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THE THEORY OF RENT

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In this article I shall not be concerned with the rent theory *per se* but with the implications for the labour theory of prices. Labour is now no longer the only primary factor; »land« emerges as another one. The term »land« may be used to denote all non-producible factors. Thus, we have to play with three balls at the same time: with two primary factors and one produced factor.

EXTENSIVE AND INTENSIVE MARGINS OF CULTIVATION

If plots of land are of different »quality« (fertility, location), the same technology applied to different plots will produce different outputs (differential fertility), or will generate different costs (different locations) or both. More generally: different technologies, each of them minimizing costs, will produce different net outputs on different plots of the same size. If not all land is used, the worst plot will generate no rent. Compared with the worst plot, others will produce more output and these productive differentials will be measured by rent. If demand increases (and technology does not change), even less productive plots will be brought to cultivation and intramarginal plots will generate increased rents. The worst plot represents the extensive margin of cultivation. Marx called the corresponding rent Differential Rent I.

The other analytical case is represented by fully used land of uniform quality. Two institutionally determined variants may be distinguished: one normal and one pathological. In the former case land is *socially owned* and producers will meet the increased demand by introducing a hitherto inferior technology which is capable of increasing yields per acre at higher unit costs. This higher cost technology marks the intensive margin of cultivation and the rent that is now generated by the original technology corresponds to Marx's Differential Rent II. This rent is not permanent. As demand continues to grow, more intensive technology will gradually replace the original

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one until it is applied over the entire area. At this point DRII will be wiped out.

In the second, pathological case, the land is *privately owned.* Once rent emerges, private landowners have no reason to deprive themselves of such a delightful source of income. Consequently, more intensive technology will not be introduced before increased price — due to increased demand — covers not only increased costs (including normal profits) but also the already established rent. This rent, which does not arise from differential productivity of either land or technology but is the product of private ownership pure and simple, corresponds to Marx's Absolute Rent. When the more intensive technology has completely replaced the original technology, we have again the case of homogeneous technology being applied to homogeneous land and so, by definition, differential rent disappears. But *not rent as such:* DRII disappears by being transformed into absolute rent. The new round, with still more intensive technology, starts with an increased absolute rent and the process repeats itself. This is clearly a very inefficient arrangement and so it need not detain our attention much longer.[1] We may note in passing that as long as land-

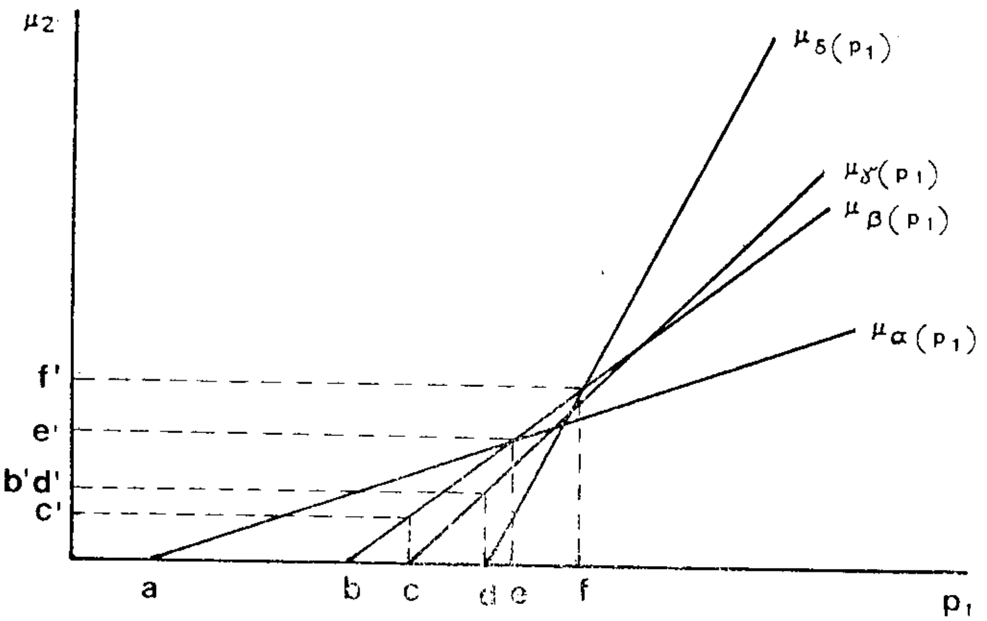

Fig. 1. Differential Rent II as dependent on techniques, product prices, and demand under social and private ownership regimes

owner and entrepreneur are different agents, absolute rent is bound to appear at the extensive margin as well.

