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THE BEHAVIOUR OF THE LABOUR MANAGED FIRM AND DISUTILITY FROM SUPPLYING FACTOR SERVICES

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I. INTRODUCTION

The theory of the labour-managed firm (LM-firm) is now well-known for predicting perverse behaviour and grave problems. The response to an output price change of a firm maximising income per worker with respect to work force size alone is an example of the former. When product price goes up, such a firm will wish to employ less labour and thus produce less output (see for example Meade, 1972). The property rights aspects of the use of capital (see Furubotn and Pejovich, 1973) and labour market inefficiencies (see for example Ireland and Law, 1978) constitute further examples of grounds for doubting the wisdom of organising production in terms of LM-firms.

However, most empirical work has been concerned with demonstrating the higher productivity achieved by LM-firms and labour-participating firms. Some of this work is summarised by Blumberg (1975). It is argued in the empirical literature that a result of moving towards labour management or participation in management is a reduction in the alienation of the labour force from the firm. A Marxist explanation has been that LM-firms may benefit from two factors: that their workers do not feel exploited by capitalists and also that their labour is not simply exchanged for a wage in the labour market as just one of many economic relations and with a corresponding lack of dignity (see Selucky, 1975).

In this paper we will take the view that the LM-firm's advantage is not simply that incentives are such that workers are prepared to work harder in LM-firms, a proposition well-known in the literature and well-discussed in Vanek (1970, Chapter 12), but rather that workers

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gain utility both from the mutual cooperation and spirit of teamwork encouraged by such incentives and from the lack of confrontation with employers in LM-firms. They then gain utility directly from an LM-firm environment as compared with a conventional firm's environment. We will take the conventional firm to be an entrepreneurial firm which we will specify more fully in Section II and denote EP-firm. We will model the shift in the utility function for identical workers in one type of firm as compared with another, and we will see that in our model it is the reduced *marginal* disutility from *extra* work effort that is important for predicting firm behaviour. We will also argue that there may exist relative economies in supervision costs in the LM-firms. As Vanek (1970, p. 238) says," if the private employer wants to produce anything, while paying a fixed contractual wage, the contract must explicitly or at least implicitly contain a provision regarding a minimum acceptable performance standard". Such a standard has to be enforced and the costs of such enforcement may be less (although of course not eliminated) in an LM-firm than in an EP-firm. The mechanism by which these assumed efficiency advantages feed through to influence productivity and other aspects of firm performance will be the primary target of our analysis, although we will also consider the distribution of gains from increasing worker participation within the context of an EP-firm.

One of the major reasons why a discussion of the direct environmental effects on individuals' utilities appears desirable is the common practice of largely ignoring the alienation of labour and related questions in theoretical comparisons of LM-firms with conventional firms. Domar (1966), after comparing LM- and other firms assuming a common relationship between inputs and outputs ends with a final caveat (p. 49): "Judged by strictly economic criteria the coop has not come out well. But even on these grounds, it is quite possible that a coop may be more efficient than a capitalist or state-owned firm in societies where membership in the coop, as contrasted with hiring out for a job, has a strong positive effect on workers incentives...". Although Domar is almost certainly referring here to the incentive effects of an income schedule (rather than a given wage) on the level of effort, our point is that in addition to this there is a direct environmental effect from membership in the coop or LM-firm. When some specific attention has been paid to the alienation — firm type — productivity question it has usually been conceived of as an upward shift in the production function for the LM-firm, which blurs the direct environmental and incentive effects. For example, Carsen (1973) considers a reduction in X-inefficiency as an advantage of the LM-firm. Neither the basis for assuming such a result from alienation reduction nor the implications for firm behaviour are pursued. One of our objectives here is to consider the extent to which this approach can be justified within the context of a simple model where a given supply of labour services, the supply of supervision services (by the entrepreneur in the EP-firm and the worker-managers in the LM-firm) and supply of risk-bearing services all create disutilities which are not independent of firm type.

One interesting observation that can be made here very quickly, however, is that if the result of switching from a conventional firm to an LM-firm organisation is purely an upward shift in production by a multiple $\theta > 1$ for all input levels, then the effect on the LM-firm's behaviour when just membership is variable would be the same as that of an increase in product prices, referred to above, except that output may increase or decrease dependent on the amount membership is reduced and the value of θ.

In Section II we present a specific model which links directly the productivity gain to the assumed utility function change as a result of an improved working environment. In Section III we extend the partial equilibrium analysis of Section II by considering general equilibrium aspects of the production sector. Section IV contains a summary of results and some discussion concerning gains from worker-participation within conventional entrepreneurial firms. Although this latter subject has been treated to some extent by Steinherr (1977), his work related mostly to the optimal level of worker participation in worker-manager contracts.

In all the above analysis we take the role of the entrepreneur to be that of a manager supplying managerial services alone. If, however, the firm exists in a risky environment then the entrepreneur (in the EP-firm) or the worker-managers (in the LM-firm) are supplying the service of risk-bearing. In Section V, we conclude our analysis by adapting the model to incorporate this possibility by the use of Arrow-Pratt risk premiums.

II

We will assume in this Section the same given capital stock for both LM-firms and EP-firms. Also product prices and fixed costs are independent of firm type, and all parameters are known with certainty. A firm's net revenue R is defined as the given product price (p) times output (Q) minus fixed costs, and is a strictly concave function R(E) of the total supply of labour in efficiency units (E). The functional form of the net revenue function is again independent of firm type. What is not independent of firm type is firstly the firm's objective and secondly the utility function of individuals associated with the firm. We will assume for simplicity that all individuals are identical, and each individual seeks to maximise his utility which is dependent on his income (y) and on his own supply of both entrepreneurial and work effort. A particularly simple form of utility function will be used partly to avoid problems of income effects in labour supply and partly to ensure an equal ranking of the two types of firm in a utilitarian assessment in the absence of direct environmental advantages.

