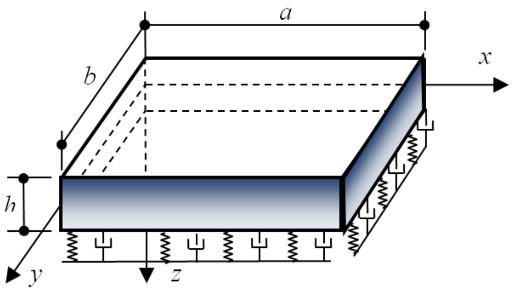

Fig. 1. The geometry of the FG plate.

The viscoelastic foundation is modeled as a Winkler model with stiffness coefficient *K* and damping coefficient *C*. The displacement fields at an arbitrary point (x,y,z) in the plate are shown below. Using the classical plate theory we have:

$$
\begin{aligned}
U(x,y,z,t) &= u(x,y,z,t) - z\frac{\partial w}{\partial x} \\
V(x,y,z,t) &= v(x,y,z,t) - z\frac{\partial w}{\partial y} \\
W(x,y,z,t) &= w\ (x,y,t)
\end{aligned} \tag{1}
$$

Using higher-order shear deformation plate theory proposed by Shimpi [30] we have:

$$
\begin{aligned}
U(x,y,z,t) &= u(x,y,z,t) - z\frac{\partial w_b}{\partial x} + \left[\frac{1}{4}z - \frac{5}{3}z\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \\
V(x,y,z,t) &= v(x,y,z,t) - z\frac{\partial w_b}{\partial y} + \left[\frac{1}{4}z - \frac{5}{3}z\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y} \\
W(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t)
\end{aligned} \tag{2}
$$

The mechanical properties of FGM, such as Young's modulus *E* and mass density ρ are assumed as:

$$
\begin{aligned}
E(z) &= (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + E_m \\
\rho(z) &= (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + \rho_m
\end{aligned} \tag{3}
$$

where the subscripts *m* and *c* represent the metallic and ceramic constituents, respectively, and *n* is the power index of the volume fraction. The linear constitutive relations of plate can be written as:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{4}
$$

where:

$$
\begin{aligned}
&Q_{11} = \frac{E(z)}{1-v^2},\ Q_{22} = Q_{11},\ Q_{12} = \frac{vE(z)}{1-v^2} \\
&Q_{44} = G_{23},\ Q_{55} = G_{13},\ Q_{66} = G_{12}
\end{aligned}
$$

Using Hamilton's principle we obtain the equations of motion of a plate as follows:

- Using the Classical Plate Theory (CPT):

$$
\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u} - I_1\frac{\partial \ddot{w}}{\partial x} \\
&\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v} - I_1\frac{\partial \ddot{w}}{\partial y} \\
&\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - Kw - C\dot{w} + f \\
&= I_0\ddot{w} + I_1\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) - I_2\nabla^2\ddot{w}
\end{aligned} \tag{5}
$$

- Using the Higher-order Shear Deformation Plate Theory (HSDT):

$$
\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u} - I_1\frac{\partial \ddot{w}_b}{\partial x} - J_1\frac{\partial \ddot{w}_s}{\partial x} \\
&\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v} - I_1\frac{\partial \ddot{w}_b}{\partial y} - J_1\frac{\partial \ddot{w}_s}{\partial y} \\
&\frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - K(w_b + w_s) \\
&-C(\dot{w}_b + \dot{w}_s) + q = I_0(\ddot{w}_b + \ddot{w}_s) + I_1\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) \\
&-I_2\nabla^2\ddot{w}_b - J_2\nabla^2\ddot{w}_s\ \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} \\
&+\frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} - K(w_b + w_s) - C(\dot{w}_b + \dot{w}_s) + q \\
&= I_0(\ddot{w}_b + \ddot{w}_s) + J_1\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) - J_2\nabla^2\ddot{w}_b - K_2\nabla^2\ddot{w}_s
\end{aligned} \tag{6}
$$

where:

$$
\begin{aligned}
&I_i = \int_{-h/2}^{h/2}\rho(z)z^i dz,\ J_i = -\frac{1}{4}I_i + \frac{5}{3h^2}I_{i+2} \\
&K_2 = \frac{1}{16}I_1 - \frac{5}{6h^2}I_4 + \frac{25}{9h^4}I_6 \\
&\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}
\end{aligned}
$$

