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THE PURE LABOUR THEORY OF PRICES AND TECHNOLOGICAL CHANGE

*Branko HORVAT**

We start from a dual system of quantity and price equations

Quantity equations:
$$\lambda_1 X_1 + \lambda_2 X_2 = L$$
$$rK_1 X_1 + rK_2 X_2 = rK = X_2$$

Price equations:
$$p_2 r\kappa_1 + p_0\lambda_1 = p_1$$
$$p_2 r\kappa_2 + p_0\lambda_2 = p_2$$

X_1 and X_2 are homogeneous consumer (baskets of standard assortment) and capital goods (machines). L is the number of workers, K is the number of identical machines, $\lambda_j = L_j/X_j$ is labour coefficient, $\kappa_j = K_j/X_j$ is capital coefficient. The rental rate $r = 1 + g$ is a sum of the variable replacement rate ($\nu^{-1} = f(n, g)$) and the rate of growth (g); n is the fixed life-span of machines. By solving the equations we obtain the quantities of output and labour prices (for $p_0 = 1$):

$$X_1 = \frac{(1 - r\kappa_2)}{\lambda_1(1 - r\kappa_2) + \lambda_2 r\kappa_1}, \quad X_2 = \frac{r\kappa_1 L}{\lambda_1(1 - r\kappa_2) + \lambda_2 r\kappa_1},$$

$$\frac{X_1}{X_2} = \frac{1 - r\kappa_2}{r\kappa_1} \tag{0.1}$$

$$p_1 = p_2\frac{\lambda_1(1 - r\kappa_2) + \lambda_2 r\kappa_1}{1 - r\kappa_2}, \quad p_2 = p_0\frac{\lambda_2}{1 - r\kappa_2},$$

$$\frac{p_1}{p_2} = \frac{\lambda_1 + r(\lambda_2\kappa_1 - \lambda_1\kappa_2)}{\lambda_2} \tag{0.2}$$

* University of Zagreb.

1. EXPANDING ECONOMY: TECHNOLOGICAL PROGRESS

A) Some Conceptual Issues

Our economy is characterized by five technical coefficients: two labour coefficients, two capital coefficients and the durability of machines. The change of any of these coefficients has a definite economic meaning. If κ_2 is reduced, one old machine (and proportionally more labour) will produce more *new* machines. The new machines may be the same as the old ones but be running longer per year due to improved repair. Or the design of the machines may be changed. As soon as new machines are installed, *new* machines will begin to produce *new* machines. In the transition period technology will be mixed. After all old machines have been scrapped, there will be again a pure technology of identical machines. Even if these machines are different from old machines, they may be equally well counted as the old ones. If κ_1 is reduced, one machine produces more baskets. And if n is increased, for given output, replacement cost (the number of machines scrapped) in both industries decreases.

An increase in labour productivity — λ_1 and λ_2 decline — implies less live labour per unit of output (equipped with more machines since κ_j remain intact). Whether λ's are reduced or κ's are reduced and n increased, the change always implies an increase in the productivity of labour, in the former case of direct labour, in the latter of indirect labour. Consequently, technological progress is always labour saving.

Reduction of capital coefficients (given n) implies an economy in embodied labour, of labour coefficients and economy of live labour. Yet, this distinction is different from the distinction between embodied and unembodied[1] technological progress. Embodied technological progress implies a new design of machines; new machines are different. Under unembodied progress machines remain the same but are used more efficiently (because of an improved organization, greater experience of the workers, better repair, and the like) and also the productivity of live labour may increase. It is, of course, possible to treat workers as machines and to consider any change in labour productivity (machines remaining the same) as a change in human capital. I do not find this latter approach paticularly enlightening for the purpose at hand. Both embodied and unembodied progress may change any of the five technical coefficients.

Machines may be both more efficient or less costly or both. This may be called the double character of fixed assets (fixed capital) which has important analytical consequences.

Since machines produce machines, the two effects cannot be easily disentangled. That presents a nightmare for social accountants trying to measure machine output or the stock of machines in constant

[1] In the literature usually called »disembodied«. But only ghosts are disembodied.

prices. A possibility of precise distinction is relatively simple in the consumer goods industry. If a machine produces more baskets, it is more efficient (κ_1 reduced). If at the same time it embodies less labour, it is less costly (p_2 is reduced). The combined effect is measured by the reduction of engaged embodied labour per unit of physical output ($p_2\kappa_1$). The combined effect may remain the same (κ_1 and p_2 need not) even if machine is changed physically as long as the basket remains the same. Thus, a change in design does not represent a special problem. Per analogiam, in machine industry κ_2 measures efficiency, and p_2 cost and $p_2\kappa_2$ the combined effect *if the machine remains unchanged*. If is changes, $\kappa_2 = K_2/X_2$ still measures efficiency because the same new machines are in the numerator (after the old capital stock has been replaced) and in the denominator and so the machine dimension cancels out. But p_2 is no longer comparable and neither is $p_2\kappa_2$. Thus, while $p_2\kappa_1/n$ has a straightforward interpretation as embodied labour expended per unit of consumer good, $p_2\kappa_2/n$ does not have such an interpretation. This indicates that we have to take into account yet another effect of price changes.

A changed machine may require a changed amount of labour time for its production. It follows that in order to be able to count machines, we must measure our machines in invariant units. We may take the standard basket of consumer goods as a numéraire which implies dividing all prices by p_1. In this case value capital coefficient in the first industry will be

$$\frac{p_2/p_1 \cdot K_1}{p_1/p_1 \cdot X_1} = \frac{p_2 K_1}{p_1 X_1} = \frac{p_2}{p_1}\kappa_1$$

and in the second industry

$$\frac{p_2/p_1 \cdot K_2}{p_2/p_1 \cdot X_2} = \kappa_2$$

In other words, machines of constant efficiency in terms of baskets—which is the same as labour embodied in machine per unit of labour embodied in output it produces—are measured simply by value capital coefficients with baskets as a numéraire. For the machine industry value and physical capital coefficients are, obviously, the same.

The foregoing considerations indicate that technical coefficients fall into two groups which should be sharply distinguished: λ_1 and κ_2 are comparable across techniques, λ_2 and κ_1 depend on how machines are measured.

If labour force is growing, $\frac{1}{n}$ will be replaced by $r = \frac{1}{\nu} + g$. Since $r > \frac{1}{n}$ this will have the same effect as an increase in κ_j (or a reduction in n) as already mentioned before. The faster the growth, the more »adverse« its effect on TP, ceteris paribus. However, since growth tends to speed up TP, which is know as Verdoorn Law [Verdoorn, 1980], the overall effect is not known in advance. If the employment and Verdoorn effect cancel out, we have accumulation and consumption increasing simultaneously — something considered contradictory by neoclassical interest theory.