Thus the utility function of an individual who is employed as a worker in an EP-firm but undertakes no entrepreneurial activities, will be written.

$$U_w = w - \beta(x); \quad \beta'(x) > 0, \ \beta''(x) > 0 \qquad (1)$$

where w is the wage income and $\beta(x)$ is the disutility incurred by the individual from supplying to the firm x efficiency units of labour. In general x measures "effort", while a more limited interpretation would be "hours worked".

Individuals in an economy of EP-firms (an EP-economy) can also become entrepreneurs, in which case they gain additional income, profits (π) but incur additional costs in terms of the disutility of hiring, organising and supervising labour. We assume this disutility to be of the form $\alpha(x) \cdot L$, where L is the number of workers and x the common number of efficiency units they each supply. Total effort supplied (E) is simply $x \cdot L$.[1] The worker-enterpreneur's utility is thus:

$$U_e = w - \beta(x) + \pi - \alpha(x).L \ ; \ \alpha'(x) \geq 0 \quad \alpha''(x) \geq 0 \qquad (2)$$

Note that because of the absence of income effects entrepreneurs will also wish to work providing $U_w > 0$ and workers will wish to become worker-entrepreneurs provided $U_e > U_w$.[2] Also, $\beta(x)$ is the disutility of his work effort borne by the worker and $\alpha(x)$ that borne by the entrepreneur. The entrepreneur seeks to maximise (2) by choosing w, x and L, but we will assume that he is faced with a competitive labour market which implies that workers will only accept employment provided $U_w \geq \bar{u}$, the competitive reservation utility, that is money wage minus disutility of work effort. Substituting $\bar{u} = U_w$ for w from (1) (as the entrepreneur will only wish to offer the minimum worker's utility) we can reformulate the entrepreneur's problem as

maximise with respect to x, L

$$U_e = \bar{u} + R - (\pi + \beta(x) + \alpha(x))\, L \qquad (3)$$

Thus the entrepreneur maximises net revenue minus the full labour costs, incorporating a breakdown of the wage rate (into a base "wage" ($\bar{u}$) and a compensation payment $\beta(x)$ and also the entrepreneurial costs of employment.

In the alternative LM-firm, the entrepreneurial role is assumed to be divided equally among the worker-members. Thus each worker has a share $\frac{1}{L}$ of profit but also bears his own entrepreneurial cost $\alpha(x)$.

[1] Note that bö assuming that R (.) is a function of E and that E = xL we are imposing a symmetry assumption on the production of labour services. A more general formulation would be to describe R as a function of x and L separately. This leads to added difficulties in analysis that remain to be explored.

[2] The fact that entrepreneurs 'work' implies that there are no 'idle rich' class in the entrepreneurial economy, and thus denies one possible efficiency advantage of a labour-managed economy. The absence of non-workers in the entrepreneurial economy is a logical consequence of the assumed utility function and allows other efficiency questions to be considered in isolation of the 'idle rich' phenomenum. It also allows a simplification in the analysis.

Then, in the absence of direct benefits from the LM-firm environment, an LM-firm's worker has utility

$$U_m = R/L - \beta(x) - \alpha(x) \qquad (4)$$

The particular utility functions we have used allow us to write from inspection of (1), (3) and (4) that for *given x and L*, we have

$$(L-1)\, U_w + U_e \equiv L \cdot U_m \qquad (5)$$

so that for given x, L, aggregate utility would be the same in the two systems, although neither the distribution of income nor utility need be. Established theory of the differences between LM-firms and profit-maximising firms lead us to suppose, however, that the choice of x, L will not be the same in the two types of firm except in long-run competitive equilibrium where profits are zero. For our model of the EP-firm this long-run competitive equilibrium is interpreted as $U_e^* = \bar{u}$, where U_e^* is the maximum value of (3). Then no individual would be better off in terms of utility by becoming or ceasing to be an entrepreneur. Results can be found relating the behaviour of EP-and LM-firms which are analogous to the established comparisons of LM- and profit-maximising firms, and some will be noted below. However, here we will proceed to the case where less alienation of labour occurs in the LM-firm which reduces either or both of $\alpha(x)$ and $\beta(x)$. In fact we will see that we will need to be rather more specific and assume that it is the disutility of marginal work effort that is reduced. Writing $g(x) = \alpha(x) + \beta(x)$ for the entrepreneurial firm, we will state that a corresponding expression for the LM-firm is $g_m(x)$, such that the marginal disutility of work effort is less everywhere, i.e.

$$g'_m(x) < g'(x) \qquad \text{all } x \qquad (6)$$

Now let us consider the conditions for optimal choice of x and L in the two types of firm. For the EP-firm, first order necessary conditions for maximising (3) are:

$$R'(E) - g'(x) = 0 \qquad (7)$$

$$R'(E).x = \bar{u} + g(x) \qquad (8)$$

Equation (7) states that the level of effort (x) should be chosen so as to equate the marginal net revenue product of an efficiency unit of labour with its marginal disutility. Equation (8) states that the marginal net revenue product of an additional worker should be equal to his full cost.

In the LM-firm, workers choose x,L to maximise their utility (4) and necessary conditions are:

$$R'(E) - g'_m(x) = 0 \qquad (9)$$

$$R'(E) = \frac{R(E)}{E} \tag{10}$$

If U_m^* is the maximum value of (4), then if $\bar{u} = U_m^*$ and $g(x) \equiv g_m(x)$ all x, (9) and (10) are identical to (7) and (8). It follows then by taking comparative statics of (7) and (8) for a change in $\bar{u}$ that $\frac{dx}{d\bar{u}} > 0$ and $\frac{dE}{d\bar{u}}, \frac{dL}{d\bar{u}} < 0$ for the EP-firm while the LM-firm is unaffected. Thus for $\bar{u} \gtreqless U_m^*$ which implies $\bar{u} \gtreqless U_e^*$ we have $x \gtreqless x_m$, $L \lesseqgtr L_m$ and $E \lesseqgtr E_m$, where the m subscript distinguishes LM-firm optimal values. These results conform to the standard analysis of the LM-firm when hours worked are variable, see for instance Berman (1977) and Bonin (1977).[3]

Now suppose that (6) holds and $g(x)$, $g_m(x)$ are different functions. Write x as E/L, and we can see that (9) and (10) are functions of E and L alone. Also if $U_e^* = \bar{u}$ and the EP-firm is in long-run competitive equilibrium then from (3), (8) can be rewritten as (10).