In fig. 1 a comparison of the effects of the two ownership regimes is presented. Under both regimes at low product prices land

[1] Besides being inefficient, it is also indeterminate. CF. H. Kurz, »Rent Theory in a Multisector Model«, *Oxford Economic Papers,* 1978, 16—37, pp. 31—34. The diagram in the text, with necessary adaptations, is borrowed from Kurz.

is not used. When price reaches the level $p_1 = a$, technique α is introduced. Further increase in price generates rent. Under *socialism*, technique β is introduced at $p_1 = b$. At that price technique α generates differential rent $\mu_2 = b'$. As demand increases, technique β gradually replaces technique α, product price remains unchanged and total rent decreases. When α is completely displaced, rent is reduced to zero and from this point onwards further increase in demand generates price increase and the emergence of new rent. At $p_1 = c$ the new rent reaches its peak, which happens to be lower than the former peak, $\mu_2(c) < \mu_2(b)$. At this point it becomes profitable to introduce technique γ which starts a new round. At $p_1 = d$ rent reaches a new peak which happens to be the same as the first one, $\mu_2(d) = \mu_2(b)$.

The pattern of *capitalist* development is very different. Technique β cannot be introduced at $p_1 = b$ because at that point it cannot cover the already established rent $\mu_2 = b'$. That will be possible only at a higher price $p_1 = e$ and as a consequence rent will have risen in the meantime to $\mu_2 = e'$. Technique γ, although less costly than technique δ, will never be introduced because it is not sufficiently rent efficient. Technique δ will not be introduced at $p_1 = d$ but at $p_1 = f$ and that will further inflate rent to $\mu_2 = f'$. In short, while under socialism rent oscillates between zero and some point b′ and new more intensive techniques are introduced as soon as they become profitable, under capitalism the introduction of new techniques is delayed until both profit and already established rent are covered. As a result, rent increases all the time and prices are always higher.

THE EQUATIONS

Let us consider an economy with two industries: Industry II produces machines and Industry I produces baskets of consumption goods. There are k qualities of land which is used by the basket industry only. Select for each of them the cost-minimizing technique. The case is described by the following system of price equations.

$$p_2 r \kappa_2 + w\lambda_2 = p_2 \tag{1}$$

$$p_2 r \kappa_1^{(i)} + w\lambda_1^{(i)} + \mu_1 z_1 = p_1' \quad i = 1, \dots k$$

where p's are prices, κ's and λ's are capital and labour coefficients, r is the capital rental rate, μ's are rents and z's are land input coefficients (acreage per basket). Observe that $\mu_i = gp_3^{(i)}$ is rent per acre of land of type *i*, $p_3^{(i)}$ is the price of that land and g is the rate of growth of economy with no technological progress; g plays the role of interest rate. $T_j = T_i(\kappa_1^{(i)}, \lambda_1^{(i)})$ are the cost-minimizing techniques applied to *i* plots of land. Since we are here examining the *extensive* margin of cultivation, the worst plot will bear no rent. Suppose that this is the plot number one and so

$$\mu_1 = 0, \qquad \mu_i > 0, \qquad i = 2, \dots k \tag{1a}$$

The system is now closed: the first two equations determine the prices of the two products and the last k — 1 equations determine that many rents (i. e., that many prices of land).[1] Prices of products are labour prices determined at the extensive margin of cultivation. Rents (and, consequently, prices of land) depend on the quality of land *and* the price of product and the capitalization rate g. If p and g change, the ranking of rents may change since the economic quality of land is only partly determined by its natural fertility and in the other part by the economic parameters of the system.

Consider now the *intensive* margin of cultivation. The quality of land is uniform but the demand for baskets surpasses the supply and part of the land must be more intensively cultivated. At any time there can be only two different techniques applied (unless two techniques just happen to produce the same output at the same cost): the original technique α which produces less at lower costs and the new technique β which produces more at higher costs. As the demand increases, technique β will gradually replace technique α until eventually only β will be used and Differential Rent II will be wiped out. Further increase in demand will initiate a new cycle with the introduction of a still more intensive technique γ.

