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OPTIMIZATION OF BUSINESS PROCESSES BY MIXED INTEGER PROGRAMMING

*Ivan MEŠKO**

The business process can usually be decomposed into purchase, production and sales activities so that the consumed and the obtained quantities of the elements which are connected with the examined activity are independent from other activities. In such a case it is possible to represent the business process by a model maximizing the function

$$z = \sum_{i \in S} \sum_{k} f_{ik}(s_{ik}) - \sum_{i \in T} \sum_{h} g_{ih}(t_{ih}) - \sum_{j} c_j(x_j) \qquad (1)$$

subject to non-negative variables s_{ik}, t_{ih} and x_j and

$$e_i = \sum_{j \in R_i} r_{ij}(x_j) + \sum_{h} t_{ih} - \sum_{j \in Q_i} q_{ij}(x_j) - \sum_{k} s_{ik} \geqq 0 \qquad \forall_i \quad (2)$$

and for some k and h we can have

$$d'_{ik} \leqq s_{ik} \leqq d_{ik} \qquad (3)$$

$$b'_{ih} \leqq t_{ih} \leqq b_{ih} \qquad (4)$$

where

- z — the net income,
- s_{ik} — the quantity of the i-th element sold to the k-th user,
- t_{ih} — the quantity of the i-th element purchased in the h-th source,
- x_j — the quantity of the j-th production activity,
- $f_{ik}(s_{ik})$ — the revenue relating to the sale of s_{ik} units of the i-th element to the k-th user,
- $g_{ih}(t_{ih})$ — the purchase cost of the t_{ih} units of the i-th element in the h-th source,
- $c_j(x_j)$ — the production cost of the j-th production activity excluding the costs of elements considered in the model,

* High Economic — Commercial School, Maribor.

e_i — the unallocated quantity of the i-th element,

$r_{ij}(x_j)$ — the quantity of the i-th element obtained by the j-th production activity,

$q_{ij}(x_j)$ — the consumed quantity of the i-th element with the j-th production activity,

d'_{ik} — the lower limit for the sale quantity of the i-th element to the k-th user,

d_{ik} — the upper limit for the sale quantity of the i-th element to the k-th user,

b'_{ih} — the lower limit for the purchase quantity of the i-th element in the h-th source,

b_{ih} — the upper limit for the purchase quantity of the i-th element in the h-th source,

S — the index set of elements with users,

T — the index set of elements with sources,

Q_i — the index set of the production activities consuming the i-th element,

R_i — the index set of production activities producing the i-th element.

We assume that the function $c_j(x_j)$ referred to (1) can be for $0 \leqq x_j \leqq d_m$ approximated by the piecewise linear function, defined by

$$c_j(x_j) = \begin{cases} 0 & x_j = 0 \\ \sum_{k=1}^{i} b_k + \sum_{k=1}^{i-1} v_k (d_k - d_{k-1}) + v_i (x_j - d_{i-1}) & d_{i-1} < x_j \leqq d_i \end{cases} \tag{5}$$

for $i = 1, \ldots, m$, where

b_1 — the fixed costs caused by the activization of the j-th production activity,

b_k — the fixed costs caused by the j-th production activity when surplusing d_{k-1} units, $k > 1$,

v_k — the unit variable cost of the j-th production activity subject to $d_{k-1} < x_j \leqq d_k$ caused by the consumption of elements which are not considered in the model.

As the function $c_j(xj)$ cannot be used in the form (5) in can be written in the form

$$c_j(x_j) = \sum_{k=1}^{m} b_k u_k + \sum_{k=1}^{m} v_k y_k \tag{6}$$

subject to

$$(d_k - d_{k-1}) u_k - y_k \geqq 0 \qquad k = 1, 2, \ldots, m \tag{7}$$

$$u_k - (d_k - d_{k-1}) u_{k+1} \geqq 0 \qquad k = 1, 2, \ldots, m-1 \tag{8}$$

$$y_k \geqq 0 \qquad k = 1, 2, \ldots, m \qquad (9)$$
$$u_k = 0 \text{ or } 1 \qquad k = 1, 2, \ldots, m \qquad (10)$$
$$x_j = y_1 + y_2 + \ldots + y_m \qquad (11)$$

The constraints (7)—(10) assure that, when x_j is increasing from 0 to d_1, y_1 increases, and, when x_j is increasing from d_1 to d_2, y_2 increases, and so on. At the same time the summand b_k is included in the sum (6) exactly when y_{k+1} begins to increase.

In the model (1)—(4) it is possible to consider the function (5) so that we put (6) into (1), to the constraints (2)—(4) we add the constraints (7)—(10) and (11) substitute in (2). We get a special case if we take $b_k = 0$ $(k = 1, \ldots, m)$ [2]. When $v_{k-1} < v_k$ $(k = 2, \ldots, m)$ and $b_k = 0$ $(k = 1, \ldots, m)$ the constraints (7) and (8) are not needed and there is no need for integer variables in the model [4]. If $m = 1$ we get a case already known where there is no need to substitute (11). Instead of (6) we get in this case

$$c_j(x_j) = b_1 u_1 + v_1 x_j$$

subject to

$$d_1 u_1 - x_j \geqq 0 \qquad u_1 = 0 \text{ or } 1$$

where b_1 means fix costs caused by the activization of the j-th production activity, v_1 is unit variable cost of the j-th activity neglecting the costs of the elements considered in the model and d_1 means the maximum possible quantity of the j-th production activity.

In the same way it is possible to represent the function $g_{ih}(t_{ih})$. Let us take an example where at the beginning of the purchase of the i-th element in the h-th source the costs b_1 arise. If we purchase less than d_1 units, the related price will be v_1 and the price decreases to v_2 if we purchase d_1 units or more (see fig. 1).