For a fixed L, denoted L_0, the functions $R'(E)$, $R(E)/E$, $g'(E/L_0)$ and $g_m'(E/L_0)$ can be drawn as functions of E, to form Figure 1. Note that $R(E)/E$ will always be intersected at its maximum by $R'(E)$. Also $g'(E/L)$ will shift upwards with a reduction in L. Now the EP-firm is in long-run equilibrium at an input of E_0 total labour efficiency units and a workforce of L_0, as at these values (7) and (10) are satisfied. The LM-firm is not in equilibrium at E_0, L_0 as (9) does not hold. The adjustment of the LM-firm to its equilibrium can be considered in two stages. In the short run, the number of worker-members is fixed and workers find it to their advantage to supply more efficiency units of labour: E_1 in total and E_1/L_0 per worker. At E_1, (9) holds but (10) does not. There would then be a tendency in the medium term for members who leave the LM-firm not to be replaced and the number of worker members would shrink. As this happens, $g'_m(E/L)$ would shift upwards until the number of workers reached L_1 such that (10) held. During this adjustment period the short-run condition (9) would continue to hold, and workers will supply more and more effort as the labour force contracts. Note that $g'_m(E/L_1)$ is not necessarily identical to $g'(E/L_0)$. However they both intersect with the $R(E)/E$ function at E_0. Thus after membership adjustment, the LM-firm will supply the same total work effort, earn the same net revenue and produce the same output as the entrepreneurial firm, but with less workers. A simple result which comes directly from the fact that (10) is independent of both the $g(x)$ and $g_m(x)$ functions and all variables other than E.

[3] The model is developed in these directions, for instance by considering comparative statics of price changes, as well as a number of other issues in N. J. Ireland and P. J. Law (1981).

Although the analysis here has assumed no income effects, the results above concerning effort and membership level in LM-firms for an improvement in work environment are unchanged if this assumption is dropped. Replace $g'_m(x)$ by MRS (x,y) in (9) to allow for income effects in the marginal rate of substitution (MRS) between effort and income. Equation (10) is still appropriate and is unchanged by a shift downwards in the MRS(x,y) function. Thus E remains unchanged with the *shift* of the MRS(x,y) function as does $R'(E)$, $R(E)/E$ and (from (9)) the optimal *value* of MRS(x,y). Thus a compensating change in x, dx, must have occurred such that

$$\frac{d\,MRS\,(x,y)}{dx} > 0$$

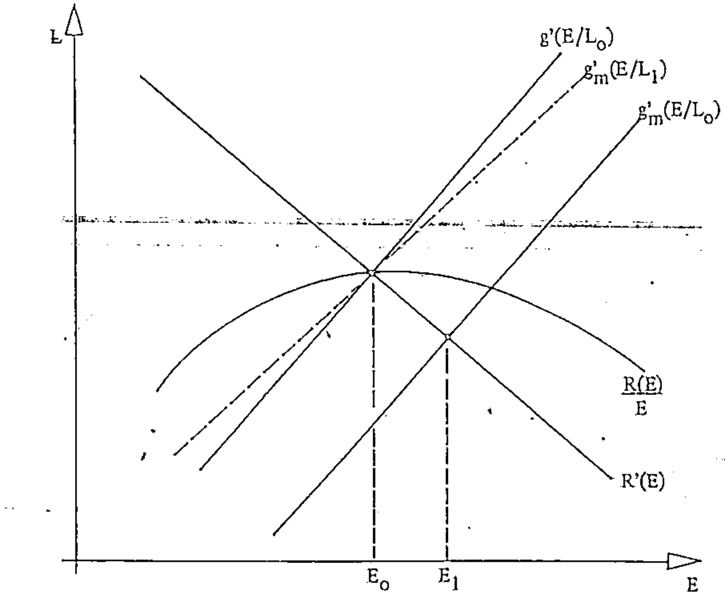


Figure 1

that is, as $y = x.R/E$ and R/E is constant, we have

$$\left[\frac{\partial MRS}{\partial x} + \frac{\partial MRS}{\partial x}\frac{R}{E}\right] dx > 0$$

The term in square brackets is positive from second-order conditions for x to be optimal for fixed E. Thus $dx > 0$ and as E is unchanged $dL < 0$, thus confirming the previous results for this more general case.

III

Although the analysis in the last section was of a partial equilibrium nature, some aspects carry through to a general equilibrium approach. Consider an economy where production takes place in either type of firm, but where the EP-firm sector is in long-run equilibrium ($\bar{u} = U_c^*$), all LM-firms are in equilibrium as defined by (9) and (10), and where all prices ar given constants, perhaps determined by a dominant foreign sector. Take two extreme situations: one where identical EP-firms constitute the entire economy (the EP-economy), and the other where there are no EP-firms, only identical LM-firms (the LM-economy). In both economies we also assume that full employment is achieved. In the absence of productivity differences arising from different disutilities of work the two extreme situations would give rise to exactly the same outputs supplied and inputs demanded by firms. Now suppose disutility of work is less in the LM-firms as described by (6). The results of Section II tell us that each LM-firm will employ less members but produce the same as an EP-firm in the EP-economy. However this would mean that more LM-firms would exist in order to satisfy full-employment. Each firm would still demand the same level of fixed inputs such as capital unless the prices of their inputs changed. Thus it is in the aggregate demand by the economy for, say, capital that general equilibrium considerations need to be taken into account, and it is this topic that will concern us here. Of course we could have focussed interest on the consumption side by not taking product prices as exogenous to the economy, but it seems reasonable to fully consider the production sector before making such extensions.