The force and moment resultants of the plate are defined by:

$$\begin{aligned}
&[N]=\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_{xy}\end{Bmatrix}dz\\
&\left[M^b\right]=\begin{Bmatrix}M_x^b\\M_y^b\\M_{xy}^b\end{Bmatrix}=\int_{-h/2}^{h/2}z\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_{xy}\end{Bmatrix}dz\\
&\left[M^s\right]=\begin{Bmatrix}M_x^s\\M_y^s\\M_{xy}^s\end{Bmatrix}=\int_{-h/2}^{h/2}\hat{f}\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_{xy}\end{Bmatrix}dz\\
&\begin{Bmatrix}Q_x\\Q_y\end{Bmatrix}=\int_{-h/2}^{h/2}\hat{g}\begin{Bmatrix}\sigma_{xz}\\\sigma_{yz}\end{Bmatrix}dz
\end{aligned}\tag{7}$$

III. ANALYTICAL SOLUTION FOR THE DYNAMIC RESPONSE OF THE FGM PLATE

The sinusoidal function based on the Navier approach is chosen to satisfy all boundary conditions, as follows:

$$\begin{aligned}
u(x,y,t)&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{U_{mn}(t)\cos\alpha x\sin\beta y\right\}\\
v(x,y,t)&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{V_{mn}(t)\sin\alpha x\cos\beta y\right\}\\
w_b(x,y,t)&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{W_{bmn}(t)\sin\alpha x\sin\beta y\right\}\\
w_s(x,y,t)&=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{W_{smn}(t)\sin\alpha x\sin\beta y\right\}
\end{aligned}\tag{8}$$

where $\alpha=\dfrac{m\pi}{a},\beta=\dfrac{n\pi}{b}$.

Substituting (8) into (5) and (6) the forced vibration of the functionally graded plate can be written as follows:

- Using the CPT:

$$\begin{aligned}
&\begin{bmatrix}s_{11}&s_{12}&s_{13}\\s_{12}&s_{22}&s_{23}\\s_{13}&s_{23}&s_{33}\end{bmatrix}\begin{Bmatrix}U_{mn}\\V_{mn}\\W_{mn}\end{Bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&\hat{c}_{33}\end{bmatrix}\begin{Bmatrix}\dot{U}_{mn}\\\dot{V}_{mn}\\\dot{W}_{mn}\end{Bmatrix}\\
&+\begin{bmatrix}m_{11}&0&m_{13}\\0&m_{22}&m_{23}\\m_{13}&m_{23}&m_{33}\end{bmatrix}\begin{Bmatrix}\ddot{U}_{mn}\\\ddot{V}_{mn}\\\ddot{W}_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\F_{mn}\end{Bmatrix}
\end{aligned}\tag{9}$$

- Using the HSDT:

$$\begin{aligned}
&\begin{bmatrix}s_{11}&s_{12}&s_{13}&s_{14}\\s_{12}&s_{22}&s_{23}&s_{24}\\s_{13}&s_{23}&s_{33}&s_{34}\\s_{14}&s_{24}&s_{34}&s_{44}\end{bmatrix}\begin{Bmatrix}U_{mn}\\V_{mn}\\W_{bmn}\\W_{smn}\end{Bmatrix}+\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&\hat{c}_{33}&0\\0&0&0&\hat{c}_{44}\end{bmatrix}\begin{Bmatrix}\dot{U}_{mn}\\\dot{V}_{mn}\\\dot{W}_{bmn}\\\dot{W}_{smn}\end{Bmatrix}\\
&+\begin{bmatrix}m_{11}&0&m_{13}&m_{14}\\0&m_{22}&m_{23}&m_{24}\\m_{13}&m_{23}&m_{33}&m_{34}\\m_{14}&m_{24}&m_{43}&m_{44}\end{bmatrix}\begin{Bmatrix}\ddot{U}_{mn}\\\ddot{V}_{mn}\\\ddot{W}_{bmn}\\\ddot{W}_{smn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\F_{mn}\\F_{mn}\end{Bmatrix}
\end{aligned}\tag{10}$$

where:

$$F_{mn}=\frac{4}{ab}\int_0^b\int_0^a q\sin\alpha x\sin\beta y\,dxdy\tag{11}$$

We consider uniformly distribution loads as follows:

$$q=q_0\hat{F}(t)\tag{12}$$

The function of time $\hat{F}(t)$ of dynamic loadings is considered as step loading:

