Reliability of Buried Pipes in Heterogeneous Soil Subjected to Seismic Loads

Hichem Benzeguir
Department of Civil Engineering
University Djillali Liabes of Sidi Bel Abbes
Sidi Bel Abbes, Algeria
h_benzeguir@yahoo.fr

Sidi Mohamed Elachachi
Civil and Environmental Engineering Department
University of Bordeaux
Talence, France
sm_elachachi@yahoo.fr

Djamel Nedjar
Department of Civil Engineering
University of Sciences and Technology of Oran
Oran, Algeria
djamel_nedjar@yahoo.fr

Mohamed Bensafi
Department of Civil Engineering
University of Sciences and Technology of Oran
Oran, Algeria
moha_bensafi@yahoo.fr

Abstract- Dysfunctions and failures of buried pipe networks, like sewer networks, are studied in this paper from the point of view of structural reliability and heterogeneity of geotechnical conditions in the longitudinal direction. Combined soil spatial variability and Peak Ground Acceleration (PGA) induce stresses and displacements. A model has been developed within the frame of geostatistics and a mechanical description of the soil–structure interaction of a set of buried pipes with connections resting on the soil by a two-parameter model (Pasternak model). Structural reliability analysis is performed considering two limit states: Serviceability Limit State (SLS), related to large "counter slope" in a given pipe, and Ultimate Limit State (ULS), corresponding to bending moment.

Keywords-reliability index; soil-structure interaction; spatial variability; SLS; ULS; seismic action

I. INTRODUCTION

Pipes that carry various substances need to be designed in a way capable of reducing the damage caused by ground displacements induced by earthquakes. The geoenvironmental effect has attracted concern on the performance of the buried pipes because of the associated hazards [1]. Most buried pipelines in seismic areas have sustained substantial damage in the past due to earthquake events [2]. Dysfunctions and failures of buried pipe networks, like sewer networks, are mainly caused by the heterogeneity of geotechnical conditions in the longitudinal direction and of the applied (seismic) action. Combined soil defects (differential settlements along the pipe, landslides, voids surrounding the pipe, etc.) and Peak Ground Acceleration (PGA) induce stresses (which lead to an Ultimate Limit State-ULS) and displacements (which constitute a violation of the Serviceability Limit State-SLS). It is worth noting that the influence of the variability of the soil is not reflected in current European standards [3]. Authors in [4] presented a comprehensive literature review on the seismic behavior of buried pipelines and underground structures

summarizing the recent research. Structural response to ground motion during earthquake cannot be accurately predicted because of the complexity of the structural properties and ground motion parameters [5]. In this paper, the response of buried pipe is investigated considering the seismic excitation by selecting real earthquake data.

II. SOIL PIPE SYSTEM MODELING

A. Pasternak Model

Among the models that describe the behavior of a beam resting on a soil and their interaction, the Pasternak model [6] attracts the most interest. In the soil-pipe interaction, the soil opposes on the components of a sewer network (pipes) a distributed force $R(x)$ (in N/m) given by (1):

$$R(x) = p(x).D_{ext} \quad (1)$$

where $p(x)$ is the stress under the pipe (Pa), and D_{ext} the external diameter of the pipe (m). According to the Pasternak model the stress is expressed as:

$$p(x) = k_w.w(x) - k_s\frac{d^2w}{dx^2} \quad (2)$$

where k_w is the coefficient of subgrade reaction (or Winkler coefficient in N/m³ or Pa/m), k_s is the shear coefficient (N/m) and $w(x)$ the vertical pipe displacement (and thus the settlement of the soil).

