

# Two-dimensional Analysis of Cracks Propagation in Structures of Concrete

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**Abstract**— In this article we present a numerical simulation that allows describing and studying the damage of concrete works subjected to various stress types. In this study, the propagation of cracks in the concrete is analyzed as of their appearance, which requires having a thorough knowledge of the mechanical behavior of the material. The mechanical approach which leads to better understanding of fracture phenomena and can give satisfactory results, is that of the elastic linear mechanics of failure. The interest in this study is focused in the development processes of the cracks from a phenomenological point of view. The analysis is carried out by using fracture criteria while being based on the determination of the critical stress intensity factors, for each case of the several elaborate tests of indirect tensile per bending and Brazilian Disc. The analysis is carried out in a two-dimensional medium by the finite element method by using the ABAQUS software. The results obtained are compared with experimental data obtained analytically from other authors.

**Keywords**- crack propagation; stress intensity factor; three-points bending beam test; Brazilian disc; fractures mechanics; two-dimensional medium.

## I. INTRODUCTION

The analysis of the behavior of any structure under different loads until its fracture, is an essential component of structural design. The classic methods of collapse designing are based on the research of acceptable fracture diagrams, and on the calculation of ultimate balance in the critical sections. Thus, dimensioning of concrete works is done considering ultimate constraints corresponding to the crushing of the concrete or the plasticization of reinforcement's steels. With the development of numerical calculation techniques and finite element methods, it is currently possible to analyze the details of state of equilibrium in various points of the structure. Models using the criterion of crack propagation based on the going beyond of the maximum stress of traction have been developed but are not sufficient to give an account of the harmfulness of the defect [1, 2, 3]. Other models have also been developed [2, 4-8] using the criterion of the breaking process that give a report on the modeling of the cracks by redefining the networks of the elements to each step of propagation. The elastic linear

mechanics of the failure is applied to study the crack propagation in concrete not reinforced with massive structures. This approach is adapted to analyze large-sized structures focusing on the influence of structure's dimension on its behavior [9, 10, 11]. The utilization of a criterion of crack propagation, based on linear mechanics of the failure, imposes the determination of the stress intensity factors.

This paper studies crack propagation in concrete structures, using the criterion of failure based on the determination of the stress intensity factors. Two approaches were proposed, an approach based on an analytical approach where the stress intensity factor is calculated starting from the functions weight [12] and a numerical approach where the stress intensity factor is calculated by the finite element method using the ABAQUS software.

## II. MODEL USED

### A. Hypothesis of bases

The numerical model used within the framework of our study, combines a criterion of the linear elastic mechanics of the failure and a distributed model of cracking that leads to taking into account the singularity of the stress field at the peak of a crack and the influence of the geometrical parameters on the development of cracking and fracture behavior.

This model adopts a certain number of basic assumptions:

- The not fissured concrete is regarded as an isotropic homogeneous material with an elastic linear behavior describing its mechanics.
- The micro crack zones and the nonlinear behavior of the material at the point of the cracks are neglected.

### B. Analytical and numerical analyses

It is a question of determining, for the load patterns considered in this work, the values of the stress intensity factor  $K_I$  and  $K_{II}$  and which are expressed [9] starting from the following relation:

$$K_I = \sigma F \sqrt{\pi \cdot \alpha} \quad (1)$$

$$K_{II} = \tau F \sqrt{\pi \cdot \alpha} \quad (2)$$

Where:

$\alpha$  is the length or half-length of the crack,  
 $\sigma$  and  $\tau$  are respectively the normal and shear stresses applied to the structure,  
 $F$  is the geometrical Functions.

Two types of tests are used:

- *Three-points bending test*: The geometrical data and the principle of the test are schematized in Figure 1:

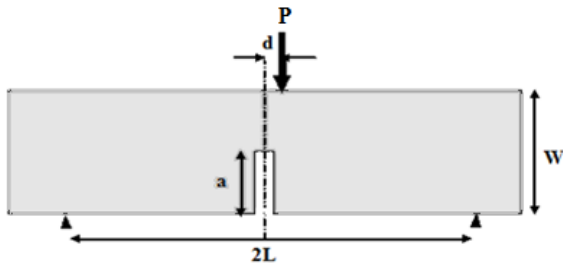


Fig. 1. Three Bending Beam Tests.

The specimen subjected to the bending test is a not reinforced concrete beam where its length is  $2L$  and its width is  $w$ , receiving a load  $P$  with an eccentricity  $d$ .

The beam is supposed to have an initial crack of length  $\alpha$  located in the middle of the beam.

- *Brazilian disc test*: The geometrical data are illustrated in Figure 2.

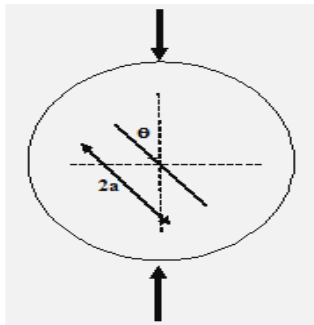


Fig. 2. Description of the Brazilian disc.