The intensive margin of cultivation may also be described by system (1) taking into account that land is homogeneous and that two different techniques, say α and β, are necessary to determine the rent in the production of one product. Consequently

$$\mu_1 = 0, \quad \mu_2 \begin{cases} > 0, & \kappa = 2 \\ = 0, & \kappa = 1 \end{cases} \tag{1b}$$

where *k* is the number of techniques used on the land of uniform quality, β corresponding to μ_1 and α to μ_2. As technique β generates no rent, it enters the system of price equations replacing the original equation corresponding to technique α. Once product price is determined, this new price is substituted into the original equation which then determines the rent. If there are more than one land-using industries, land will have to be distributed among them in such a way to *minimize* total rent (production costs being kept at minimum by the competition among firms). If rent were higher than minimum, that would mean that demand for certain consumer good could have been satisfied but was not. There will be as many equations as there are industries, plus possibly an additional equation for an alternative technique for the industry that happened to determine rent. As excess demand develops in various markets, more intensive high-cost high-yield techniques will gradually spread over the area until rent is wiped out. After that a rise in prices in one of the markets may temporarily generate new rent and so initiate a new round of intensification of production.

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[1] In this and the next few paragraphs the reader will notice the influence of P. Sraffa *(Production of Commodities by Means of Commodities*, Cambridge University Press, 1960, ch. XI).

The introduction of joint products would not change the foregoing analysis except that prices would be determined in a more complicated fashion. More than two joint products would require additional equations for all prices to be determined. Also, the specific condition for the non-negativity of prices would have to be observed. Since in neither case the determination of rents is affected, we may stay with simpler case of one-product industries. In that case cultivation at the extensive margin implied production of one product at several plots of different qualities. Cultivation at the intensive margin implied production of one product on homogeneous land by pairs of specifically different techniques (higher production associated with higher unit costs).

THE WAGE-RENTAL-RENT SURFACE

Consider the case in which two different techniques are used to produce two different products on rent-bearing lands of equal quality.[1] Equal quality is an economic description of the fact that rent per acre is the same. In this case both basket and machine industry use land and pay rent

$$p_2 r\kappa_1 + w\lambda_1 + \mu z_1 = p_1 \tag{2}$$

$$p_2 r\kappa_2 + w\lambda_2 + \mu z_2 = p_2$$

Solving the equations we obtain prices and their ratios

$$p_1 = w\left(\lambda_1 + \frac{r\kappa_1}{1 - r\kappa_2}\lambda_2\right) + \mu\left(z_1 + \frac{r\kappa_1}{1 - r\kappa_2} z_2\right) \tag{3}$$

$$p_2 = \frac{w\lambda_2}{1 - r\kappa_2} + \frac{\mu z_2}{1 - r\kappa_2}$$

$$\frac{p_1}{p_2} = \frac{w[\lambda_1 + r(\lambda_2\kappa_1 - \lambda_1\kappa_2)] + \mu\,[z_1 + r(z_2\kappa_1 - z_1\kappa_2)]}{w\lambda_2 + \mu z_2} \tag{4}$$

As machines continue to be inputs in both industries, while baskets are not inputs, p_2 is determined by the second equation only, while p_1 depends on both equations. The introduction of rent adds fully symmetrical terms to both price equations which reflects the fact that land appears as a primary factor symmetrical with labour. For $\mu = 0$ the two expressions are reduced to equations determining

[1] G. Abraham-Frois and E. Berrebi use the same model but are apparently not aware of the intricacies involved (*Théorie de la valeur, des prix et de l'accumulation*, Economica, Paris, 1976, pp. 216—23).

labour prices. Equal organic composition of capital, $\lambda_2\kappa_1 - \lambda_1\kappa_2$, is no longer sufficient to make the price ratio equal to the ratio of direct labour coefficients. Even the addition of an analogous equal organic composition of capital with respect to land, $z_2\kappa_1 - z_1\kappa_2$, will not do. However, if we impose these two conditions on the expression for p_1, we obtain a linear relation between real wages, rental rate and real rent

$$w\lambda_1 + r\kappa_2 + \mu z_1 = 1, \qquad p_1 = 1 \tag{5}$$

Moreover, if the extreme values for the variables

$$W = \frac{1}{\lambda}, \qquad r = \mu = 0$$

$$R = \frac{1}{\kappa_2}, \qquad w = \mu = 0$$

$$M = \frac{1}{z_1}, \qquad w = r = 0$$

are substituted into (5), the three-factor version of Sraffa's equation emerges (with $r = \pi$)

$$\pi = \Pi \left(1 - \frac{w}{W} - \frac{\mu}{M}\right) \tag{6}$$

The interpretation is classical: the higher wages and rents are, the lower are profits.