Pasternak's idealization considers the soil as being a system of identical but mutually independent, closely spaced, discrete, linearly elastic springs related by an incompressible "shear layer" which is defined as a layer of linear-elastic material of unit thickness that resists vertical shear forces only [6]. Thus it is a refinement of the well known Winkler model which suffers from not describing shear influence. The use of a two parameter model to characterize the soil's response under loading can appear a too simplified concept. However, such a

Corresponding author: Hichem Benzeguir

simplification seems coherent if one takes into account the variability and uncertainties related to the characterization of the soil. The spatial correlations which will be introduced further will ensure, in fact, a coherence of displacements, like it exists in a continuous medium. One should note that the two coefficients k_w and k_s are not soil-specific parameters. They are also affected by the rigidity of the pipe. In fact, these stiffness parameters depend on several factors, such as the length and/or the width (or diameter) of the pipe, the laying depth, the type of material used, and the type of the pipe bed. The value of these coefficients can only be approached by semi-empirical methods. For a same set of values, a parametric study, conducted in [7], illustrated the fact that k_w varies from 1 to 3. To the best of our knowledge the literature has not given ways on how to identify k_s in practice from physical and geometrical data [8]. In order to integer the damping of the system, the Pasternak model is adapted and expanded to a Kelvin-Voigt model by adding dashpots. The stiffness matrix of the soil–pipe system has been derived in [9] by using the energy method. We will consider two relative soil-pipe stiffness ratios, r_p and r_{ks} which are defined by (3) and (4) and can be considered as governing parameters:

$$r_p = \frac{1}{L} \sqrt[5]{\frac{E_p I}{k_w}} \quad (3)$$

$$r_{ks} = \frac{k_s}{k_w D_{ext}^2} \quad (4)$$

where E_p, I, D_{ext}, and L and are respectively the pipe's Young modulus, the pipe's moment of inertia, the exterior diameter, and the length of the pipe.

B. Joint-connection Stifness

Sealing between pipes is ensured by joints made of cement mortar or more frequently of elastomer. The rigidity of these joints is as variable as the technologies and geometries employed: it can be very weak (flexible joints) or very high (welded joints). It is difficult to identify realistic numerical values of the joint stiffness even if some laboratory experiments have focused on this question [10]. In this work, we assume a continuity of vertical displacements at joint connection and contrary to a model of continuous beam, a model was developed which enables introducing discontinuities of rotation between the ends of the pipes. The joints between two adjacent pipes are assimilated to rotation springs with stiffness R_j relating to the proportionality between the bending moment M applied to the joint and the variation of the angle of rotation $\Delta \varphi$:

$$M = R_j . \Delta \varphi \quad (5)$$

To take into account this joint stiffness, the pipe-joint stiffness ratio r_{joint} is introduced which is defined by:

$$r_{joint} = \frac{R_j L}{E_p I} \quad (6)$$

To summarize, the soil-pipe interaction system, one can say that it is governed by a geometric parameter L (pipe's length) and three relative (dimensionless) stiffness ratio parameters: r_{ks}

(compression to shear), r_p (soil to pipe), and r_{joint} (connection to pipe).