In a situation where firms buy capital with their own or their members' finance, lack of ownership rights in some versions of the LM-firm (such as the Yugoslav firm, see Furubotn and Pejovich (1973)) suggest reduced demand for capital by these firms. With such internal finance, comparison of the demand for capital goods between the LM-economy and the EP-economy is bound to be ambiguous. We will proceed, however, by assuming that in each economy there is an identical rentier institution which owns all the capital and rents this out to firms at a rental which equates supply and demand of capital. If there is a perfectly elastic supply of capital, then, as the same E holds for both types of firm and thus the marginal revenue product of capital is equated to the given rental at the same level of capital, the analysis of Section II holds. Alternatively, we might assume that the rentiers have the same fixed supply of capital in each economy and fix the rentals to clear the respective markets. Of course, the rentier may distribute the proceeds to individuals, but, providing this is done in a non-distortive way, this will not complicate the analysis. The rental on capital will be such as to distribute the available capital equally between the identical firms. Thus firms in the LM-economy will have less capital each than those in the EP-economy. This will affect both the marginal net revenue product and the average net revenue product of an efficiency unit of labour, and will feed back into the LM-firm's decisions as regards membership size and work effort. It is not a priori obvious that work effort per worker would still be higher in the LM-economy as a result of lower disutility of work, nor that aggregate output would be higher. If we can establish these points with fixed aggregate capital, however, it would seem reasonable a fortiori that they would also hold if aggregate capital could respond positively to the higher demand.

The general equilibrium is assumed to be defined by the following conditions holding for each identical LM-firm, where $pQ(E, K)$ is the strictly concave revenue fuction with negative-definite hessian, r the capital rental and p the given product price, so that income per nember is $(pQ - rK)/L$. Note that the assumption of concavity is not sufficient for an interior global optimum to exist. For a discussion of related points concerning dividend maximisation when all factors are variable see Ireland and Law (1980).

$$p\frac{\partial Q}{\partial E} = g'_m(E/L) \tag{9a}$$

$$p\frac{\partial Q}{\partial E} = (pQ - rK)/E \tag{10a}$$

$$p\frac{\partial Q}{\partial K} = r \tag{11}$$

$$K = hL \tag{12}$$

where (9a) and (10a) are just restatements of (9) and (10), (11) is the condition for optimal capital and (12) is the fixed total capital condition expressed as a fixed capital to labour ratio h. By substituting (11) and (12) into (9a) and (10a) the following comparative statis results are derived in the neighbourhood of equilibrium for a small change in the parameter Φ when

$$g'_m(E/L) \equiv \Phi\, g'(E/L) \tag{13}$$

$$\frac{dE}{d\Phi} = C\left\{\frac{\partial^2 Q}{\partial K^2}K + \frac{\partial^2 Q}{\partial K \partial E}E\right\} \tag{14}$$

$$\frac{dK}{d\Phi} = -C\left\{\frac{\partial^2 Q}{\partial E^2}E + \frac{\partial^2 Q}{\partial E \partial K}K\right\} \tag{15}$$

and $C = h\,g'(E/L)/(E.\Delta)$

$$\text{when}\quad \Delta = pK^2 \left\{ \frac{\partial^2 Q}{\partial E^2} \frac{\partial^2 Q}{\partial K^2} - \left(\frac{\partial^2 Q}{\partial E \partial K} \right)^2 \right\} / L - \Phi\, g''(E/L)$$

$$\left[2 \frac{\partial^2 Q}{\partial E \partial K} EK + E^2 \frac{\partial^2 Q}{\partial E^2} + K^2 \frac{\partial^2 Q}{\partial K^2} \right] / L^2 \qquad (16)$$

As C is positive from the negative definiteness of the hessian of the production function, the signs of (14) and (15) depend upon the signs of their bracketed terms. At least one, but not necessarily both, of the brackets will be negative by concavity of the production function. Whatever the signs of (14) and (15) we can relate them to the change in optimal K and E due to a change in the product price p. We have:

$$\frac{dE}{dp} = -\left\{ \frac{\partial Q}{\partial E} / g'(E/L) \right\} \frac{dE}{d\Phi}$$

$$\text{and} \quad \frac{dK}{dp} = -\left\{ \frac{\partial Q}{\partial E} / g'(E/L) \right\} \frac{dK}{d\Phi}$$

Thus qualitatively the effects of a reduction in Φ are the same as those of an increase in product price.

Similar expressions to $\frac{\partial^2 Q}{\partial E^2} \cdot E + \frac{\partial^2 Q}{\partial E \partial K} \cdot K$ have appeared in other areas of analysis. For instance Baumol and Klevorick (1970) focussed attention on the sign of such an expression which needed to be negative for the capital-labour ratio to increase in a regulated firm when the regulation was tightened (maximum rate of return on capital reduced). They took the view that such negativity "is obviously not necessarily true, nor is it even easily interpretable". (Baumol and Klevorick 1970, p. 179). However, a simple interpretation is that if $\frac{\partial Q}{\partial E}$ *decreases* for a one per cent *increase* in both K and E then negativity holds. Similary if $\frac{\partial Q}{\partial K}$ *decreases* then $\frac{\partial^2 Q}{\partial K^2} K + \frac{\partial^2 Q}{\partial K \partial E} E$ is negative. We argue that marginal products reducing with scale increases provides a reasonable basis for assuming negativity and thus allocate signs for (14) and (15) of negative and positive respectively. Of course, there do exist production functions which would invalidate this allocation. (An example would be $Q = K \log(E + a) + b \log(K + a)$ for $K < b - a$, $b > a > 0$. Here (15) would be negative. Note, however, that the example of Baumol and Klevorick (1970, p. 179) is inadmissable as revenue is negative for all non-trivial input levels.)

All we will do here is to simply note the existence of perverse production functions while continuing the analysis on the assumption that production is well-behaved in that all marginal products decrease along rays from the origin.