In general the accumulation of capital in value terms (embodied labour time) and in physical terms (expansion of output capacity) need not move together under the conditions of technological change. It is impossible for the value of an identical machine to increase while its output does not because that would imply technological regress. But it is entirely possible that a decumulation of value capital be accompanied by an accumulation of physical capital. Beside, a reduced value of capital stock may make possible not only larger physical output but also larger value output. This will happen if total labour force is growing, but labour employed in the machine industry is suffiiciently reduced.

Unembodied technological progress generates instantaneous effects and poses no special problems. Embodied TP generates different vintages of machines operated simultaneously. Since historical costs are of no interest for price formation, the synchrony rule implies that prices be determined by *current reproduction costs.* In other words, prices will be determined by the current year technology.

As far as equipment is concerned, its changing shape is of no special concern. We must only be aware of implications. However, unless explicitly specified, I shall assume unembodied TP, i. e., unchanged design of machines. Also, for the time being, baskets will be assumed to remain identical in terms of quantities, proportions, and qualities of consumer goods.

B) Once-and-for-all Technological Change
Output and price changes

If in a stationary economy positive technological change occurs, the output will begin to incease. After a transitional period, during which necessary adjustments will be made, economy will reach a new, higher, stationary level. We then compare the old and the new stationary level. Since two stationary levels are compared, we may use equations characterizing simple reproduction ($g = 0$, $\nu = n$).

As already mentioned, the technology of our economy is described by three sets of parameters, κ_j, λ_j, and n. If any of these parameters change, both production and prices must change. For two industries and three parameters, of which two are specific for each industry, there will be $2 \times 2 \times 2 + 2 = 10$ effects on output and the same number of effects concerning prices. It will be useful to systematize these effects. For this we use equations (0.1) and (0.2).

(1.1) *Output Changes*

Reduction of labour coefficients:

$$-\frac{\partial X_1}{\partial \lambda_1} > 0, \quad -\frac{\partial X_1}{\partial \lambda_2} > 0; \quad -\frac{\partial X_2}{\partial \lambda_1} > 0, \quad -\frac{\partial X_2}{\partial \lambda_2} > 0$$

Reduction of capital coefficients:

$$-\frac{\partial X_1}{\partial \kappa_1} > 0, \quad -\frac{\partial X_1}{\partial \kappa_2} > 0; \quad -\frac{\partial X_2}{\partial \kappa_1} < 0, \quad -\frac{\partial X_2}{\partial \kappa_2} < 0$$

Increase in durability:

$$\frac{\partial X_1}{\partial n} > 0; \quad \frac{\partial X_2}{\partial n} < 0.$$

(1.2) *Price Changes*

Reduction of labour coefficients:

$$-\frac{\partial p_1}{\partial \lambda_1} < 0, \quad -\frac{\partial p_1}{\partial \lambda_2} < 0; \quad -\frac{\partial p_2}{\partial \lambda_1} = 0, \quad -\frac{\partial p_2}{\partial \lambda_2} < 0$$

Reduction of capital coefficients:

$$-\frac{\partial p_1}{\partial \kappa_1} < 0, \quad -\frac{\partial p_1}{\partial \kappa_2} < 0; \quad -\frac{\partial p_2}{\partial \kappa_1} = 0, \quad -\frac{\partial p_2}{\partial \kappa_2} < 0$$

Increase in durability:

$$\frac{\partial p_1}{\partial n} < 0; \quad \frac{\partial p_2}{\partial n} < 0.$$

As far as production is concerned, an increase in labour productivity (reduction of labour coefficients) leads to an increase in all outputs, given the labour force L. An increase in the productivity of machines (reduction of capital coefficients) increases output of consumer goods but, understandably, reduces output of capital goods. The same is the effect of an increase in the durability of machines.

Price effects are independent from the availability of the primary resource L and depend exclusively on technological changes. Improvements in productivity of labour and machines and increased durability of machines diminish labour-time prices. Since sector No. 1 does not participate in the production of machines, technological improvements in that sector (reduced λ_1 and κ_1) have no impact on prices of machines. Thus, prices of machines are determined exclusively by technological changes in sector No. 2 which produces machines.

Three measures of capital intensity

Technological progress will affect differently various structural parameters used in economic analysis.
The three are relevant here:

Capital coefficients in value terms (capital-output ratios):

$$\hat{\kappa}_1 = \frac{p_2 K_1}{p_1 X_1} = \kappa_1 \frac{p_2}{p_1}, \quad \hat{\kappa}_2 = \kappa_2, \quad \hat{\kappa} = \frac{p_2 K}{p_1 X_1 + p_2 X_2} = \frac{n p_2 X_2}{p_1 X_1 + p_2 X_2} \tag{1.3}$$

Technical composition of resources (machine-worker ratios):

$$k_1 = \frac{K_1}{L_1} = \frac{\kappa_1}{\lambda_1}, \quad k_2 = \frac{\kappa_2}{\lambda_2}, \quad k = \frac{K}{L} = \frac{\kappa_1 X_1 + k_2 X_2}{\lambda_1 X_1 + \lambda_2 X_2} \tag{1.4}$$

Organic composition of resources (capital-labour ratios):

$$\omega_1 = \frac{p_2 K_1}{L_1} = p_2 k_1, \quad \omega_2 = p_2 k_2, \quad \omega = \frac{p_2 K}{L} = p_2 k \tag{1.5}$$

Prices used are, of course, exact labour time prices and therefore, $w = 1$. Expression (1.5) needs a slight elaboration. Using (1.3) and remembering $p_1 X_1 = L$, $n X_2 = K$, we obtain organic composition as a function of value capital coefficients and durability

$$\omega = \frac{n \hat{\kappa}}{n - \hat{\kappa}} \tag{1.5a}$$

For sufficiently large n — the average life-span of fixed assets in the modern economy is 30 years or more — the magnitude of organic composition is approximately equal to that of capital coefficient in value terms, $\omega \doteq \hat{\kappa}$.

It is of some interest to note the difference between Marx's »organic composition of capital« and my »organic composition of resources.« Marx defined his concepts for an analysis of specific, capitalist, institutions. Thus, his organic compostion is a ratio between constant (C = fixed and circulating capital) and variable capital (V = wages advanced).

$$\omega_M = \frac{C}{V} = \frac{C}{wL}$$

Since wL are wages paid out for the perior before the product was finished and ready for sale, wL is a flow, ω_M has time dimension and represents stock/flow ratio. My concepts are designed to analyse conditions for optimal use of resources and are not institutionally restricted. Thus, my organic composition is a ratio of embodied and live labour

$$\omega = \frac{C}{L}$$

Constant capital is evaluated in labour prices, C and L are both stocks, ω is a pure number. The two ratios are numerically equal, $\omega_M = \omega$, if $w = 1$, i. e., if the turnover period of variable capital is defined as a time unit. Note also that an alternative definition of the organic composition for the entire economy is capital — net output ratio, $\omega = p_2K/p_1X_1$, since $p_1X_1 = L$.

