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OPTIMIZING MULTIPHASE BUSINESS PROCESSES

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When optimizing the multiphase business process one may use the method of linear optimization. For this reason one must break down the process into several elementary processes and represent it by a graph to facilitate the construction of a mathematical model. If such a model is linear it can be used for the optimization of the processes of large systems, it not if should be firstly linearized.

1. BASIC MODEL

Let us consider a business process consisting of the purchase and transport of the production elements, the production, the sale and the transport of the products. Usually production represents the largest part of the business process. In order to get a good survey of the business process it is therefore necessary to break down production into several elementary production processes or, as it may be said, into modes of production at which the products are obtained from the production elements. These products may appear as production elements at the technological procedures of the following production phases, as final products which are sold, or they can be partially returned as elements into previous phases. To simplify the expressions let us denote production elements, semiproducts and final products as elements.

The mathematical model is constructed so that to each elementary process, in which the modes of production and modes of purchase

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and sale of the single element are included, a non-negative decision variable is associated. Denote by x_j the wanted quantity of production for the j-th mode of production, by e_i the quantity purchased of the i-th element and by e'_i the quantity sold of the i-th element. In most cases we may suppose that the normatives of consumption and the normatives of production of the elements are independent of the values of decision variables. Denote by a_{ij} the normative of the consumption of the i-th element at the j-th technological procedure and by a'_{ji} the normative of the production of the i-th element in the j-th technological procedure. In that case we can associate the following linear inequation to each element:

$$\sum_{j \in P_i} a'_{ji} x_j - \sum_{j \in N_i} a_{ij} x_j + e_i - e'_i \geqq 0 \qquad (1)$$

where P_i presents the index set of the technological procedures at which the i-th element is produced and N_i the index set of the technological procedures at which the i-th element is consumed.

By (1) the condition is stated that there cannot be more of the i-th element consumed or sold than is produced or purchased.

Bounded capacities of sources of the i-th element are considered in the inequation:

$$e_i \leqq b_i \qquad (2)$$

where b_i represents the available quantity of the i-th element. Bounded needs of consumers of the i-th element are considered in the inequation:

$$e'_i \leqq b'_i \qquad (3)$$

where b'_i represents the needs of consumers of the i-th element.

If we denote by c_i the purchase price of the i-th element reduced for the marginal cost of purchase, by c'_i the sale price of the i-th element reduced for the marginal cost of sale and by m_j the marginal costs of the j-th elementary process which are not included in the already considered elements, then we get the difference between revenues and expenditures in the form of the function:

$$z = \sum_i c'_i e'_i - \sum_i c_i e_i - \sum_j m_j x_j \qquad (4)$$

where the first sum refers to all elements which should be sold, the second sum to the elements which should be purchased, and the third sum to all elementary processes. When the prices c_i, c'_i and marginal costs m_j are independent of the decision variables, then (4) is a linear function.

From the conditions stated the mathematical model is obtained in the form of the maximum of the function (4) subject to the constraints (1), (2), (3) and subject to the non-negativity constraints.

2. LINEARIZATION OF THE MODEL

Let us consider a more general problem in which the normatives a_{ij} and a'_{ji} also depend on the values of the decision variables. In practice the most important case is when the normatives a_{ij} and a'_{ji} depend only on the variable x_j. The quantity produced and quantity consumed of the i-th element is in that case expressed by the sum of integrals, so instead of (1) we get the inequation:

$$\sum_{j \in P_i} \int_0^{x_j} a'_{ji}(t)\,dt - \sum_{j \in N_i} \int_0^{x_j} a_{ij}(t)\,dt + e_i - e'_i \geqq 0 \qquad (5)$$

Introduce the substitution:

$$x_j = x_{j1} + x_{j2} + \ldots + x_{jk_j} \qquad (6)$$

$$0 \leqq x_{jk} \leqq d_{jk} \qquad k = 1, \ldots, k_j \qquad (7)$$

$$x_{jk} < d_{jk} \Rightarrow x_{j,k+1} = 0 \qquad k = 1, \ldots, k_j - 1 \qquad (8)$$

Let us divide the admissible interval for x_j into k_j subintervals and let us by (8) ensure the correct increase of values of the variables x_{jk}. Let us denote by $a'^{(k)}_{ji}$ the smallest value of the normative a'_{ji} and with $a^{(k)}_{ij}$ the biggest value of the normative a_{ij} on the k-th subinterval.