III. MODELING THE SPATIAL VARIABILITY OF SOIL AND ITS EFFECTS: THE CORRELATION LENGTH

In many common geotechnical problems, the variability of soil is only one among many sources of uncertainty (others are for example reduced sampling measurement or model errors). It can be accounted for by taking conservative values of the soil parameters, even if geotechnicians need long practice to be able to justify the choice of these values [11]. In fact, the variability of soils cannot be reduced to case-by-case variability – as a result of its natural or man-made fabric (deposit processes, compaction processes). The soil properties can be considered as spatially structured. Thus, tools like autocorrelation functions or semi-variograms appear to be appropriate for modeling. One must then identify the standard deviation or coefficient of variation of the studied property as well as the correlation length l_c (i.e. the distance above which the local properties at two points can be assumed to be independent). The first consequence of the spatial correlation is that the representative value of any soil property depends on the volume concerned by the problem to be solved. This question has been analyzed in detail during the drafting of Eurocode-7 but the code writers have limited themselves to general considerations, without prescribing any formal method: the representative value is only said to be a characteristic value defined as a cautious estimate of the parameter governing the studied limit state [12]. Nevertheless, accounting for spatial correlation has direct consequences on the safety of designs. A simple illustration of the effects of spatial variability is that of the rotation (tilting) of a foundation of length L resting on a heterogeneous elastic soil and supporting a uniform loading. It was shown [13] that the magnitude of the rotation depends on the correlation length in the horizontal direction: it tends towards zero when l_c is very small (which corresponds to very quickly varying properties, thus homogeneous at L scale) or very large (which corresponds to very slowly varying properties, thus also homogeneous at L scale) and it is maximum for an intermediate range. This illustrates a consequence of the spatially correlated variation of soil properties: tilting occurs only when the soil is not homogeneous and its magnitude depends both on the scatter in the soil properties (linearly) and on the correlation length, with a "worst case" for a particular range of l_c values. Since the role of the longitudinal variability of the filling appears essential, we chose to model it by using the theory of the local average of a random field developed in [14]. The random field of the coefficients k_w or k_s is defined by three properties: its average value $\overline{k_w}$ (resp. $\overline{k_s}$), its variance $\overline{\overline{k_w}}^2$ (resp. $\overline{\overline{k_s}}^2$) and its scale (or length) of correlation l_c (resp. l_{cs}). These scales are related to a function of correlation $\rho(\tau)$ in (7) where τ points out the distance between two points, and which describes the spatial structure of correlation of the properties: $\rho(\tau)$ differs whether the properties vary more or less quickly while deviating from a given point. This correlation length (length from which the correlation between soil properties tend to disappear) depends on the characteristic (modulus, porosity, water content, etc.) and on the direction (horizontal or vertical).

$$\rho(\tau) = \exp\left(-2\frac{|\tau|}{l_c}\right) \quad \text{for } \tau \le l_c \quad (7)$$

The soil is subdivided in several zones. The random field value in each zone is thus a random variable whose value is estimated by the average of the space field over the zone. The local average and the variance in zone i of length D_i fulfil (8):

$$E[k_w(D_i)] = \overline{k_w} \quad (8a)$$

$$\text{Var}[k_w(D_i)] = \overline{\overline{k_w}}^2 \gamma(D_i) \quad (8b)$$

Equation (8b) shows that the local variance Var $[k_w(D_i)]$ depends on the length D_i of zone i while following a variance reduction function $\gamma(D_i)$.

$$\gamma(D_i) = 2\left(\frac{l_c}{D_i}\right)^2 \left(\frac{D_i}{l_c} - 1 + \exp\left(-\frac{D_i}{l_c}\right)\right) \quad (9)$$

$\gamma(D_i)$ is a measure of the variance reduction due to the averaging of the random process according to the length of the studied zone. In this work, k_w and k_s follow a lognormal distribution. In order to show the effect of both the mean values of the Winkler and shear coefficients and their correlation length, six cases were considered (Table I) and applied to an example of a sewer section (set defined between two manholes) and which will be presented in further detail. The section is made of 20 pipes of unit length L=3m. For each computation, the outputs which are linked to limit states are processed all along the section and their worst value is kept. Finally the Cumulative Density Function (CDF) of these parameters (stress, displacement, bending moment) is drawn and used for comparison. Figure 1 shows the CDF of the maximum bending stress in a concrete sewer's section.

TABLE I.   VALUES OF k_w, k_s, l_c AND l_{cs}

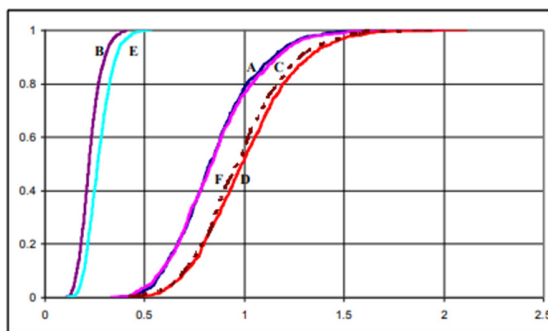| parameters	k_w(KN/m³)	l_c (m)	ks (KN/m)	l_{cs} (m)
Case A	10	3	1	30
Case B	10	30	1	3
Case C	10	3	1	3
Case D	10	3	10	3
Case E	10	30	10	3
Case F	10	3	10	30

Fig. 1.   CDF bending stress for the 6 different cases presented in Table I.