Patterns of Technological Change

There are several special patterns of technological progress worthwhile exploring (see Table 1.1).

Neutral Technological Progress of Type One. Let both labour coefficients decrease at the rate of γ, $\lambda_i = (1 + \gamma)^{-1} \lambda^0_i$, which means that labour productivity increases $(1 + \gamma) = \Gamma$ times. It follows from (0.2) that prices of both commodities will also be reduced Γ times. It follows from (0.1) that output of both industries will increase Γ times. Thus, value of output will remain unchanged. $p_jX_j = p^0_jX^0_j$. Real wage will increase in the same proportion as output, $\overline{w} = X_1/L = (\Gamma)\overline{w}^0$. Since technical change has not affected capital coefficients, after an adjustment period the old structural relationship will be reestablished. It follows from (1.3)—(1.5) that capital coefficients in value terms will remain unchanged ,technical composition of resources will be double (if machines remain physically unchanged), while organic composition of resources also remains invariant to change. Because of the last effect, this pattern of technological progress may be called Marx-neutral. It may be also called labour-augmenting TP. Since capital-

Table 1.1. Effects of a Single Technological Change

	Technological Progress: Pure Neutral I	Pure Neutral II	Pure Capital Saving I	Pure Capital Saving II	Mixed Neutral	Mixed Capital Using I	Mixed Capital Using II	Mixed Capital Using III	Technological Change without TP: Capital Saving I	Capital Saving II	Neutral I	Neutral II	Capital Using I	Capital Using II
	$\lambda_j \Gamma^{-1}$	$\lambda_1 \Gamma^{-1}$ $\kappa_1 \Gamma^{-1}$	$\lambda_j \Gamma^{-1}$ $\kappa_j \Gamma^{-1}$ $n \Gamma^{-1}$	$\lambda_j \Gamma_\lambda^{-1}$ $\kappa_j \Gamma_\kappa^{-1}$ $n \Gamma_\kappa^{-1}$	$\lambda_1 \Gamma^{-1}$ $\lambda_2 \Gamma^{-2}$ $\kappa_1 \Gamma$	$\lambda_j \Gamma^{-1}$ $\kappa_j \Gamma$ $n\Gamma$	$\lambda_1 \Gamma^{-1}$ $\kappa_2 \Gamma$ $n\Gamma$	$\lambda_j \Gamma_\lambda^{-1}$ $\kappa_j \Gamma_\kappa$ $n \Gamma_\kappa$	$\kappa_j \Gamma^{-1}$ $n \Gamma^{-1}$	$\lambda_2 \Gamma^{-1}$ $\kappa_2 \Gamma^{-1}$ $n \Gamma^{-1}$	$\lambda_2 \Gamma^{-1}$ $\kappa_1 \Gamma$	$\lambda_2 \Gamma$ $\kappa_1 \Gamma^{-1}$	$\lambda_2 \Gamma$ $\kappa_2 \Gamma$ $n \Gamma$	$\kappa_j \Gamma$ $n \Gamma$
X_1	$X_1\Gamma$	$X_1\Gamma$	$X_1\Gamma$	$X_1\Gamma_\lambda$	$X_1\Gamma$	$X_1\Gamma$	$X_1\Gamma$	$X_1\Gamma_\lambda$	same	same	same	same	same	same
X_2	$X_2\Gamma$	same	$X_2\Gamma$	$X_2\Gamma_\lambda$	$X_2\Gamma^2$	$X_2\Gamma$	same	$X_2\Gamma_\lambda$	same	$X_2\Gamma$	$X_2\Gamma$	$X_2\Gamma^{-1}$	$X_2\Gamma^{-1}$	same
p_1	$p_1\Gamma^{-1}$	$p_1\Gamma^{-1}$	$p_1\Gamma^{-1}$	$p_1\Gamma_\lambda^{-1}$	$p_1\Gamma^{-1}$	$p_1\Gamma^{-1}$	$p_1\Gamma^{-1}$	$p_1\Gamma_2^{-1}$	same	same	same	same	same	same
p_2	$p_2\Gamma^{-1}$	same	$p_2\Gamma^{-1}$	$p_2\Gamma_\lambda^{-1}$	$p_2\Gamma^{-2}$	$p_2\Gamma^{-2}$	same	$p_2\Gamma_\lambda^{-1}$	same	$p_2\Gamma^{-1}$	$p_2\Gamma^{-1}$	$p_2\Gamma$	$p_2\Gamma$	same
$p_1 x_1$	same	same	same	same	same	same	same	same	same	same	same	same	same	same
$p_2 x_2$	same	same	same	same	same	same	same	same	same	same	same	same	same	same
$\hat{\kappa}_1$	same	same	$\hat{\kappa}_1\Gamma^{-1}$	$\hat{\kappa}_1\Gamma_\kappa^{-1}$	same	$\hat{\kappa}_1\Gamma$	$\hat{\kappa}_1\Gamma$	$\hat{\kappa}_1\Gamma_\kappa$	$\hat{\kappa}_1\Gamma^{-1}$	$\hat{\kappa}_1\Gamma^{-1}$	same	same	$\hat{\kappa}_1\Gamma$	$\hat{\kappa}_1\Gamma$
$\hat{\kappa}_2$	same	same	$\hat{\kappa}_2\Gamma^{-1}$	$\hat{\kappa}_2\Gamma_\kappa^{-1}$	same	$\hat{\kappa}_2\Gamma$	$\hat{\kappa}_2\Gamma$	$\hat{\kappa}_2\Gamma_\kappa$	$\hat{\kappa}_2\Gamma^{-1}$	$\hat{\kappa}_2\Gamma^{-1}$	same	same	$\hat{\kappa}_2\Gamma$	$\hat{\kappa}^2\Gamma$
k_1	$k_1\Gamma$	same	same	$k_1\Gamma_\lambda\Gamma_\kappa^{-1}$	$k_1\Gamma^2$	$k_1\Gamma^2$	$k_1\Gamma$	$k_1\Gamma_\kappa\Gamma^\lambda$	$k_1\Gamma^{-1}$	same	$k_1\Gamma$	$k_1\Gamma^{-1}$	same	$k_1\Gamma$
k_2	$k_2\Gamma$	same	same	$k_2\Gamma_\lambda\Gamma_\kappa^{-2}$	$k_2\Gamma^2$	$k_2\Gamma^2$	$k_2\Gamma$	$k_2\Gamma_\kappa\Gamma_\lambda$	$k_2\Gamma^{-1}$	same	$k_2\Gamma$	$k_2\Gamma^{-1}$	same	$k_2\Gamma$
ω_1	same	same	$\omega_1\Gamma^{-1}$	$\omega_1\Gamma_\kappa^{-1}$	same	$\omega_1\Gamma$	$\omega_1\Gamma$	$\omega_1\Gamma_\kappa$	$\omega_1\Gamma^{-1}$	$\omega_1\Gamma^{-1}$	same	same	$\omega_1\Gamma$	$\omega_1\Gamma$
ω_2	same	same	$\omega_2\Gamma^{-1}$	$\omega_2\Gamma_\kappa^{-1}$	same	$\omega_2\Gamma$	$\omega_2\Gamma$	$\omega_2\Gamma_\kappa$	$\omega_2\Gamma^{-1}$	$\omega_2\Gamma^{-1}$	same	same	$\omega_2\Gamma$	$\omega_2\Gamma$