Often we may suppose that the normatives are piecewise constant. In that case the interval for x_j may be divided into subintervals so that:

$$\int_0^{x_j} a'_{ji}(t)\,dt = \sum_{k=1}^{k_j} a'^{(k)}_{ji} x_{jk} \qquad (9)$$

$$\int_0^{x_j} a_{ij}(t)\,dt = \sum_{k=1}^{k_j} a^{(k)}_{ij} x_{jk} \qquad (10)$$

Even when the normatives are not piecewise constant it is possible to divide the interval x_j so that in practical cases the integrals may be substituted by sums considering (9) and (10). If the normative is not continuous at many points, but is bounded, it is possible to define the intervals so that in (9) and (10) the integral differs from the sum by less than any positive value. From the definition of the coefficients $a'^{(k)}_{ji}$ and $a^{(k)}_{ij}$ it follows that subject to the constraint

$$\sum_{j \in P_i} \sum_{k=1}^{k_j} a'^{(k)}_{ji} x_{jk} - \sum_{j \in N_i} \sum_{k=1}^{k_j} a^{(k)}_{ij} x_{jk} + e_i - e'_i \geqq 0 \qquad (11)$$

(5) always holds. So it follows that in practical cases nonlinear constraint (5) may be substituted by constraints (6), (7), (8) and (11). The constraints (6), (7) and (11) are linear, only the constraint (8) cannot be used in the usual linear model. When the sequence $a'_{ji}{}^{(k)}$ is non-increasing and the sequence $a_{ij}{}^{(k)}$ non-decreasing, the constraint (8) may be cancelled. From (11) then (5) always follows. That is to say, if the set of solutions of the inequation (5) is convex, then the conditions (8) do not influence the optimal values of the decision variables x_j. In that case the nonlinear model can be approximated by the linear one. Only the number of inequations and variables is increased. Equations (6) may be cancelled if x_j is substituted by the corresponding sum everywhere.

Similarly we proceed when the coefficients c_i and c'_i in (4) depend on the corresponding decision variables. At linearization of the objective function the conditions analogous to (8) may be cancelled when the function is concave or it may be approximated by the concave function. The most frequent case is when there are several consumers and for each of them there is a different sale price and marginal cost of sale. Linearization of the model is then very simple. If we denote by e'_{ik} the quantity of i-th element sold to the k-th consumer and by h_i the number of consumers of the i-th element, then we get:

$$e'_i = e'_{i1} + e'_{i2} + \ldots + e'_{ih_i} \tag{12}$$

$$b'_{ik} \leqq e'_{ik} \leqq b''_{ik} \qquad k = 1, \ldots, h_i \tag{13}$$

where b'_{ik} represents the minimal need of k-th consumer which must in any case be satisfied and b''_{ik} the total need of the k-th consumer. If there is no prescription as to which consumer should be satisfied first, then there is also no need to consider (8), so the consumer who offers the best price is satisfied first. If by c'_{ik} the sale price achieved by the k-th consumer reduced for the marginal cost of sale is denoted, then in the objective function (4) the following must be considered:

$$c'_i e'_i = c'_{i1} e'_{i1} + c'_{i2} e'_{i2} + \ldots + c'_{ih_i} e'_{ih_i} \tag{14}$$

We proceed in a similar way when there are several sources for an element with different prices of purchase or different marginal costs of purchase. If there are for the i-th element g_i different sources and if we denote by d'_{ik} the minimal, and by d''_{ik} the maximal quantity of the i-th element which could be purchased at the k-th source, then we get:

$$e_i = e_{i1} + e_{i2} + \ldots + e_{ig_i} \tag{15}$$

$$d'_{ik} \leqq e_{ik} \leqq d''_{ik} \qquad k = 1, \ldots, g_i \tag{16}$$

$$c_i e_i = c_{i1} e_{i1} + c_{i2} e_{i2} + \ldots + c_{ig_i} e_{ig_i} \tag{17}$$

The variables e_i and e'_i are not needed, so they are substituted in (11) by (12) or (15) and so we get:

$$\sum_{j \in P_i} \sum_{k=1}^{k_j} a'_{ji}{}^{(k)} x_{jk} - \sum_{j \in N_i} \sum_{k=1}^{k_j} a_{ij}{}^{(k)} x_{jk} + \sum_{k=1}^{g_i} e_{ik} - \sum_{k=1}^{h_i} e'_{ik} \geqq 0 \tag{18}$$