$$\hat{F}(t)=\begin{cases}1 & 0\le t\le t_1\\0 & t\ge t_1\end{cases}\tag{13}$$

In this work, we use the state-space method to solve (9) and (10), which must be rewritten in order to find a solution, as follows:

$$\dot{\boldsymbol{Z}}=\boldsymbol{A}\boldsymbol{Z}+\boldsymbol{b}\tag{14}$$

where:

$$\begin{aligned}
\boldsymbol{Z}&=\left\{U_{mn}\quad V_{mn}\quad W_{bmn}\quad W_{smn}\quad \dot{U}_{mn}\quad \dot{V}_{mn}\quad \dot{W}_{bmn}\quad \dot{W}_{smn}\right\}^T\\
\boldsymbol{b}&=\left\{0\quad 0\quad 0\quad 0\quad b_1\quad b_2\quad b_3\quad b_4\right\}^T
\end{aligned}$$

The solution of (14) is obtained as:

$$\boldsymbol{Z}(t)=e^{A(t-t_0)}\boldsymbol{Z}(t_0)+\int_{t_0}^{t}e^{A(t-\tau)}\boldsymbol{b}(\tau)d\tau\tag{15}$$

where t_0 is the initial time, $\boldsymbol{Z}(t_0)$ is the initial response, and $e^{A(t-t_0)}$ is the exponential matrix. This exponential matrix can be formulated in terms of the matrix of eigenvectors and eigenvalues associated with matrix A.

IV. NUMERICAL EXAMPLES

We consider a simply supported rectangular FGM plate with side-to-thickness ratio a/h=10, rectangular dimensions of a=0.3m, b=0.5m, and power index p=3. The elastic moduli and mass density are chosen to be the same as in [31]: ρ_m = 2707kg/m^3, E_m = 70GPa, E_c = 380GPa, ρ_c = 3800kg/m^3, with Poisson's ratio being 0.3. The uniform load is written as $q_0=100\left(\mathrm{kN}/\mathrm{m}^2\right)$ and the duration of load application time is t_1 = 0.002s. We note the normalized stiffness coefficient and the normalized damping coefficient of the foundation as follows:

$$k=\frac{Ka^4}{D_m},\quad c=\frac{Ca^2}{D_m}\tag{16}$$

where $D_m=\dfrac{E_m h^3}{12\left(1-\nu^2\right)}$ is the flexural rigidity of a full-metal plate.

The first two mode shapes are shown in Figure 2.

(a)
(b)

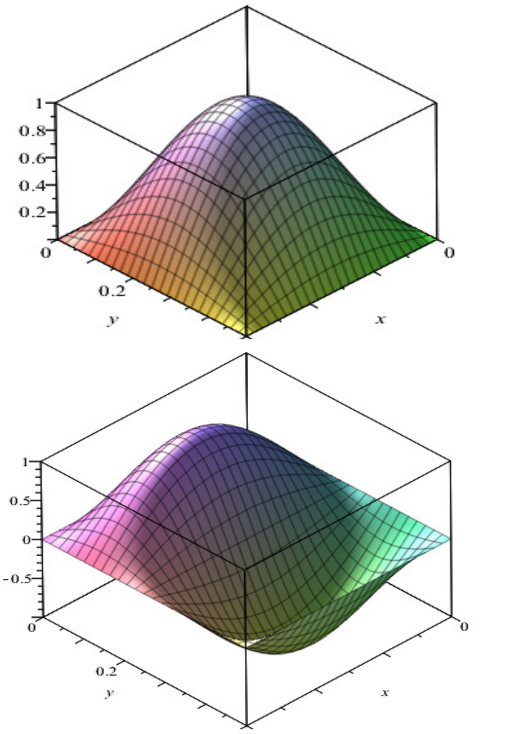

Fig. 2. (a) First and (b) second mode shapes of the simply supported rectangular FGM plate.