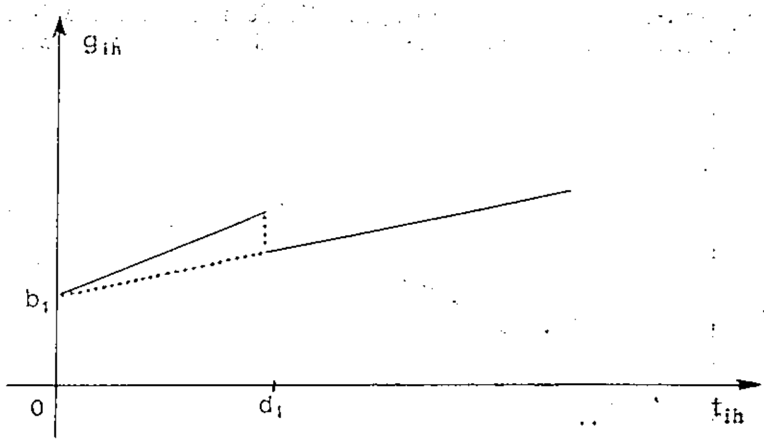

Figure 1

Using (6)—(11) we get

$$g_{ih}(t_{ih}) = b_1 u_1 - b_2 u_2 + v_1 y_1 + v_2 y_2 \qquad (12)$$
$$d_1 u_1 - y_1 \geqq 0 \qquad (13)$$

$$y_1 - d_1 u_2 \geqq 0 \tag{14}$$
$$d_2 u_2 - y_2 \geqq 0 \tag{15}$$
$$y_k \geqq 0 \qquad k = 1, 2 \tag{16}$$
$$u_k = 0 \text{ or } 1 \qquad k = 1, 2 \tag{17}$$
$$t_{ih} = y_1 + y_2 \tag{18}$$

where d_2 is a suitable constant and b_2 denotes a discount arising when the purchased quantity exceeds d_1 and, owing to the reduction of the price for the whole purchased quantity, we have

$$b_2 = (v_1 - v_2)\, d_1$$

Because of (13) and (17) we get

$$y_1 > 0 \Rightarrow u_1 = 1$$

That means that at $y_1 > 0$, in (12) we get the summand b_1 and in this way the initial fixed costs are considered. Because of (14)—(17) it follows:

$$y_1 < d_1 \Rightarrow u_2 = 0 \Rightarrow y_2 = 0 \tag{19}$$

and because of (18) it follows:

$$y_1 < d_1 \Rightarrow t_{ih} = y_1$$

The discount b_2 is considered in (12) if $u_2 = 1$. This can be because of (19) only when $y_1 = d_1$.

It is possible to put the function (12) into (1), to add the constraints (13)—(17) and to consider (18). Therefore the function presented by fig. 1 is correctly considered in the model.

Similarly, it is possible to linearize the constraints (2) if the functions $r_{ij}(x_j)$ and $q_{ij}(x_j)$ can be replaced by piecewise linear functions.

Suppose that the normative of the i-th element with the j-th production activity if $x_j \leqq d_1$ is equal v_1, if $x_j > d_1$ is equal v_2 (see fig. 2).

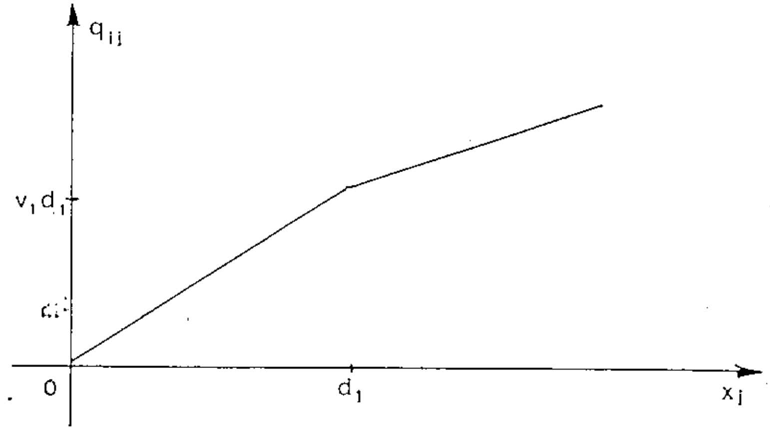

Figure 2

The consumption of the i-th element with the j-th activity can be written in the form:

$$q_{ij}(x_j) = v_1 y_1 + v_2 y_2 \qquad (20)$$

subject to

$$y_1 - d_1 u_1 \geqq 0 \qquad (21)$$
$$d_2 u_1 - y_2 \geqq 0 \qquad (22)$$
$$y_j \geqq 0 \qquad j = 1, 2 \qquad (23)$$
$$u_1 = 0 \text{ or } 1 \qquad (24)$$
$$x_j = y_1 + y_2 \qquad (25)$$

where d_2 is a suitable constant. Because of (21)—(24) it follows:

$$y_1 < d_1 \Rightarrow u_1 = 0 \Rightarrow y_2 = 0$$

So in (25) y_1 increases first in spite of the greater normative v_1. But y_2 can increase only after $y_1 = d_1$. It is possible to put the function (20) into (2), to add the constraints (21)—(24) and replace x_j by means of (25).

All cases cannot be described. Some of them can be found in [5]. It is commonly possible to replace the model, representing the business process by a linear mixed integer model if the functions considered in the model are separable. This is useful because of the rapid development of integer programming [3] and due to the possibility of computer processing of large models [1].

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