$$X_1 = \frac{n - \kappa_2}{\lambda_2\kappa_1 + \lambda_1(n - \kappa_2)} L \qquad p_1 = \frac{\lambda_2\kappa_1 + \lambda_1(n - \kappa_2)}{n - \kappa_2} \qquad r = 1 + \gamma$$

$$X_2 = \frac{\kappa_1}{\lambda_2\kappa_1 + \lambda_1(n - \kappa_2)} L \qquad p_2 = \frac{n\lambda_2}{n - \kappa_2} \qquad L = \text{const.}$$

$$\hat{\kappa}_1 = \kappa_1 \frac{p_2}{p_1} \qquad k_1 = \frac{\kappa_1}{\lambda_1} \qquad \omega_1 = p_1 k_1$$

$$\hat{\kappa}_2 = \kappa_2 \qquad k_2 = \frac{\kappa_2}{\lambda_2} \qquad \omega_2 = p_2 k_2$$

output ratio remains unchanged, it may also be considered Harrod-neutral TP if γ is interpreted as interest rate. Finally, in labour prices capital-labour ratio is equal to organic composition. Thus, it is also Hicks-neutral because the rate of technical substitution remains constant along p_2K/L ray (and is equal to zero). Since in real world economies rapid increases in labour productivity are accompanied by relatively low changes in capital-output ratios in value terms, Marx-neutral technological progress of type one may be taken as a reasonable approximation of reality.[2]

Neutral Technological Progress of Type Two. If technological change occurs in the consumer goods industry, that will leave prices of machines unchanged. Let the change be such that both technical coefficients in consumer goods industry be reduced in the same proportion, $\lambda_1 = \Gamma^{-1}\lambda^0_1$ $\kappa_1 = \Gamma^{-1}\kappa^0_1$. As a result x_1 will increase and p_1 decrease Γ times leaving the value of consumer goods unchanged, $p_1x_1 = p^0_1x^0_1$. The output of machines and their prices remain unaffected. Consequently, structural characteristics of the system κ_j, k_j and ω_j remain unchanged.

As the stock of machines remains constant through time, the growth of net output occurs without any new investment.

Capital Saving Technological Progress I and II. In the first variant all technical coefficients (λ_j, κ_j, and n) are reduced by the same factor Γ which leaves relative prices unchanged. Both outputs are increased by Γ which leaves relative outputs. unchanged. Values of sectoral outputs remain unchanged. However, value capital coefficients are reduced. Since κ_j and λ_j are reduced in the same proportion, machine-labour ratios, $k_j = \kappa_j/\lambda_j$, also remain unchanged. But the organic composition of capital is reduced by factor $\Gamma = (1+\gamma)$: $\omega = \Gamma^{-1}p^0_2k = \Gamma^{-1}\omega^0$. In the second variant labour coefficients decrease by factor $\Gamma\lambda$, capital coefficients and durability by $\Gamma\kappa$. It turns out that $\Gamma\lambda$ affects both outputs and prices, $\Gamma\kappa$ affects value capital coefficients and organic composition while machine-worker ratios are affected by both TP factors. A reduction in n may be justified by the effects of strong TP in λ_j and κ_j which make for an earlier scrapping of machines. Shorter n makes this type of TP mixed.

Mixed Technological Progress Generally. It is possible that labour productivity increases while machine productivity decreases (or vice versa) but that gains outweight losses. Under conditions of simple reproduction that will happen when real wage increases. Given the number of workers, that implies only an increase in X_1. To satisfy this requirement we must have, with respect to (0.1).

$$X_1 = \frac{(1 - a\rho^0_2)\, L}{c\lambda^0_2 b\rho^0_1 + d\lambda^0_1\,(1 - a\rho^0_2)} > \frac{(1 - \rho^0_2)\, L}{\lambda^0_2\rho^0_1 + \lambda^0_1\,(1 - \rho^0_2)},$$

[2] Marx himself expected a biased-capital using-technological progress, i. e., an increasing ω.

$$\rho_i = \kappa_j / n$$
$$a,\ b > 1$$
$$c > 0,\ 0 < d < 1 \qquad (1.6)$$

If replacement coefficients (ρ_j) increase, a, b > 1, at least one labour coefficient must sufficiently decrease so that (1.6) remains satisfied. In general, whenever TP is resource-using it must also be (other) resource-saving, i.e., it must be mixed. That also accounts for the other type of mixed TP, namely regress in machine industry and progress in basket industry. If κ_2 and λ_2 increase, either κ_1 or λ_1 or both must decrease in order to expand X_1 and, consequently, real wage.

In this conte: t it will be of some interest to explore the economic meaning of an ir rease or decrease in value capital coefficients. We may imagine that technological change proceeds in two stages. From (1.3) we have

$$\frac{n}{\hat{k}} = \frac{p_1 X_1}{p_2 X_2} + 1$$

In order for $\hat{\kappa}$ to increase (decrease), for given durability of machines, p_2X_2 must increase (decrase) because $p_1X_1 = L$ remains constant. Now, in the first stage we increase value capital coefficients. (Mixed TP cannot start with an improvement in labour productivity because it would never reach the second stage and would not be mixed). Since in the machine industry value and technical coefficients are the same, $\hat{\kappa}_2 = \kappa_2$, we have to increase the latter in any case. In the consumer goods industry $\hat{\kappa}_1 = \kappa_1 \dfrac{p_2}{p_1}$, which means that κ_1 may remain unchanged, provided the price ratio increases. An increase in κ_2 increases X_2, p_2 and p_1, but reduces X_1, (cf. (0.1) and (0.2)). In the second stage labour productivity must be improved in order to raise X_1 to at least the original level, and possibly more, to make the change worthwhile. A decrease of λ_1 increases X_1 and increases X_2 even further, and also increases further the ratio p_2/p_1 (0.2). Thus, a sufficiently strong increase in labour productivity in the consumer goods industry completes the change. If productivity increases also in the machine industry, both X_1 and X_2 will increase, but their ratio remains unchanged (0.1). The ratio of prices may change (0.2). But to satisfy the requirement that value capital-output ratio in the consumer goods industry also increases given an unchanged κ_1, the price ratio must increase, which implies a bigger productivity increase in production of consumer goods, $\lambda_1/\lambda_2 < \lambda^0_1/\lambda^0_2$. If after all these changes the requirements have just been satisfied, and we then increase κ_1 as well, this will have an additional impact on machine intensity because p_2X_2 will increase even more.