Two observations can be made:

• Not taking into account the shear effect is not conservative (case C compared to case F) being given that the mean stress varies more than 20%,

• The correlation length of the Winkler coefficient seems to have more importance than the correlation length of the shear coefficient (case B compared to A and C cases or case E compared to D and F cases).

IV.   EQUATIONS OF MOTION

The soil resistance to the pipe motion is generated by the relative motion u between the pipe and the soil. The pipe resistance comes from the absolute displacement U (Figure 2). The governing equation of the system is [15]:

$$[M_p + M_{sol}]\{\ddot{U}\} + [C_p + C_{sol}]\{\dot{U}\} + [K_p + K_w + K_s]\{U\} =$$
$$[M_{sol}]\{\ddot{u}_S\} + [C_{sol}\dot{u}_S] + [K_p + K_s]\{u\} \quad (10)$$

in which M_p and M_{soil} are respectively the mass matrix of the pipe and of the soil, K_p, K_w and K_s the stiffness matrices of the pipe and the soil, and C_p and C_{soil} the damping matrices of the pipe and of the soil respectively. We note that the damping matrices are constructed from the Rayleigh damping. U, $\dot{U}$, $\ddot{U}$, are respectively the vectors of absolute displacement, velocity and acceleration, u, u, $\dot{u}$, and $\ddot{u}$ are the vectors of relative displacement, velocity and acceleration, and u_s, $\dot{u}_s$, and $\ddot{u}_s$ u_s are the vectors of ground displacement, velocity, and acceleration of the soil. Denoting the total stiffness as $[K] = [K_p] + [K_w] + [K_s]$, the total mass as $[M] = [M_p] + [M_{soil}]$, the total damping as $[C] = [C_p] + [C_{soil}]$, and the absolute displacement as $\{U\} = \{u_s\} + \{u\}$, the equations of motion in terms of the relative displacement can be rewritten as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} =$$
$$-[M_p]\{\ddot{u}_S\} - [C_p\dot{u}_S] - [K_p]\{u_S\} \quad (11)$$

These equations of motion are solved for each time history of the ensemble of real earthquake records. The boundary conditions can be considered either as fixed ends or as free ends if we want to eliminate their effects.
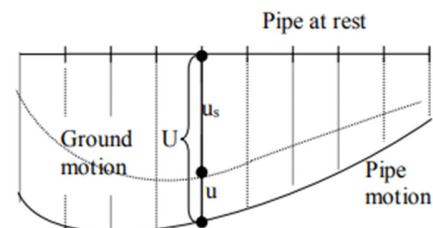

Fig. 2.   Pipe and ground motion.

V.   PERFORMANCE FUNCTIONS AND LIMIT STATES

Two performance functions are defined. One relates to the counterslope and corresponds to the SLS, by relating the sewer's hydraulic performance to the local counterslope. The presence of too high counterslopes harms the flow of the effluents and facilitates the clogging of the pipes by the sedimentation of the suspended particles. The corresponding limit state function is:

$$G_{CS} = CS_R - CS_S \quad (12)$$

where CS_R is the maximum acceptable counterslope and CS_S is the maximum simulated counterslope. The other performance function relates to the stresses and defines the equivalent of an ULS, by relating the cracking state (for a concrete pipe) to the bending stresses:

$$G_\sigma = \sigma_R - \sigma_S \quad (13)$$

where σ_R is the yield tension stress of the concrete and σ_S is the maximum bending stress. We assume that CS_S and σ_S also follow lognormal distributions, whose means and standard deviations are obtained from coupling Monte-Carlo simulations and finite element method, and that CS_R as σ_R also follow lognormal distributions of means and standard deviations of $4\% \pm 0.8\%$, and 2 ± 0.3MPa respectively.