The effect of a change in Φ on the firm's output level can be characterised by the relative responses of the marginal products to a scale increase in the factor inputs K and E. Define

$$\frac{\partial Q}{\partial K} = Q_K(\lambda E, \lambda K) = Q_K(\lambda)$$

$$\frac{\partial Q}{\partial E} Q_E = (\lambda E, \lambda K) = Q_E(\lambda)$$

then as

$$\frac{dQ}{d\Phi} = \frac{\partial Q}{\partial K} \frac{dK}{d\Phi} + \frac{\partial Q}{\partial E} \frac{dE}{d\Phi}$$

we can write

$$\frac{dQ}{d\Phi} = C \left\{ \frac{dQ_K}{d\lambda} \frac{\partial Q}{\partial E} - \frac{dQ_E}{d\lambda} \frac{\partial Q}{\partial K} \right\}$$

$$= C \frac{\partial Q}{\partial E} \frac{\partial Q}{\partial K} \left\{ \frac{d \ln Q_K}{d\lambda} - \frac{d \ln Q_E}{d\lambda} \right\}$$

and

$$\frac{dQ}{d\Phi} \gtreqless 0 \quad \text{as} \quad \frac{d \ln Q_K}{d\lambda} \gtreqless \frac{d \ln Q_E}{d\lambda}$$

That is output will increase (decrease) for a reduction in Φ if the percentage change in the marginal product of an efficiency unit of labour is greater than (less than) the percentage change in the marginal product of capital for a one per cent scale increase in input levels. The same characterisation relates to the effects of a product price increase.

We also obtain from $x = hE/K$ that

$$\frac{dx}{d\Phi} = C\left\{\frac{\partial^2 Q}{\partial K^2}K^2 + 2\frac{\partial^2 Q}{\partial K \partial E}KE + \frac{\partial^2 Q}{\partial E^2}E^2\right\} \tag{17}$$

which is negative as $C > 0$ from (16) and negative-definiteness of the hessian of Q(.). Note also that $\frac{dK}{d\Phi}$ and $\frac{dL}{d\Phi}$ have the same signs from (12). Aggregate output over all firms changes in proportion to average productivity, and

$$\frac{d(Q/L)}{d\Phi} = \left[\frac{\partial Q}{\partial E}\frac{dE}{d\Phi}L + \left(\frac{\partial Q}{\partial K}K - Q\right)\frac{dL}{d\Phi}\right]/L^2 \tag{18}$$

Using (10a) and (11) this simplifies to

$$\frac{d(Q/L)}{d\Phi} = \frac{\partial Q}{\partial E}\frac{dx}{d\Phi}$$

which is clearly negative.

Also we can show that $\frac{dr}{d\Phi} < 0$ independent of whether factors are substitutes or complements. Using (14) and (15), we have

$$\frac{dr}{d\Phi} = p\frac{d}{d\Phi}\left(\frac{\partial Q}{\partial K}\right) = -E\,C\,p\left\{\frac{\partial^2 Q}{\partial E^2}\cdot\frac{\partial^2 Q}{\partial K^2} - \left[\frac{\partial^2 Q}{\partial K \partial E}\right]^2\right\} < 0$$

The final comparative static result concerns the utility of a member. We have

$$\frac{dU_m}{d\Phi} = \frac{-d\,g(x, \Phi)}{d\Phi} - h\frac{dr}{d\Phi} \tag{20}$$

where $\frac{dg(x, \Phi)}{d\Phi}$ is the change in the total disutility of supplying factor services when the *marginal disutility* shifts according to (13).

There is obviously no assurance that (20) is negative, unless the extra rental is distributed back to the members of the firms by the rentier. If the extra rental is used to buy more capital goods for the next period's production then the members may eventually benefit

from what is in effect forced savings. However if the rentier distributes his profit abroad or consumes it himself, then his monopoly position may be such as to allow him to appropriate all the efficiency gain and more beside.

Thus as Φ decreases from unity, moving the LM-economy away from the long-run EP-economy equilibrium, effort per worker and average productivity per worker increases. Also if aggregate capital is fixed, and marginal products decline along linear paths from the origin, then total efficiency units of labour per firm increase but capital and number of workers per firm falls. Also the worker-members will only be definitely better-off if all the extra rental generated by the increased demand for capital (and demand for capital will always be increased) is distributed to the worker-members.

If income effects are allowed to enter the model, then the effort level may move either way (in contrast to the fixed r case). However we can use 10A, 11 and 12 to show how E and r will change in the same, and L and R in the opposite, direction to x, provided that both marginal products decline along linear paths from the origin.

IV

We have been concerned so far in a consideration of the effects of an improved work environment as a result of reorganising firms under collective rather than individual entrepreneurship. The results of this analysis are summarised in the first row of the Table. In the first set of columns, results from Section II are reported. These relate to responses of a single firm to such reorganisation from an initial situation of an EP-firm in long-run equilibrium. They also relate to an economy-wide reorganisation provided the supply of capital is perfectly elastic. This is of course because, in the absence of a change in capital rental, the labour input (in efficiency units) per firm is unchanged and so the same capital level solves (11). The right-hand set of columns constitute the results of Section III where capital in the economy is assumed fixed, so that reorganisation of the firms in the economy under collective entrepreneurship, which implies more firms with less members each, leads to an increase in the equilibrium capital rental.

An alternative way of improving work environment may be by maintaining individual entrepreneurship but involving some worker participation and self-supervision, counterbalancing this with incentives in terms of profit shares, etc., but retaining the overall objective of maximising the entrepreneur's utility. Such systems may involve problems of the agent-principal kind (see Ross, 1974). In such an environment reduced disutility of work may occur, and this could be due to a reduction of either or both constituent parts of g(x). A partial equilibrium analysis in such a case is both simple and instructive, if we continue to assume a competitive labour market with equilibrium worker's utility of $\bar{u}$. The entrepreneur again chooses the number of workers (L) and the effort level required from each (x), in order to maximise his utility subject to

$$\bar{u} = y - \beta^*(x)$$

where y is worker's generalised income which may include a profit share, etc., and $\beta^*(x)$ is the worker's disutility of supplying x efficiency units in this improved participatory environment. Obviously, the competitive labour market, if sustained, means that workers cannot improve their utility above ū: all gains from reduced workers' and entrepreneur's disutility of effort are available to the entrepreneur. Thus workers in an EP-firm would not be keen to initiate or agree to such a change in organisation unless it was accompanied by a measure of worker-control, which would approximate the firm to an LM-firm, or was part of a general economy-wide movement. Even in the latter case, workers will only unambiguously gain in the long run where $U_e = \bar{u}$ and when capital is in perfectly elastic supply. This is because the response of each firm to the improved disutility is ambiguous in respect to the demand for number of workers employed. If aggregate demand for labour were to fall ū would be forced downwards and workers would be worse off in the short run, when the number of entrepreneurs, and thus firms, is fixed at the initial level. Furthermore, the presence of barriers to becoming entrepreneurs may make the long run heavily discounted by workers.