We may now summarize the preceding somewhat involved story. An increase of capital-output ratios in value terms in all industries and the economy as a whole implies an increase of the technical capital coefficient in machine production while the technical capital coefficient of consumer goods production may or may not increase. In order to make the change worthwhile, this worsening of machine productivity must be more than compensated by an improvement in labour productivity which implies a relative increase of labour productivity in the production of consumer goods if κ_1 remains invariant and some productivity increase or relatively more in machine production if κ_1 increases. The final result of the change (with the durability of machines remaining constant) is a rise of GNP measured in labour time,

$$p_1X_1 + p_2X_2 > p^0_1X^0_1 + p^0_2X^0_2,\ \ p_1X_1 = p^0_1X^0_1 = L.$$

In other words, besides the given direct labour, there is now more indirect labour embodied in commodities. Biased technological progress which increases value capital-output ratios—and thereby increases the organic composition of resources (1.5) — is such as to embody more labour in current production. Because of symmetrical relationships, a capital-saving TP — reduced value capital output ratios — implies a reduction of labour time embodied in current production (with n = const.). These are the changes in the absolute volume of labour, direct (live) labour remaining unchanged.

An increase in κ_2 and/or κ_1 means a greater number of machines per unit of output. With L unchanged, that also means more machines per worker, an increase in the technical composition of resources (κ_j). If in the original situation, each worker was operating one machine, after technical change, due to better organization, each worker can operate, say, two machines. Given the labour force, total output increases and an improved productivity of labour more than compensates for the reduced productivity of machines so that net output is larger than before. Since, in order to increase labour productivity, the stock of machines must be increased—while under simple reproduction X_2 only replaces worn-out machines leaving the stock unchanged—there will be a transitional period during which part of the labour force will be relocated from the production of consumer goods to the production of consumer goods to the production of machines. During this period net output will be reduced until a sufficient number of new machines becomes available and higher labour productivity begins to generate additions to X_1 which will eventually surpass the original level of production. Thus, an increase of capital output ratios (or the organic composition of resources) generates transitional accumulation which increases the stock of embodied labour required by the new technology and increases GNP in labour time as observed above. Symmetrically, a reduction of capital-output ratios (of organic composition resources) leads to an immediate increase in X_1 and makes possible a temporary decumulation which adjusts the stock of machines to the new proportions between the embodied and the current labour with a resulting fall in GNP.

Technological Change without Technological Progress. Technological progress implies increasing real wages (X_1 for fixed labour). If an alternative technology leaves X_1 unchanged, there is no progress but simply a technological change. Such a change may be neutral, capital saving or capital using and it is always mixed. Several such patterns are shown in Table 5.1. At any one time a book of blueprints may contain a number of such alternative technologies. All of them are equally eligible because they make possible the same per capita output of consumer goods with the same input of live and embodied labour.

Final comments. Table 1.1 does not exhaust the list of all possible simple paterns of TP. Yet, it is sufficiently inclusive to make possible some generalizations. A change in X_1 (L = const.) determines whether there is technological progress or not

$$\frac{\Delta X_1}{\Delta t} \begin{cases} > 0, & \text{technological progress} \\ = 0, & \text{technological change without TP} \\ < 0, & \text{technological regress} \end{cases}$$

By its very definition TP implies that output increases without any previous sacrifice of consumption. There may even be no new investment, as in Neutral II TP, or new investment may even be negative, if efficiency of machine production sufficiently increases.

A change in X_2 does not indicate anything about changes in capital intensity. Increased output of machines, with *unchanged durability* and capital coefficients, will, of course, increase worker-machine ratios (κ_j). If also capital coefficients improve, organic composition may fall while κ_j remain unchanged, as in Capital Saving I TP. If the life-span of machines is lengthened, even κ_j may be reduced.

Value capital coefficients and organic composition change in the same way, which also follows from (5a). It is, therefore, sufficient to consider only changes in the organic composition

$$\frac{\Delta \omega}{\Delta t} \begin{cases} > 0, & \text{capital using TP} \\ = 0, & \text{neutral TP} \\ < 0, & \text{capital saving TP} \end{cases}$$

If technological progress occurs only in the production of consumer goods (Neutral II), all indicators of capital intensity remain unchanged. If it occurs in the production of machines, the pattern is not so simple. If at the same time the durability of machines is shortened to $n\Gamma^{-1}$, there will be no progress, but simply a capital saving change. If n remains unchanged, with TP affecting only the production of capital ($\lambda_2\Gamma^{-1}$ and $\kappa_2\Gamma^{-1}$), it follows from (0.1) and (0.2) that all prices decrease, X_1 increases while X_2 may change either way. Next, k_1 and k_2 remain unchanged, while $\omega = p_2k$ decreases. Thus in both cases capital-worker ratios remain unchanged, but TP in consumer goods production leaves ω undisturbed, while if it occurs in machine production, TP is capital saving.

A similar analysis may be conducted with respect to changes of technical coefficents. Improvements in both λ_j increase outputs and reduce prices. Improvements in both κ_j reduce prices, increase output of X_2. On the other hand improvements in λ s increase k s but leave ω s undetermined. Reductions in κ s diminish both κ s and ω s. As a general orientation, it may be said that improvements in labour coefficients reduce prices and increase outputs, while improvements in capital coefficents reduce capital intensity measured either by κ_j or ω_j. Effects of individual changes in technical coefficents are given in (1.1) and (1.2).

In Table 1.1 the value of sectoral outputs remains the same. This is not necessarily so but is a result of implied assumptions. Net product is always the same, $p_1x_1 = 1$, $x_1 = X_1/L$ but the value of machine output per capita

$$p_2x_2 = \frac{n\lambda_2\kappa_1}{(n - \kappa_2)\,[\lambda_2\kappa_1 + \lambda_1 \;\; (n - \kappa_2)]} \qquad\qquad x_2 = X_2/L$$

need not remain the same, as is easily seen if only one of technical coefficients is changed. It will remain the same only if the shares of direct and indirect labour in gross national product do not change. As all ten patterns of technological change satisfy this condition, it does not appear very strong and it also approximates to the empirical world. In particular, organic composition may change in any way and leave the »factor« share constant.