It should be noted that there is not any regulation limiting the counterslope, that's why we have chosen a 4% arbitrary limit based on the design considerations: trespassing such a value leads to damages or dismantling of the joints and to defects of sealing. The $\pm 0.8\%$ can be seen as a model uncertainty on this SLS criterion. The target values of the reliability index to be reached are 3.8 for the ULS and 1.5 for the SLS with a service life of 50 years. In the following, we will consider two reliability indexes β_{ULS} and β_{SLS}.

VI. CASE STUDY

A. Caracteristics of the Sewer's Section

Lets consider a reinforced concrete material for the pipe (E_p = 25000MPa) with an exterior diameter D_{ext} = 1m, a thickness of 0.08m, made of a set of 20 pipes with L = 3m unit length. It rests on a soil whose Winkler coefficient is 10kN/m^3 and shear coefficient is 10kN/m. This type of configuration corresponds to a pipe-soil ratio r_p = 0.678. It must be noted that the pipe geometry intervenes in the expression of inertia I and in that of the Winkler coefficients. The pipes are subjected to a deterministic uniform loading of 50kN/m and to the earthquake of El-Asnam (Algeria, 1980). The ratio between the length of correlation l_c and the pipe length L is defined as λ. In the following sections are analyzed:

- the effect of the length of correlation on the response of the system,

- the effect of the relative soil-pipe stiffness, for an unfavorable length of correlation (λ # 1),

- and the effect of the relative joint-pipe stiffness.

The values of the obtained reliability β indexes (ULS or SLS) must be considered rather comparatively that in an absolute way because of:

- the arbitrary character of the values retained for CS_R as σ_R,

- the type of the probabilistic distribution,

- model uncertainties on soil characteristics (k_w, k_s).

B. Effect of Correlation Length

The analysis is carried out for lengths of correlation ranging from 0.03m to 1500m, ($10^{-3} < \lambda < 5 \times 10^2$). Two types of joints are considered: rigid (r_{joint}=10^{+5}) and flexible (r_{joint}=10^{-5}).

Figures 3 and 4 show the ULS and SLS reliability indexes. It can be seen that there is a critical length of correlation and thus a critical value of λ which causes the most unfavorable effects in the structure. The evolution of reliability is not monotonous. The existence of such a limit value is one of the invariants of the problems of (heterogeneous) soil-structure interaction [13]. Reliability indexes grow when λ tends to zero or infinity. In these two situations, the soil tends to be homogeneous at the scale of the analysis, either because its variations are very fast are filtered by the pipe, or because its variations are very slow, inducing almost uniform values between close elements. We had seen that the rigid joints, and for a high coefficient of variation of k_w, do not satisfy the ULS for λ between 0.01 and 7, while the flexible joints provide much better results. On the other hand, if we consider the SLS, the flexible joints provide results a little worse than the rigid joints. One thus notes at this level a conflict in the choice of the "good pipe length" to answer the ULS and the SLS.
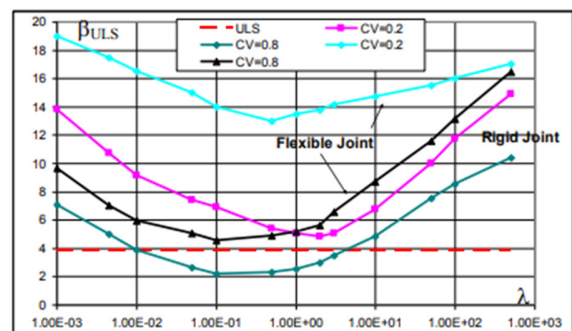