The source of the short-run ambiguity of the change in ū to an improved work environment can be seen by using a parametric shift in the g(x) function. Let this shift to $\Phi g(x)$, $\Phi < 1$, with the improved work environment. Following from (7) and (8) we have the EP-firm's equilibrium defined by:

$$p\frac{\partial Q}{\partial E} = \phi\, g'(E/L) \tag{7a}$$

$$p\frac{\partial Q}{\partial E}\cdot E/L = \bar{u} + \phi\, g(E/L) \tag{8a}$$

If capital is also a variable we also have (11) and *either* the rental on capital is a fixed price (under the assumption of perfectly elastic supply of capital) *or* (12) holds under the assumption of a fixed aggregate stock of capital in the economy. Also in the short run when the number of entrepreneurs is fixed but full-employment is still required, entrepreneurs will not change their employment of workers ex post, although their demand schedules may have shifted. Thus the equilibrium system is completed by the requirement that in each firm L is constant in the short run:

$$L = \bar{L} \tag{21}$$

while in the long run (21) is replaced by $U_e^* = U_w$. As we have argued, long-run equilibria are indistinguishable from those of the comparable (same Φ) LM-economy. We shall therefore confine ourselves to comparative statics of the short run case, defined by (7a), (8a), (11), (21) and either r as an exogenous constant or (12). Consider the fixed aggregate capital stock assumption (12) first. From (12) and (21), both K and L are constant. (7a), (8a) and (11) can then be totally differentiated to find the response of E, ū and r to a change in Φ. We obtain

$$\frac{dE}{d\Phi} = g'(E/L)/(p\frac{\partial^2 Q}{\partial E^2} - \phi\, g'(E/L)) < 0 \tag{22}$$

(and thus $\frac{dx}{d\phi} < 0$)

$$\frac{dr}{d\phi} = p\frac{\partial^2 Q}{\partial K \partial E}\frac{dE}{d\phi} \tag{23}$$

which is negative if $\frac{\partial^2 Q}{\partial K \partial E} > 0$, i.e. inputs are complements,

and

$$\frac{d\bar{u}}{d\phi} = (\frac{\partial^2 Q}{\partial E^2} pg'(E/L)\cdot E)\,/\,(\frac{\partial^2 Q}{\partial E^2} pL - \phi\, g''(E/L)) - g(E/L)) \tag{24}$$

Note that (24) is ambiguous in sign. One factor tending to depress $\bar{u}$ is the diminishing returns to labour, for as workers supply more effort, this diminishes the product of the marginal worker and thus the demand for workers. However, against this is the fact that, the bigger is $g(x)$, the bigger is the reduction in the cost of employing that marginal worker, caused by the reduction in ϕ.

The case where capital is in perfectly elastic supply can be considered by taking r and L as fixed and solving (7a), (8a) and (11) for changes in K, E and $\bar{u}$ in response to the change in ϕ. This yields the same qualitative results for E, x and $\bar{u}$ and also $\frac{dK}{d\phi}$ has the same sign as $\frac{dr}{d\phi}$ in (23).

The comparative static results are summarised in the middle row of the Table. While these results relate to an economy-wide change in ϕ and thus change $\bar{u}$, the case of a single firm adopting a better work environment is reported in the bottom row of the Table. Here $\bar{u}$ is exogenous and fixed, as is r, and (7a), (8a) and (11) are used to solve for $\frac{dE}{d\phi}$, $\frac{dK}{d\phi}$ and $\frac{dL}{d\phi}$.

In the analysis of the EP-firm as opposed to the LM-firm, the shift in $g(x)$ rather than $g'(x)$ has had to be considered. This is because $g(x)$ appears in (8a) and $U_e^* \neq U_w$ after the change in ϕ. The parametric shift we have considered means that both $g(x)$ and $g'(x)$ change in the same proportion. This may be significant as the change in $g'(x)$ effects the change in x and thus, through the diminishing marginal revenue product of labour, the reduction in the marginal revenue product of a worker, while $g(x)$ effects the amount the marginal cost of a worker is reduced, due to reduced disutility of work and hence less required compensation. It is the interplay of these two effects which determine the sign of $\frac{d\bar{u}}{d\phi}$ in the general equilibrium analysis and $\frac{dL}{d\phi}$ in the partial equilibrium analysis of the EP-firm case.

V

The model is capable of adaptation to analyse a number of specific extensions and complications. We will pursue only one rather obvious extension here: that of incorporating risk and risk aversion into the model. We can only indicate an approach here that may be thought interesting. It is likely however that the question of risk in comparative economic systems is dominated by the infra structure for risk-sharing — stock markets, insurance, etc. We will ignore such possibilities and assent that the entrepreneur in the EP-firm and the members of the LM-firm bear all the risk. Assume that the source of the risk is the product price p. Also that production takes place prior to the price being revealed, but that in both kinds of firm decision makers have a common subjective probability distribution function concerning the value of p. In particular the mean and variance of price are known to be $\bar{p}$ and σ^2. We will further assume that capital is fixed in both firms at the same level and ignore capital market considerations. However both x and L are decided before the price is revealed, and it is the joint decisions concerning these variables that must be the subject of our attention.