C) Regular Unembodied Technological Progress Changes in rental

Continuous technological progress generates growth. Labour productivity increases which, given L, increases output. The patterns of technological progress will remain the same, except that now formulas for growing economies must be applied (Table 1.2).

Because of higher productivity, some of the workers will be displaced. In order to re-employ them, gross investment must increase. Thus, in general, stationary n will be lengthened into dynamic ν and a new term will appear in the rental rate

$$r = \frac{1}{\nu} + \gamma_2$$

where γ_2 determines the number of machines (γ_2K) necessary to equip the displaced workers. It is important to note that γ_2 depends on technological progress in the machine industry and not on the average technological progress. If technological progress is Mixed Neutral, γ_2 will be $\gamma_2 = \Gamma^2 - 1 = (1 + \gamma)^2 - 1 = \gamma\,(2 + \gamma)$, while the average productivity growth rate will lie between that rate and γ.

The essential precondition for technological progress is that X_1 increases (L = const.). This means that X_2 may remain unchanged or even decrease. Suppose TP is Neutral II. Then TP in the two sectors amounts to (Table 1.2).

$$\frac{p^0_1 X_1}{Y_1} = \frac{p_1 X_1 \Gamma}{p_1 X_1} = 1 + \gamma_1, \quad \frac{p^0_2 X_2}{Y_2} = \frac{p_2 X_2}{p_2 X_2} = 1 + 0, \ \gamma_2 = 0$$

where p_1 are base year prices, and Y_j are sectoral gross national products. Quantity equations are

$$(\lambda_1 \Gamma^{-1})(X_1 \Gamma) + \lambda_2 X_2 = L$$
$$(\kappa_1 \Gamma^{-1})(X_1 \Gamma) + \kappa_2 X_2 = K$$

Labour and machines become redundant at the same rate in the first sector and do not change in the second. Full employment is achieved when redundant workers are equipped with redundant machines and then produce additional γX_1. Prices of baskets fall, prices of machines remain unchanged. Since total capital stock does not change, there are no costs over and above stationary replacement

$$r = \frac{1}{n}$$

In terms of machines, the economy is stationary and $\gamma_2 = 0$. In terms of consumer goods the economy is growing ad $\gamma_1 > 0$.

Measuring Technological Change

When technology, i.e., technical coefficients, change that affects output and prices in different ways. So far we have been examining the consequences of specified changes of technical coefficients. In practical statistical work the task is usually defined in the reverse fashion. The changes of technology are not specified but we have data on quantities and prices. The task is to derive a measure of techno logical progress from such data assuming unembodied TP).

We start from the familiar social accounting identity of value of outputs and input being the same

$$p_1 X_1 + p_2 X_2 \equiv wL + rp_2 K \tag{1.7}$$

If w = 1, the value of final output (GNP) is exactly equal to the labour time embodied in the commodities at the current productivity of labour.

Increase in productivity means higher output given the resources or smaller expenditure of resources given the output. The increase in aggregate output is established if physical products are evaluated in last year prices. Thus, the overall rate of technological progress in our economy is given by

Table 1.2.

Effects of Continuous Technological Progress

		Technological Progress				
		Pure	Neutral	Capital	Saving	Mixed
		I	II	I	II	Neutral
		$\lambda_j\Gamma^{-1}$	$\lambda_j\Gamma^{-1}$	$\lambda_j\Gamma^{-1}$	$\lambda_j\ \Gamma_\lambda{}^{-1}$	$\lambda_1\Gamma^{-1}$
			$\kappa_j\Gamma^{-1}$	$\kappa_j\Gamma^{-1}$	$\kappa_j\Gamma_\kappa{}^{-1}$	$\lambda_2\Gamma^{-2}$
				$r\Gamma$	$r\Gamma_k$	$\kappa_1\Gamma$
Outputs						
Baskets	X_1	$X_1\Gamma$	$X_1\Gamma$	$X_1\Gamma$	$X_1\ \Gamma_\lambda$	$X_1\Gamma$
Plants	X_2	$X_2\Gamma$	same	$X_2\Gamma$	$X_2\ \Gamma_\lambda$	$X_2\Gamma^2$
Labour prices: $w = 1$						
Baskets	p_1	$p_1\Gamma^{-1}$	$p_1\Gamma^{-1}$	$p_1\Gamma^{-1}$	$p_1\ \Gamma_\lambda{}^{-1}$	$p_1\Gamma^{-1}$
Plants	p_2	$p_2\Gamma^{-1}$	same	$p_2\Gamma^{-1}$	$p_2\ \Gamma_\lambda{}^{-1}$	$p_2\Gamma^{-2}$
Capital intensity						
Machine-worker ratios	κ_1	$\kappa_1\Gamma$	same	same	$\kappa_1 \dfrac{\Gamma_\lambda}{\Gamma_\kappa}$	$\kappa_1\Gamma^2$
	κ_2	$\kappa_2\Gamma$	same	same	$\kappa_2 \dfrac{\Gamma_\lambda}{\Gamma_\kappa}$	$\kappa_2\Gamma^2$
Organic composition of resources	ω_1	same	same	$\omega_1\Gamma^{-1}$	$\omega_1\Gamma_k{}^{-1}$	same
	ω_2	same	same	$\omega_2\Gamma^{-1}$	$\omega_2\Gamma_k{}^{-1}$	same
Basket prices: $\bar{p}_1 = 1$						
Real wage	$\bar{w} = \dfrac{w}{p_1}$	$\bar{w}\Gamma$	$\bar{w}\Gamma$	$\bar{w}\Gamma$	$\bar{w}\Gamma$	$\bar{w}\Gamma$
Plant price	$\bar{p}_2 = \dfrac{p_2}{p_1}$	same	$\bar{p}_2\Gamma$	same	same	$\bar{p}_2\Gamma^{-1}$

$$X_1 = \frac{1 - r\kappa_2}{\lambda_2 r\kappa_1 + \lambda_1(1 - r\kappa_2)} L, \quad X_2 = \frac{r\kappa_1}{\lambda_2 r\kappa_1 + \lambda_1(1 - r\kappa_2)} L, \qquad L = \text{const.}$$

$$p_1 = \frac{\lambda_2 r\kappa_1 + \lambda_1(1 - r\kappa_2)}{1 - r\kappa_2} w, \quad p_2 = \frac{\lambda_2}{1 - r\kappa_2} w, \qquad w = 1$$

$$\bar{w} = \frac{X_1}{L} = \frac{1 - r\kappa_2}{\lambda_2 r\kappa_1 + \lambda_1(1 - r\kappa_2)}, \quad \bar{p}_2 = \frac{\lambda_2}{\lambda_2 r\kappa_1 + \lambda_1(1 - r\kappa_2)}, \qquad \bar{p}_1 = 1$$

$$\kappa_1 = \frac{K_1}{\lambda_1}, \quad \kappa_2 = \frac{K_2}{\lambda_2}, \quad \omega_1 = p_2\kappa_1, \quad \omega_2 = p_2\kappa_2$$

$$\frac{\overset{o}{p}_1X_1 + \overset{o}{p}_2X_2}{wL + rp_2K} = 1 + \gamma \tag{1.8}$$