It may seem that we can progress even further and construct the production possibility frontier. Use (2) to express all prices in baskets remembering that $\mu = gp_3$ and $r = \frac{1}{\nu(g)} + g = f(g)$

$$1 = \frac{w}{p_1}\left(\lambda_1 + \frac{r\kappa_1}{1 - r\kappa_2}\lambda_2\right) + g\frac{p_3}{p_1}\left(z_1 + \frac{r\kappa_1}{1 - r\kappa_2}z_2\right) \tag{7}$$

The former w-r curve is now transformed into w-r-μ surface. If real wage $\frac{w}{p_1}$ is interpreted as per capita consumption as before, it looks that it will fall if g increases *and* if $\frac{p_3}{p_1}$ increases. Now, the rent component $\frac{p_3}{p_1}$ is the basket price of land, in other words, its productivity. It looks rather odd that an increase in the productivity of land could reduce wages. The contradiction is easily eliminated if two facts are clearly understood. An increase in $\frac{p_3}{p_1}$ is a spurious productivity increase. The physical productivity of the plot in equation remains as it has been, but the next worse quality of land is brought under cultivation (or the intensive margin is shifted) and that increases the *difference* in productivity. Average productivity *falls* and so does per capita consumption, though the equation does not indicate how much.

Secondly, land is not a fully symmetrical factor: it is similar to labour in being non-producible but it is also very different in generating production without consuming anything. In neo-classical economics all factors of production are treated symmetrically, and for certain analytical purposes that is satisfactory. But very soon we encounter difficulties like the one just described. Clearly, labour, capital and land are three very different factors of production and a truly general theory must account for that. It is true that if $\mu = g\frac{p_3}{p_1}$ increases, real wages fall, *ceteris paribus*. But w/p_1 is not per capita consumption. Two interpretations are possible. Either capital and land are privately owned and then w, π (which is different from g) and μ determine the distribution of income among social classes and maximizing per capita consumption is an incompatible goal. Or capital and land are socially owned and then g and μ represent parameters determining rational allocation of resources. In that case rent becomes an addition to wages, g (the production of additional machines) representing the only cost. And so we are back to the w-r curve describing the production possibility frontier.

It is, as always, helpful to consider the dual aspect of the problem. Price equations (1) are then complemented by quantity equations

$$\begin{aligned} \lambda_1 X_1 + \lambda_2 X_2 &= L \\ r\kappa_1 X_1 + r\kappa_2 X_2 &= rK = X_2 \qquad (8) \\ z_1 X_1 + z_2 X_2 &= z \end{aligned}$$

The system is clearly over-determined. If *only one* technology were available, either some land or some labour would be left unemployed in a general case. But since we have the book of blueprints, we can always select technology which will secure full employment. If Z is is abundant, there will be no rent and only the first two equations remain relevant. If there is rent, as assumed above, land is scarce and will be fully used. A selection of appropriate techniques for machine and basket production will lead to full employment of labour. (In the neo-classical framework, one would talk of substituting labour and capital for land). As long as $g > 0$, this procedure must be repeated with every investment project. Such a shifting of the intensive margin cannot continue for ever. The end is reached when the last, most intensive, technology is selected from the book of blueprints. With that technology marginal product of labour in terms of consumer goods becomes zero. Further growth of labour force increases labour-land ratio beyond the possibilities of productive employment of labour and so some labour will be left unemployed. However, long before this classical doomsday arrives, technological progress will produce a new book of blueprints in which all production functions will be shifted upwards.

The upshot of the foregoing discussion is that we do not need land as an implicit element in the construction of the technology frontier. Scarcity of land is implied in the technology selected. This being so, the first two equations of (8) determine the relation between per capita consumption $\frac{X_1}{L}$ and the gross rate of expansion of capital stock r. Greater r means greater output of machines which, given labour (and land), reduces output of consumer goods. Thus all that is necessary for the construction of the technology frontier — assuming that social goal is maximizing X_1 — is to relate $\frac{X_1}{L}$ and r. Land, being in fixed supply, incurs no labour cost and is not a variable.