Fig. 3. Effect of the correlation length on the ultimate state reliability index for two values of the variation of coefficient k_w and two of joints.
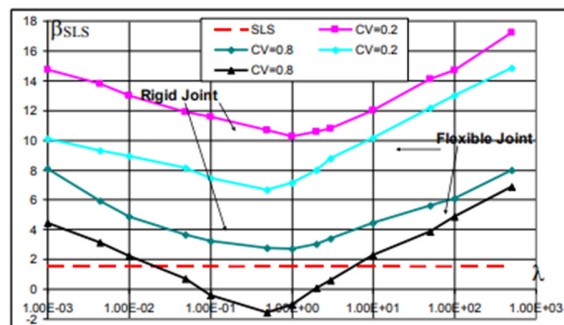

Fig. 4. Effect of the correlation length on the service limit state reliability index for two values of the variation of coefficient k_w and two of joints.

C. Effect of the Joint-pipe Stiffness Ratio

The effect of joints on the behavior of the system was analyzed by varying the joint-pipe ratio r_{joint} from 10^{-5} to 10^{+5}. Figures 5 and 6 show the evolution of the reliability indexes for different coefficients of variation of Winkler coefficient. The strong influence of the joints' stiffness should be underlined since the rigid joints are penalizing with the ULS, by the fact that they necessarily induce more significant stresses. On the other hand, the flexible joints are penalizing with the SLS. An intermediate joint stiffness must be chosen in order to carry out

the best compromise, since reliability is completely assured for both ULS and SLS. Finally, one can also note that for a given value of r_{joint}, the ULS index reliability and the SLS index reliability decrease with the increase of k_w coefficients of variation.
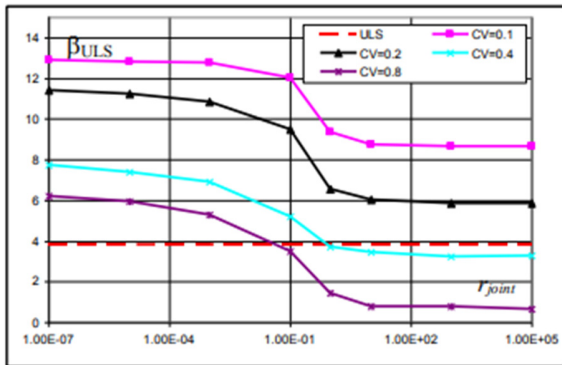

Fig. 5.   Effect of joint-pipe stiffness ratio on ultimate limit state reliability index for four values of the variation of coefficient k_w.
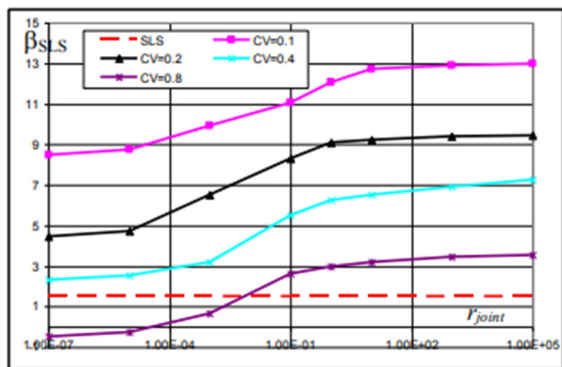

Fig. 6.   Effect of joint-pipe stiffness ratio on the service limit state reliability index for four values of the variation of coefficient k_w.