Note the difference in treatment of labour and capital. Labour, being primary factor, represents no problem. Capital, being produced factor, must be evaluated at $\overset{o}{p}_2$ when output and at p_2 when input. It would be inadmissible to add together labour input at current productivity and capital input at past productivity.

If exact labour prices are used, (1.8) has an interesting interpretation. Since $\overset{=}{p}_j$ means labour of last year productivity contained in the unit of X_{j1}, the numerator represents labour time of last year labour productivity and the demoninator represents labour time of this year productivity, both contained in the same this year final physical output. In this way $(1 + \gamma)$ measures the ratio between past and present labour time necessary to produce the same bill of goods and is thus a natural and direct measure of labour productivity change.

Technological progress may proceed at different pace in the two industries. Define the sectoral measures and some auxiliary relationships as follows

$$\frac{\overset{o}{p}_1X_1}{Y_1} = 1 + \gamma \qquad \frac{\overset{\bullet}{p}_2X_2}{Y_2} = 1 + \gamma_2 \tag{1.9}$$

$$\begin{aligned} &Y_1 = wL_1 + rp_2K_1 = wL\,, \quad Y_2 = wL_2 + rp_2K_2 = rp_2K \\ &Y = Y_1 + Y_2 = wL + rp_2K\,, \quad Y_1/Y = \alpha, \quad Y_2/Y = \beta = 1 - \alpha \end{aligned} \tag{1.10}$$

and insert (1.9) and (1.10) into (1.8)

$$\alpha\,(1 + \gamma_1) + \beta\,(1 + \gamma_2) = 1 + \gamma \tag{1.11}$$

the aggregate factor of TP is equal to the sum of sectoral factors weighted by the shares of labour and capital input in GNP. These shares represent also the shares of basket and machine industries in GNP.

The most popular production function in measuring technological progress in Cobb-Douglas function

$$Q = L^{\alpha} K^{\beta} e^{\gamma^* t}$$

If we apply Divisia indexes, we get a neat expression for the rate of TP

$$\overset{*}{\gamma} = \frac{\dot{Q}}{Q} - \alpha\frac{\dot{L}}{L} - \beta\frac{\dot{K}}{K} \tag{1.12}$$

I use starred gamma in order to indicate that this neoclassical measure of TP is logically inconsistent and, therefore, theoretically wrong. The mistake made is to treat K and L in the same fashion as given resources.

Since K underwent a technological change, K as an input is different from K as an output and part of Q. The logically consistent function will look as follows

$$Q = L^{\alpha} (Ke^{-\gamma t})^{\beta} e^{\gamma t}$$

which generates a consistent measure of TP

$$\gamma (1 - \beta) = \frac{\dot{Q}}{Q} - \alpha \frac{\dot{L}}{L} - \beta \frac{\dot{K}}{K} \qquad (1.13)$$

The relation between the two measures is given by

$$\overset{*}{\gamma} = \gamma (1 - \beta) = \alpha \gamma \ \ if \ \ \alpha + \beta = 1 \qquad (1.14)$$

The neoclassical measure is substantially lower.[3]

If the economy is decomposed into two or more industries, capital input-evaluated at constant prices must by deduced at the rate of TP in capital good industry(-ies). It follows from the numerator in (1.8).

$$p_2 = p_2^o \frac{p_2}{p_2^o}$$

and from (1.9)

$$\frac{p_2^o X_2}{Y_2} = \frac{p_2^o X_2}{p_2 X_2} = \frac{p_2^o}{p_2} = 1 + \gamma_2 \qquad (1.15)$$

Consequently, if labour-time prices are used, capital input $r\overset{o}{p_2}{}^{o}K$ must be reduced at the rate $\gamma_2 = p_2^o/p_2 - 1$.

[3] That neoclassical defect was already noticed by Reed (1968) and Rymes (1971).

Constant nominal prices

Since all values represent labour values, the shares α and β represent also the shares of live and embodied labour of final output. These shares can change in time. Consequently, average γ may change even if sectoral γ_i remain constant and also, γ may remain constant even if sectoral γ_i change.

As prices are determined by two multiplicative terms, of which the first is conditioned exclusively by technology and r, while the second consists of nominal wage rate, it is easy to see how prices can be kept stable. Sectoral prices will be stabilized if labour-time prices are inflated for the increase of sectoral labour productivity, $\overset{*}{p}_i = p_i\Gamma_i$. It follows from (1.9).

$$\frac{p_i^{\circ} X_1}{\Gamma_i Y_1} = 1$$

If all prices are inflated by the average productivity factor Γ, $p_i = p_i\Gamma$, the use of (1.10) and (1.11) gives

$$\frac{p_1^{\circ} X_1 + p_2^{\circ} X_2}{\Gamma Y} = 1$$

and the general price level will be stabilized

D) Embodied Technological Progress

Embodied and unembodied technological progress compared

If the design of machines and the composition of plants change in time, technological progress is embodied and we encounter new difficulties. Each vintage of plants will have a different efficiency.

Consider the case when, with constant labour force, the output of (standard) baskets expands by factor Γ and the number of (changing) plants constructed by factor M. We assume that the life-span of plants of all vintages is the same. If capital coefficients decline by factor Γ_2, capital in (physical) efficiency units is $K\Gamma_2$, where K is the number of plants. We shall first compare capital stocks ad now investments in efficiency units for embodied and unembodied TP.

Capital stock

Embodied TP: $$1 + M\Gamma_2 + \ldots + M^{n-1}\Gamma_2^{n-1} = \frac{(M\Gamma_2)^n - 1}{M\Gamma_2 - 1} \qquad (1.16a)$$

$$\text{Unembodied TP: } \Gamma_2^{n-1} + M\Gamma_2^{n-1} + \ldots + M^{n-1}\Gamma_2^{n-1} = \Gamma_2^{n-1}\frac{M^n - 1}{M - 1} \tag{1.16b}$$

Capital stock consists of all gross investments in the last *n* years. Initial unit investment will by the n-th year increase to M^{n-1} units and their efficiency to Γ^{n-1}. In the embodied case each vintage of plants has its own efficiency, while in the unembodied case all vintages have the same, last year, efficiency. Clearly, under the unembodied case capital stock, measured in efficiency units, is larger.