However, this cheering conclusion is not generally valid. It is strictly dependent on the special assumption made in (2): that land is homogeneous and rent is everywhere the same, a sort of Marxian Absolute Rent. This special model exclusively served for conceptual clarification. In a general case, land is heterogeneous and the former production possibility frontier — which implies fixed technical coefficients λ and κ — is rendered impossible. Every change in the proportions of X_1 and X_2 produced requires different distribution of land which, being heterogeneous and generating different outputs, changes λ_i and κ_i even if technically the same techniques are used. In order to construct the frontier, one must describe the same technology by changing input coefficients as a function of the changing proportions of production, $T = T(\frac{X_1}{X_2})$. The latter, of course, imply different distributions of land use. So defined fixed T makes it possible to derive the relation between per capita consumption and the rate of growth and the corresponding curve may have a very complicated shape. For this reason it seems advisable to forget about the production possibility frontier and consider only technology frontier. In that case a change in coefficients implies both spurious and genuine change in techniques.

A LABOUR INTERPRETATION OF RENT

It remains to sum up our findings. As Ricardo already knew, the emergence of rent does not change the determination of prices. Prices will be determined by the processes at the extensive and intensive margins of cultivation. With no rent included, such prices will be labour prices. Since every increase in labour-land and capital-land ratios — given the book of blueprints — increases labour cost, technology frontier — describing the relation between per capita consumption and the rate of growth — is a downward sloping curve.

The fact that prices are determined at the margin has one important consequence. Without rent by setting $p_0 = 1$, prices become *absolute* labour prices and the value of net output is equal to the labour time currently expended, $p_1 X_1 = L$. That is generally no longer true: one does not imply the other. Prices, being determined by marginal processes with no rent, are still *relative* labour prices, but $p_1 X_1 = L$ must be imposed as a matter of definition of net output in order to derive the appropriate value of p_0. Consider value balances corresponding to price equations (1)

$$p_2 r K_2 + p_0 L_2 = p_2 X_2$$

$$p_2 r K_1 + p_0 L_1 + \Sigma \mu_i z_i = p_1 X_1, \quad \mu_1 = 0, \quad \mu_i \geqslant 0, \quad i \geqslant 2 \tag{9}$$

where $K_1 = \Sigma K_1^{(i)}$ and $L_1 = \Sigma L_1^{(i)}$ represent capital and labour applied to all k qualities of land. The price of baskets, being determined by the worst land contains more labour than necessary on the average, $p_1 X_1 > L$. If the value of net output is to be made equal to L, p_0, and all other prices, must be multiplied by a factor $q < 1$. To find the multiplier, use the conditions for exchange between the two industries

$$p_0 L_2 = p_2 r K_1$$

Consequently

$$q p_0 (L_2 + L_1) + q \Sigma \mu_i z_i = q p_1 X_1 = L$$

$$q = \frac{L}{p_0 L - \Sigma \mu_i z_i} = \frac{L}{L - \Sigma \mu_i z_i}, \qquad p_0 = 1 \tag{10}$$

If price of labour is equal to $p_0 = q$, net output will be expressed in labour time. For $\mu_i = 0$, $q = 1 = p_0$, which is the original case of mutual implication. Total rent $\Sigma \mu_i z_i$ represents labour time saved because of the cultivation of better qualities of land. Mathematically — though not institutionally — a symmetric statement is also possible. Suppose prices are determined by the best land, which bears no rent. The cultivation of all other qualities of land is carried out under increasing costs which may be considered as negative rents. Consequently, $\Sigma \mu_i z_i$ is labour time lost due to the scarcity of good land.

In a general model, rent will appear in both groups of industries. Using the same exchange condition, the multiplier will now be reduced for additional rents $\Sigma\mu_j z_j$

$$q = \frac{L}{p_0 L + \Sigma\mu_i z_i + \Sigma\mu_j z_j} \tag{10a}$$

and the interpretation is the same as before.

Analytically, rents must be added to wages in order to exhaust the value of net product. Institutionally they are, of course, an ideal source of taxes.

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