We will build risk aversion into the model by assuming that all individuals wish to maximise the expected value (using $\in$ as the expectation operator) of a strictly concave monotonic-increasing transformation T of utility u, that is each individual wishes to maximise

$$\in T(u), \quad T'(.) > 0, \quad T''(.) < 0 \qquad (25)$$

	Partial equilibrium analysis of single firm *or* general equilibrium of economy-wide change with perfectly elastic supply of capital							General equilibrium analysis of economy-wide change assuming fixed aggregate capital in economy.						
	dx	dL	dE	dK	dQ	d(Q/L)	dū	dx	dL	dE	dK	dQ	d(Q/L)	dū
LM-firm *or* EP-firm in long-run equilibrium with $U_e^* = \bar{u}$	+	—	0	0	0	+	+	+	—*	+*	—*	?	+	?
EP-economy with fixed number of firms and full employment	+	0	+	+**	+	+	?	+	0	+	0	+	+	?
EP-firm with ū fixed	+	?	+	+**	+									

Table: Comparative Static results of a proportionate reduction in the disutility of effort (g(x)).
* Assumes both marginal products decrease along straight line paths from the origin in (K, E) space.
** Assumes capital and efficiency units of labour are complementary inputs.

We will approximate (25) with $T(\mathcal{E}(u) - \eta)$ where $\mathcal{E}(u)$ is the expected value of u, i.e. expected income minus disutility of effort, and η is the Arrow-Pratt risk premium given by

$$\eta_e = -\frac{1}{2}\frac{T''(\mathcal{E}(u))}{T'(\mathcal{E}(u))}Q^2\sigma^2 = \frac{1}{2}A_e Q^2 \sigma^2 \qquad (26)$$

for the EP-firm's entrepreneur and

$$\eta_m = \frac{1}{2}A_m\left(\frac{Q}{L}\right)^2\sigma^2 \qquad (27)$$

for the LM-firm's member where A_e, A_m are their respective coefficients of absolute risk aversion evaluated at expected income minus disutility of effort. The EP-firm's workers are not faced with any risk, so that $\eta_w = 0$.

Provided that $\eta_e > L \cdot \eta_m$ there is an efficiency gain for the LM-firm. However little should be read into this, as apart from the mechanisms for risk-sharing which may be available for EP-firms but not for LM-firms, there is also the question of income and wealth distributions. The EP-firm may occur when the entrepreneur has wealth such that he is much less risk averse than the typical LM-firm member.

It is of interest to see, however, how the existence of risk changes the decisions of the two types of firm. The objective function of the LM-firm is

$$\mathcal{E}(U_m) - \eta_m = R/L - g_m(x) - \frac{1}{2}A_m\left(\frac{Q}{L}\right)^2\sigma^2 \qquad (28)$$

where R is expected revenue net of non-labour costs. Optimal x and L are given by

$$R'(E) - g_m'(x) = B\,Q'(E)$$

$$R'(E) - \frac{R}{E} = B(Q'(E) - Q/E)$$

where $B = (A_m \frac{Q}{L}\sigma^2) / (1 - \frac{1}{2}A_m'(\mathcal{E}\,u)\,(Q/L)^2\sigma^2$

and $B > 0$ if $A_m'(\mathcal{E}\,u) \leq 0$, i.e. non-increasing absolute risk aversion.

Thus here the optimal supply of efficiency units of labour is greater than at E_0 in Figure 1, as $Q'(E) < Q/E$ from concavity. Also

$R/E > R'(E) > g_m'(x)$ implies that x is lower under uncertainty and thus L must be higher. The solution is depicted as E^* in Figure 2. The result concerning the number of workers mirrors that of Muzondo (1979), and Hey and Suckling (1979). The proposition that members will work less hard under uncertainty has some intuitive appeal given risk aversion as members are opting for the non-risky consumption of leisure.

Finally consider the EP-firm. The same approach applied to the entrepreneur's objective function of

$$\mathcal{E}(U_e) - \eta_e = \bar{u} + R - (\bar{u} + g(x))\,L - \frac{1}{2}A_e(\mathcal{E}(u))\,Q^2\sigma^2 \qquad (29)$$

yields, if $A_e(\cdot)$ is non-increasing in its argument:

$$R'(E) - g'(x) = R'(E) - \left\{\frac{\bar{u} + g(x)}{x}\right\} > 0$$

Thus $R'(E) > g'(x) = \frac{\bar{u} + g(x)}{x}$, and in the long-run equilibrium where

$\mathcal{E}\,U_e - \eta_e = \bar{u}$, we have

$$\frac{\bar{u} + g(x)}{x} = \left(R - \frac{1}{2}A_e Q^2\sigma^2\right) / E$$

so that $R'(E) > g'(x) > R/E$.

Inspection of Figure 1 shows us that the equilibrium labour input will be less than E_0, and the number of workers will be less than in the certainty case as the g' ( ) function has shifted to the left. The solution is depicted as E^* in Figure 3. However, we cannot say whether the level of effort per worker has increased or decreased, (i.e. whether a or b is higher in Figure 3). The smaller number of workers and smaller output per firm is to be expected given the results of Sandmo (1971).