We consider two cases of viscoelastic foundation to investigate the effect of the damping coefficient of the viscoelastic foundation:

$$\begin{cases} \text{Case 1}: k=5, \quad c=0.05 \\ \text{Case 2}: k=5, \quad c=0.1 \end{cases}$$

(a)
(b)

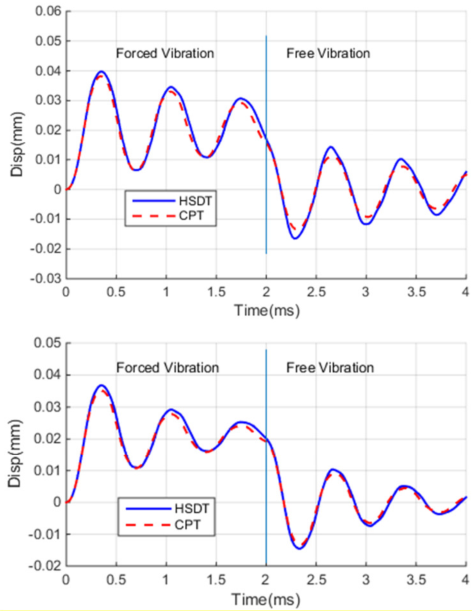

Fig. 3. The deflection response at the central plate. Cases (a) 1, (b) 2.

(a)
(b)

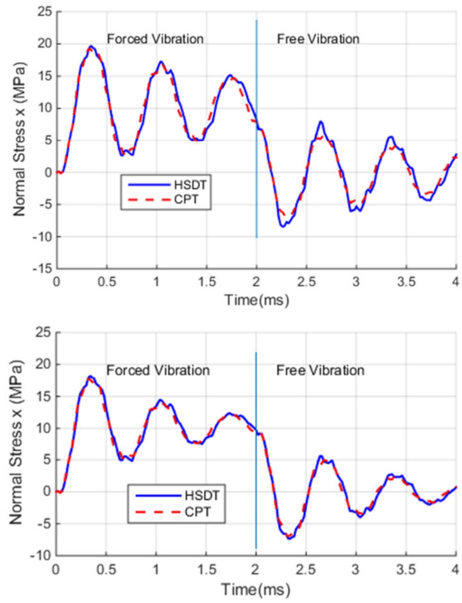

Fig. 4. The normal stress σ_x at the top of the central plate (z=h/2). Cases (a) 1, (b) 2.

(a)
(b)

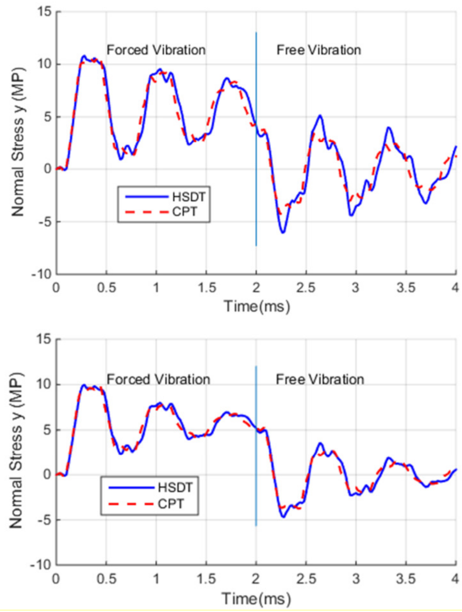

Fig. 5. The normal stress σ_y at the top of the central plate (z=h/2). Cases (a) 1, (b) 2.

Figure 2 illustrates the transverse deflection due to step loading as functions of time. Figures 3–4 show the transient response of stress at the center of the plate for the two considered cases. The amplitude of dynamic responses decreases due to the influence of viscous damping. The displacements computed by HSDT are clearly larger than those using the CPT because the plate modeled by CPT is relatively stiffer. However, the deviation of stresses σ_x, σ_y between CPT and higher order plate theories is small. So, the effect of shear deformation is more significant in predicting displacement than predicting stresses. In the forced vibration regime, it can be seen in Figures 2–4 that the deflection and stresses predicted for functionally grade plates with c=0.05 are moderately larger than for the plate with c=0.1. It is observed that when the damping coefficient increases, the deflection and stresses become smaller, as expected.

V. CONCLUSIONS

This research computed the vibration of the functionally graded plate resting on the viscoelastic foundation by the analytical method, using both classical plate theory and higher-order shear deformation plate theory. The analytical solution for the dynamic functionally graded plate was solved to be a double sine series based on the Navier approach. Dynamic responses are considered for both forced and free vibrations. The results show that the damping coefficient of the foundation has a significant influence on the dynamic response of the functionally graded plate. The damping factor of the foundation dissipates energy, reducing the ambient vibrations in forced and free vibrations. This work is significant to structural engineering, and the results can be used in practical design structures.

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