The specimen submitted for testing is not reinforced concrete, whose dimensions are shown in Table II.

The surface of the disc is supposed to be notched (half-length of the crack  $\alpha$ ). The crack forms an angle  $\theta$  with the vertical axis of the disc.

In the numerical study, the values of the stress intensity factors  $K_I$  and  $K_{II}$  are numerically given by finite element modeling in a two-dimensional medium by the ABAQUS software. The models used during the simulation are represented in Figures 3 and 4.

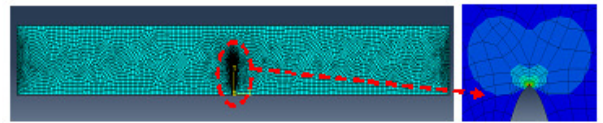


Fig. 3. Geometrical representation by ABAQUS of three points bending test .

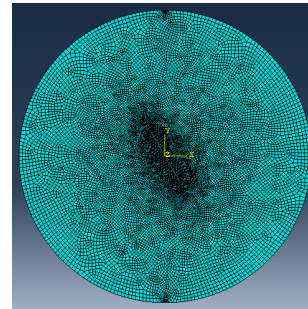


Fig. 4. Geometrical representation by ABAQUS of the Brawilian disc.

TABLE I. SUMMARY OF GEOMETRIC DATA 3-PT BB

$L$	The length of the beam	Varies from 320 to 800 mm
$w$	Beam width	equal to 160 mm
$\alpha$	length of the initial crack	Varies from 8 to 64 mm
$d$	Eccentricity of load	$0 \leq d < L$

TABLE II. SUMMARY OF GEOMETRIC DATA BRASILIAN DISK

$D$	Disc diameter $D = 160$ mm	Equal to 160 mm
$R$ <td>radius of the disk</td> <td>Equal to 80 mm</td>	radius of the disk	Equal to 80 mm
$\alpha$ <td>Half length of the initial crack</td> <td>Varies from 8 to 40 mm</td>	Half length of the initial crack	Varies from 8 to 40 mm
$\theta$ <td>angle of inclination of the initial crack</td> <td>Varies from <math>0^\circ</math> to <math>90^\circ</math></td>	angle of inclination of the initial crack	Varies from $0^\circ$ to $90^\circ$

### III. RESULTS AND DISCUSSIONS

#### A. Mechanical properties of the material (concrete)

The mechanical properties of the concrete are given in Table III [13].

TABLE III. SUMMARY OF MECHANICAL CHARACTERITICS OF CONCRETE

$E$	Elastic modulus	35982 MPa
$\sigma_c$	Compressive strength	35 MPa
$\sigma_t$	Tensile strength	2.7 MPa
$\nu$	Poisson's ratio	0.2

#### B. Comparison of the results

##### 1) Mode I: Three-points bending test

a) Variation of the stress intensity factor  $K_I$  according to the length of the crack

Figure 5 shows the variation of the stress intensity factor  $K_I$  (calculated numerically and analytically) according to the length of the crack; it is observed that the values of  $K_I$  increase when the reported  $a/w$  increases.

b) Influence of the load's position on the stress intensity factor  $K_I$

Figure 6 represents the variation of the stress intensity factor  $K_I$  according to the position of the load; it is noticed that the increase in  $d/w$  leads to a reduction of  $K_I$  and that when  $d/w = 0$  the values of  $K_I$  are maximum.

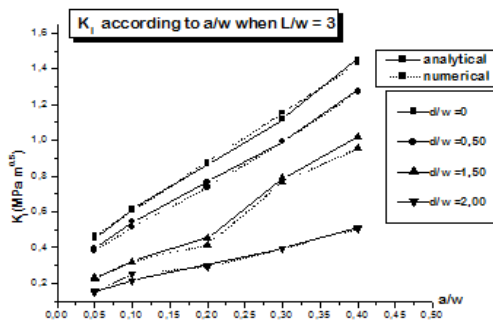


Fig. 5. Variation of the SIF  $K_I$  according to  $a/w$ .

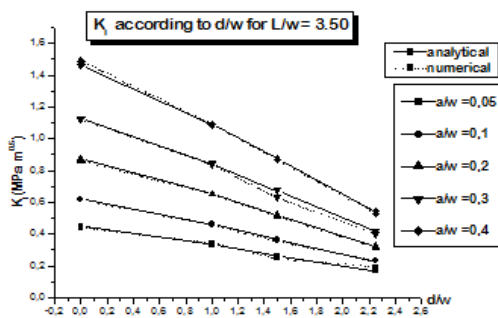


Fig. 6. Variation of the SIF  $K_I$  according to the position of the load.