Figure 2

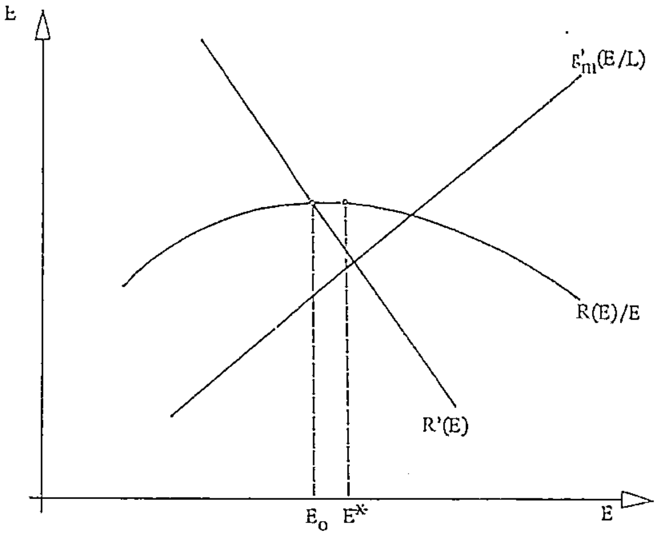
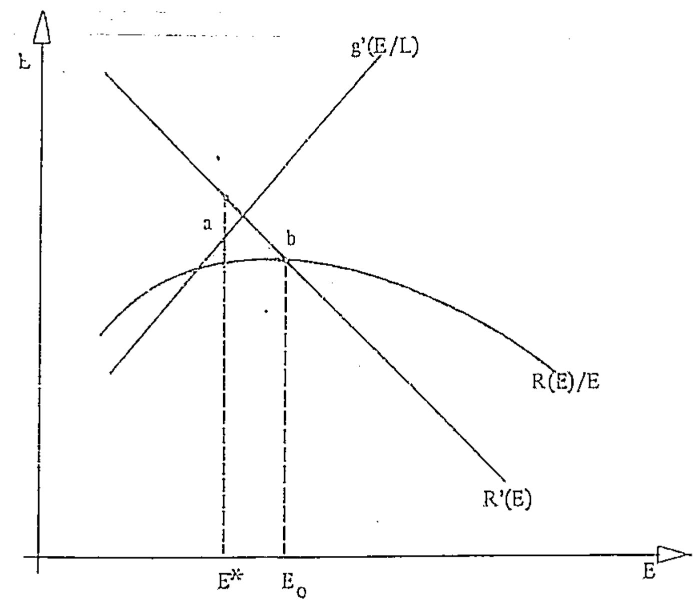


Figure 3



VI

In this paper we have tried to emphasise important aspects in the comparison of economic systems that do not appear to have attracted much attention before. The key to the model is that the supply of given labour, managerial or risk bearing services involved disutility but that this disutility may well be different as between labour-managed and entrepreneurial systems. This difference may imply different optimal decisions concerning the supply of such services, and these decisions may not conform to the traditional view that labour-managed firms produce less output than entrepreneurial firms because of their Illyrian objective function.

In order to analyse the model, a number of restrictive assumptions have been necessary, particularly the ignoring of income effects in the supply of effort. On the other hand, although the consumption sector has been treated as exogenous by the assumption of given product prices, the market for rental capital has been included in the analysis. In particular, the intuitively appealing assumption of fixed aggregate capital stock has been considered.

In the Table, the picture of firms in the LM-economy employing smaller numbers of workers but with higher productivity per worker (and thus possibly higher output per firm) is clearly seen. Also an explanation is given for the often perceived hesitancy of labour unions to invoke worker participation short of worker control, and the impact of different production systems on the market for capital is shown. Furthermore the model considers the distribution of utility in addition to that of income, as well as concepts of equilibrium.

In the final section of the paper, recent results concerning the behaviour of labour-managed and profit-maximising competitive firms are recreated for the utility maximisers in our model using the device of risk premiums. This involves a slight variation on the form of the maximand compared with the earlier sections but the results concerning comparative effort per worker and workers per firm reinforce the results for when effort per worker is fixed, particularly in the LM-firm case, where membership increases by a bigger percentage than efficiency units of labour.

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PONAŠANJE SAMOUPRAVNOG PREDUZEĆA I DISUTILITET RADA

Norman J. IRELAND

Rezime

U uslovima ravnoteže na dugi rok, različiti načini organizovanja privrede ponajčešće su ekvivalentni u odsustvu bilo kakvih razlika u tehnologiji. Tvrđeno je da radničko samoupravljanje ima veću produktivnost nego preduzetnička organizacija zbog izvesne tehnološke superiornosti. U ovom članku mi posmatramo tu tehnološku superiornost kao endogenu, koja proizlazi iz nižeg marginalnog disutiliteta individualne ponude rada. Sistemi parcijalne i opšte ravnoteže samoupravnih i preduzetničkih organizacija kontrastirani su u uslovima postojanja ove prednosti utilitetne proizvodne funkcije samoupravnog preduzeća. Pravi se razlika između disutiliteta rada radnika i disutiliteta preduzetničkog napora. Mnoge su pretpostavke postavljene radi pojednostavljenja analize u meri koja omogućava njen dalji razvoj, i u većem delu članka posmatra se samo svet izvesnosti.

Komparativna analiza sistema opšte ravnoteže izvodi se pod pretpostavkama parametarskih cena gotovih proizvoda i pune zaposlenosti i rada i kapitala. Puna zaposlenost rada u samoupravnoj privredi postiže se formiranjem i rasformiranjem preduzeća; u preduzetničkoj privredi ona se ostvaruje pomoću konkurentskog tržišta rada takvog da se svako preduzeće suočava sa minimalnim utilitetnim nivoom radnika. Dva polarna slučaja tržišta kapitala razmatraju se jedan za drugim. Prvo je agregatni kapital cele privrede fiksiran, a rentijer fiksira rental da bi uravnotežio ponudu i tražnju, a potom se pretpostavlja da je ponuda kapitala beskonačno elastična pri datom rentalu. Brojni rezultati su dobijeni; na primer, kada važi prva pretpostavka, koja se odnosi na tržište kapitala, jedinice efikasnosti rada po radniku, jedinice ukupne efikasnosti rada po preduzeću i proizvodnja po radniku verovatno su veće u samoupravnoj privredi, broj radnika i količina kapitala po preduzeću su manji, a poređenje proizvodnje po preduzeću i utiliteta po radniku je dvosmisleno.

Model je proširen da bi se razmotrila privreda preduzetničkih organizacija koja postiže manji disutilitet ponude rada (pomoću radničke participacije, na primer). Na kraju je neizvesnost uključena u model i analizirana je pomoću premija za rizik. Dobijeni su rezultati sa varijabilnim individualnim radnim naporom koji odgovaraju ranije publikovanim rezultatima sa fiksnim individualnim radnim naporom.