D. Effect of the Soil-pipe Stiffness Ratio

By maintaining the same set of pipes and soil parameters ($\lambda = 1$, $r_{ks} = 1$), we vary the soil-pipe stiffness ratio r_p in the range between 0.4 and 1.7 which corresponds to a Winkler coefficient in the range from 100kPa to 100MPa for four values of the coefficients of variation ($CV_{kw} = 0.1$, 0.2, 0.4, and 0.8). While taking a length of correlation l_c close to the length of the pipe ($\lambda \# 1$), we refer to the situation in the vicinity of the most unfavorable geotechnical conditions. Figures 7 and 8 present the values of the reliability indexes. Whatever the type of joint, reliability is lower for high soil-pipe stiffness ratio, situation corresponding to sections of very stiff pipes or low soil stiffness. In the presence of rigid joints (Figure 7) and high coefficient of variation of the soil's parameter, the Eurocode β_{ULS} is not respected (lower than 3.8). We can see that increasing the coefficient of variation induces reduction of the reliability of the pipe section. So, it is better to have flexible rather than rigid joints, it is better to privilege a lower r_p to ensure the "good" reliability of the network.
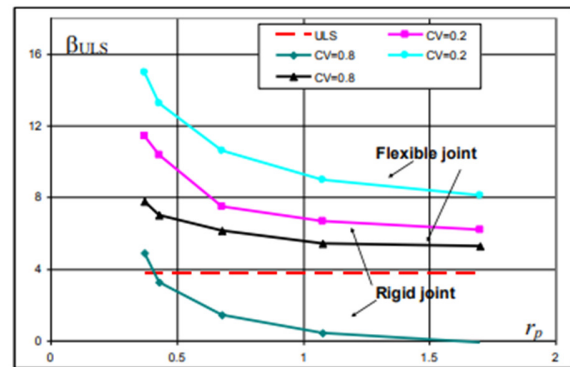

Fig. 7.   Effect of soil-pipe stiffness ratio on the service ultimate state reliability index for four values of the variation of coefficient k_w.

In terms of counterslopes (Figure 8), it is better to have rigid than flexible joints. To ensure the reliability of the sewer's section using rigid joints imposes that the ground is rather homogeneous. It can be also noted that it is more profitable to be in the presence of a relatively homogeneous soil whose mechanical characteristics are poor that in the presence of a better ground but whose heterogeneity is more pronounced.
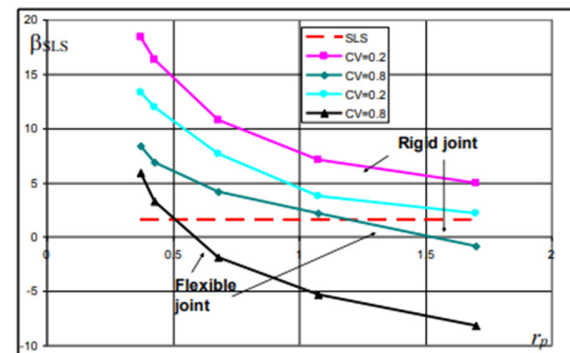

Fig. 8.   Effect of soil-pipe stiffness ratio on the service limit state reliability index for four values of the variation of coefficient k_w.

VII.   CONCLUSION

A model which includes a description of the soil's spatial variability, within the frame of geostatistics has been developed in this paper. It is based on a mechanical description of the soil–structure interaction between a set of segmented buried pipes and the soil support represented by the Pasternak model. The need to take into account the soil spatial variability is clearly recognized. Several conclusions are drawn:

Soil heterogeneity induces effects (differential settlements, bending moments, stresses, and possible cracking) that cannot be predicted if homogeneity is assumed. These effects can significantly affect the SLS or ULS reliabilities. In addition to the effects of the soil stiffness, the magnitude of the induced stresses depends mainly on four factors or governing parameters:

- The magnitude of the soil variability (i.e. its coefficient of variation).

- The soil-structure length ratio, which combines the soil fluctuation scale and a structural characteristic length (buried pipe length). A worst value, corresponding to the value leading (from a statistical point of view) to the (statistically) largest effects in the structure, can be found.

- The soil-structure stiffness ratio.

- The structure-connection (pipe-joint) stiffness ratio (relative flexibility).

The principal benefit of such an approach is that it provides some new approaches for better considering phenomena such as the geometrical irregularities in the longitudinal profile during the control of soil compaction of sewer trench filling. This kind of approach can also give the experts new tools for better calibration of safety in soil-structure interaction problems, when the soil variability is an influential parameter.