2) Mode I: Brazilian disc test:

a) Influence of the crack's size on the stress intensity factor  $K_I$

Figure 7 represents the variation of the stress intensity factor  $K_I$  according to  $a/R$  (Brazilian disc). It is noted that for angles of inclination of the crack lower than  $30^\circ$ , the value of  $K_I$  increases when  $a/R$  increases. For inclination angles larger or equal to  $30^\circ$ ,  $K_I$  decreases and become negative.

b) Influence angle of inclination of the crack on the stress intensity factor  $K_I$

Figure 8 represents the variation of the stress intensity factor according to the crack inclination angle. It is noted that for values of inclination angle  $\theta$  lower than  $22^\circ$ , the values of  $K_I$  are positives and become maximum for  $\theta = 0$ . For values of  $\theta$  larger than  $22^\circ$ ,  $K_I$  becomes significant in absolute value,

specimen is in compression and the crack remains closed [14-17].

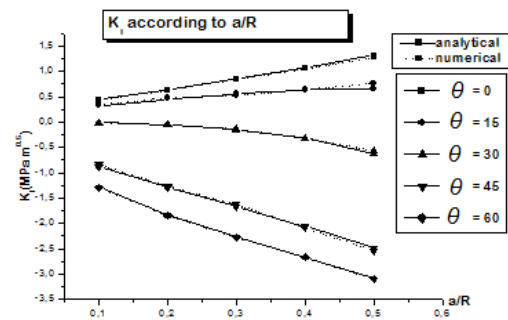


Fig. 7. Variation of the SIF  $K_I$  according to  $a/R$ .

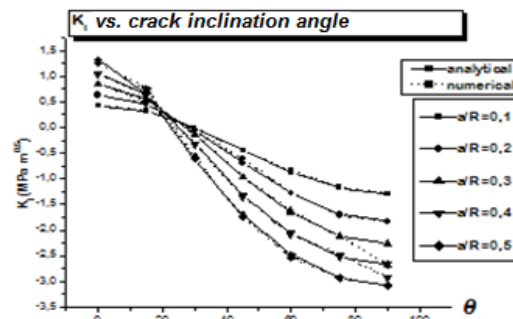


Fig. 8. Variation of  $K_I$  according to crack inclination angle.

3) Mode II

In three-points bending test, the beam is not subjected to shearing and the values of  $K_{II}$  are negligible. The study is limited only to the case of Brazilian Disc test.

a) Influence of the crack's size on the stress intensity factor  $K_{II}$

Figure 9 represents the variation of  $K_{II}$  according to the size of the crack; we can note that  $K_{II}$  varies proportionally with  $a/R$  i.e. when the length of the crack in the disc increases, the values of  $K_{II}$  increase. This means that there exist high shear stresses that facilitate the fracture in Mode II and the propagation of the cracks is carried out quickly. If the crack is not inclined or not formed at an angle of  $\theta = 0^\circ$  or  $\theta = 90^\circ$ , we can note that  $K_{II}$  values are cancelled and the increase of the crack's size does not have any effect on the variation of  $K_{II}$  because of the absence of shear stresses in these two cases.

b) Influence of the angle of inclination of the crack on the stress intensity factor  $K_{II}$

Figure 10 represents the variation of  $K_{II}$  according to the crack inclination angle; it is shown that  $K_{II}$  reaches its maximum value when the crack inclination angle  $\theta$  is in the interval of  $30^\circ$  to  $60^\circ$ , because shear stresses are highest in this zone. On the other hand if the inclination angle  $\theta$  tends towards  $0^\circ$  or  $90^\circ$ ,  $K_{II}$  decreases until it is cancelled.

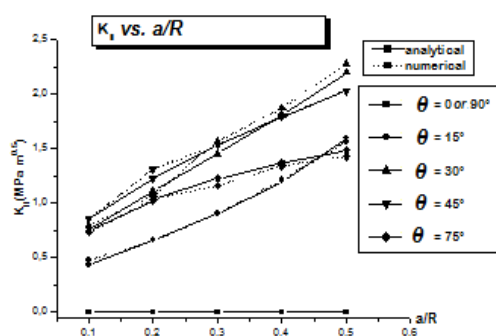


Fig. 9. Variation of the SIF  $K_{II}$  according to  $a/R$ .

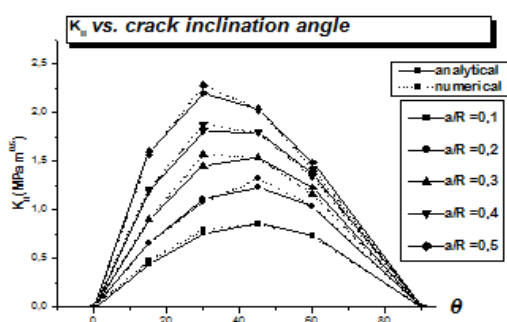


Fig. 10. Variation of the SIF  $K_{II}$  according to crack inclination angle.

#### IV. CONCLUSION

In actual position, the phenomenon of crack propagation in concrete structures can be described by the concept of the elastic linear mechanics of failure starting from the criterion of the critical stress intensity factor. This method makes it possible to quantify the effects of the presence of a crack and its influence on the fracture behavior of the structure. In this study, we compared two methods of calculating the stress intensity factor, and showed that the obtained results are comparable.

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