In the objective function (14) and (17) must be considered. The last sum in (4) which corresponds to the elementary processes should be transformed in an analogous way if this is necessary. So the model is obtained in the form of the maximum of the function:

$$z = \sum_i \sum_{k=1}^{h_i} c'_{ik} e'_{ik} - \sum_i \sum_{k=1}^{g_i} c_{ik} e_{ik} - \sum_j \sum_{k=1}^{k_j} m_{jk} x_{jk} \tag{19}$$

subject to the constraints (7), (13), (16) and (18) which must be formulated for all the elements.

Different marginal costs of purchase or sale may be a consequence of the differences in transportation costs. For this reason it is possible with the described model to optimize at the same time the transport of purchased elements, the production and the transport of the products sold, if all the producers in some region are taken into the same model. Of course in the model associated with a single producer the transport cost may be considered as well and we can optimize transport from the viewpoint of this producer.

3. REALIZATION OF THE MODEL

In practice it is possible to use only such a model which is fairly simple and clear. In spite of using the linear model some difficulties in understanding the model may occur if the process to which the model should be associated is not easy to survey. These difficulties may be lessened if a graph is associated with the business process.

This is done by means of associating with each elementary process the transformation node and with each element the allocation node. From the allocation node E_i to the transformation node X_j leads a branch with the value a_{ij} if there are a_{ij} units of the i-th element consumed per unit of the j-th elementary process. From the transformation node X_j to the allocation node E_i leads a branch with the value a'_{ji} if there are a'_{ji} units of i-th element produced per unit of the j-th elementary process. The transformation nodes are represented by squares and the allocation nodes by circles. To each elementary process and thus to each transformation node a variable representing the quantity of process — the quantity of production — is associated. For this reason for each elementary process a unit of measure is needed which in the production process is usually equal to the unit of measure of the basic raw material or of the main product.

Inequation (1), which is associated with the i-th element is general when there are linear relations, which holds for almost every practical model for the majority of the elements.

In special cases (1) can be simplified. If we do not intend to sell the i-th element, the variable e'_i is not introduced, and if we do not intend to purchase it then e_i is not introduced. If the i-th element cannot be produced, the first sum is cancelled and if it is not spent at any production process the second sum is cancelled. If the i-th element represents their own working means with the capacity b_i and if the surplus capacity cannot be sold or rented and also if no further capacities can be rented, the variables e_i and e'_i are not introduced and the first sum is cancelled as well. Instead of e_i the available capacity b_i is taken and the simple inequation is obtained:

$$\sum_{j \in N_i} a_{ij} x_j \leqq b_i$$

Even when the i-th element is a semiproduct which cannot be sold or purchased the variables e_i and e'_i are not introduced. In order to distinguish particular types of constraints, the allocation nodes can be represented by circles in different colours.

The system of inequations cannot be written down in practical cases. Even the fairly small model which was used at the Factory for Milk Powder with 151 employees has had 134 rows and 196 variables. In the larger business systems there is a need for larger models. All the technological data is therefore gathered on a graph and from the graph it is brought onto the coding list. The control of data and the analysis of results is done by graph as well.

In the meat industry PIK Belje a model with 1431 elements and 1162 elementary processes was used. Irrespective of the fact that drawing the graph was time-consuming it was more than compensated for by repeated use for different purposes. The model was treated by the program LOMP and solved on computer IBM 4341 only 36 minutes.

When the computer based information system of a process already exists it is possible to formulate the objective function and restrictions directly from the computer. This was the case for the steel-plant ŽELEZARNA RAVNE. Most of the data was carried into the optimization model by computer program and the rest manually. In this way, apart from other advantages, the possibility of errors was significantly reduced.

In the model it is also possible to consider the fixed costs. Let us consider that in the elementary process X_j fixed cost s_j monetary units is obtained when $x_j > 0$ and 0 when $x_j = 0$. From the constraint

$$x_j \leqq d_j y_j \qquad y_j = 0 \ \text{ or } \ 1 \tag{20}$$

it follows $x_j = 0$ when $y_j = 0$ and $x_j \leqq d_j$ when $y_j = 1$ where d_j is a constant and x_j a non-negative variable. The fixed cost in the elementary process X_j is included in the objective function (19) by adding the term $-s_j y_j$.

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