REFERENCES

[1] L. R. Khan and K. F. Tee, "Quantification and comparison of carbon emissions for flexible underground pipelines," *Canadian Journal of Civil Engineering*, vol. 42, no. 10, Jul. 2015, https://doi.org/10.1139/cjce-2015-0156.

[2] M. J. O'Rourke and X. Jiu, *Seismic Design of Buried and Offshore Pipelines*. Buffalo, NY, USA: MCEER, University at Buffalo, State University of New York, 2012.

[3] *Eurocode 7, Geotechnical design, Part 1: general rules*. Brussels, Belgium: CEN, 1997.

[4] G. Manolis and D. E. Beskos, "Underground and lifeline structures," in *Computer Analysis and Design of Earthquake Resistant Structures: A Handbook*, vol. 3, D. E. Beskos and S. A. Anagnostopoulos, Eds. Southampton, UK: Computational Mechanics Publications, 1997, pp. 775–837.

[5] R. A. Hakim, M. S. A. Alama, and S. A. Ashour, "Seismic Assessment of an RC Building Using Pushover Analysis," *Engineering, Technology & Applied Science Research*, vol. 4, no. 3, pp. 631–635, Jun. 2014, https://doi.org/10.48084/etasr.428.

[6] P. L. Pasternak, "On a new method of analysis of an elastic foundation by means of two foundation constants," in *Gosudarstvennoe Izdatelstro Liberaturi po Stroitelstvui Arkhitekture*, Moscow, Russia, 1954.

[7] S. M. Elachachi, M. Bensafi, M. Hamane, D. Nedjar, and D. Breysse, "Comportement longitudinal en flexion d'une conduite enterrée dans un remblai hétérogène," *Revue Française de Géotechnique*, no. 108, pp. 17–29, 2004, https://doi.org/10.1051/geotech/2004108017.

[8] K. Morfidis and I. E. Avramidis, "Formulation of a generalized beam element on a two-parameter elastic foundation with semi-rigid connections and rigid offsets," *Computers & Structures*, vol. 80, no. 25, pp. 1919–1934, Sep. 2002, https://doi.org/10.1016/S0045-7949(02)00226-2.

[9] S. M. Elachachi, D. Breysse, and L. Houy, "Longitudinal variability of soils and structural response of sewer networks," *Computers and Geotechnics*, vol. 31, no. 8, pp. 625–641, Dec. 2004, https://doi.org/10.1016/j.compgeo.2004.10.003.

[10] J. Buco, F. Emeriault, P. Le Gauffre, and R. Kastner, "Statistical and 3D Numerical Identification of Pipe and Bedding Characteristics Responsible for Longitudinal Behavior of Buried Pipe," in *Pipeline Division Specialty Conference 2006*, Chicago, IL, USA, Aug. 2006, https://doi.org/10.1061/40854(211)83.

[11] J. M. Duncan, "Factors of Safety and Reliability in Geotechnical Engineering," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 126, no. 4, pp. 307–316, Apr. 2000, https://doi.org/10.1061/(ASCE)1090-0241(2000)126:4(307).

[12] J. B. Kovarik, "A propos des valeurs caractéristiques des propriétés des sols," in *Journées ENPC de présentation des Eurocodes*, 1996.

[13] D. Breysse, H. Niandou, S. Elachachi, and L. Houy, "A generic approach to soil–structure interaction considering the effects of soil heterogeneity," *Géotechnique*, vol. 55, no. 2, pp. 143–150, Mar. 2005, https://doi.org/10.1680/geot.2005.55.2.143.

[14] E. Vanmarcke, *Random Fields: Analysis And Synthesis*. Singapore, Hackensack, NJ, USA: Wspc, 2010.

[15] D. Nedjar, M. Hamane, M. Bensafi, S. M. Elachachi, and D. Breysse, "Seismic response analysis of pipes by a probabilistic approach," *Soil Dynamics and Earthquake Engineering*, vol. 27, no. 2, pp. 111–115, Feb. 2007, https://doi.org/10.1016/j.soildyn.2006.06.001.