New investment

$$\text{Embodied TP: } (M\Gamma_2)^n - 1 \tag{1.17a}$$

$$\text{Unembodied TP: } K_t - K_{t-1} = \left(\frac{M^n - 1}{M - 1}\right)(M\Gamma_2^n - \Gamma_2^{n-1}) \tag{1.17b}$$

Under embodied TP gross investment at *t* is $M^n\Gamma_2^n$, and scrapped plant is 1, i.e., gross investment at t—n. Under unembodied TP gross investment is $M^n\Gamma_2^n$ augmented for the increase in efficiency of the entire capital stock. Scrapped output capacity is Γ_2^{n-1}.

Since $X_1 = \overset{*}{\kappa}_1$ ($\overset{*}{K}_1$, and $\overset{*}{\kappa}_1$ = const. because $\overset{*}{K}_1$ is expressed in efficiency units, $\overset{*}{K}{}_1^t = \overset{*}{K}{}^1_1\Gamma_2^t$, the rate of growth of net output (g_{x1}) is equal to the rate o fgrowth of effective capital stock (g_{k1}).

Rate of growth

(1.18)

$$\text{Embodied TP: } g_{x1} = \frac{(M\Gamma_2)^n - 1}{\dfrac{(M\Gamma_2)^n - 1}{M\Gamma_2 - 1}} = M\Gamma_2 - 1$$

$$\text{Unembodied TP: } g_{x1} = \frac{\dfrac{M^n - 1}{M - 1}(M\Gamma_2^n - \Gamma_2^{n-1})}{\Gamma_2^{n-1}\dfrac{M^n - 1}{M - 1}} = M\Gamma_2 - 1$$

The rate of growth is, of course, given by the ratio of new investment and capital stock. The result is quite pleasing, the rate of growth of net output is the same under both types of technological progress. But absolute net output and its absolute increments are higher under unembodied TP which should be obvious.

It remains to check our results by reference to Table 1.2. Under Neutral I TP, $\Gamma_2 = 1$ and $M = \Gamma_1$. Consequently, the net output grows as $X_1\Gamma_1$. Under Neutral II TP, $M = 1$, and $\Gamma_2 = \Gamma_1$, and so net output grows again at the same rate. Since $M = 1$, new investment is zero, as it should be.

Replacement

The three quantities — baskets, plants and productivity — may expand at different rates. Which one is relevant for price formation and investment evaluation? How are current prices determined?

Labour prices are cost prices. The two cost components are labour costs and capital costs. Labour costs are determined by technology exclusively (λ_j). Capital costs consist of two items: replacement and new investment necessary to maintain full employment. Both items depend on technology (κ, n) and the rate of growth of employment (g). For $g = 0$, capital costs depend exclusively on technology. If technology does not change, economy is stationary and capital costs reduce to $\frac{1}{n}K$.

If $g = 0$ and technology changes, replacement is still $R_t = I_{t-n}$ and so its current rate depends on the rate of growth of capital. But $K_t \neq K_{t-n}$ and so replacement depends also on technological progress. What has to be replaced is not physical machines but output capacity. If the stock of physical machines expands a the rate m, and their efficiency at the rate γ_2, output capacity expands at combined rate $M\Gamma_2 - 1$. Consequently, current replacement under embodied and unembodied TP will amount to

$$eR^*_{t+1} = I^*_{t-1+n} = (M\Gamma_2)^{t-n} \tag{1.19a}$$

$$uR^*_{t+1} = M^{t-n}\,\Gamma_2^{t-1} \tag{1.19b}$$

where the asterisk denotes output capacity. Current output capacity is equal to all investment in the last n years

$$eK_t^* = \sum_{t-n}^{t-1} (M\Gamma_2)^{\tau} = (M\Gamma_2)^{t-n}\frac{(M\Gamma_2)^{n-1}}{M\Gamma_2^{-1}} \tag{1.20a}$$

$$uK_t^* = \sum_{t-n}^{t-1} M^{\tau}\,\Gamma_2^{t-1} = M^{t-n}\,\Gamma_2^{t-1}\frac{M^n - 1}{M - 1} \tag{1.20b}$$

For $\Gamma_2 = 0$, K_t^* is the number of currntly operating plants of equal efficiency.

Replacement per unit of capital is

$$\frac{1}{e\nu} = \frac{eR^*_{t+1}}{eK_t^*} = \frac{M\Gamma_2 - 1}{(M\Gamma_2)^n - 1} \tag{1.21a}$$

$$\frac{1}{u\nu} = \frac{uR^*_{t+1}}{uK_t^*} = \frac{M - 1}{M^n - 1} \tag{1.21b}$$

If capital stock is measured in terms of initial number of effective plants, $(M\Gamma_2)^{t-n} = 1$, replacement ratio $1/e\nu$ is equal to the reciprocal value of capital stock under embodied TP. Further, under ETP the variable ν is a function of both: increasing number of plants (M) and changes in their efficiency (Γ_2). Under UTP, dynamic years $u\nu$ depend exclusively on the growing number of physical plants — because the efficiency of all plants in a given year is the same. ETP replacement ratio is smaller, and in general the following relation holds

$$e\nu \geqslant u\nu \geqslant n$$

where the equality sign applies only when there is no technological progress, $M = \Gamma = 1$.

If also labour force is growing, the stock of capital will have to expand further in order to provide employment for new workers. In our formulae every M will have to be replaced by GM, where $G = 1 + g$ is the growth factor of labour input.

Price formation

In order to derive price equations, we must start by considering material balances under ETP at the beginning of the current year. If labour coefficients improve, some workers will be displaced. In order to employ them, the stock of machines must increase. If also capital coefficients improve, the increase of the capital stock will be modified. If also the size of plants changes, capital stock will be modified for the third time. In general, changes in size and in design cannot be distinguished. Unrestricted changes make the analysis unmanageable and we need some simplification. Empirical data suggest that the proportions of live and embodied labour remain approximately constant. In a closed economy with costant L that implies that sectoral employments remain unchanged. The size of a plant will be assumed constant. The two assumptions make plausible the third one, namely that capital coefficients improve at the same rate. Let A_1 and A_2, $A_1 + A_2 = 1$, be sectoral gross investments at $t-n$. Material balances at the beginning of t+1 are as follows

$$L_1 \qquad + \qquad L_2 \qquad = \qquad L$$

$$A_1 \frac{(M\Gamma_2)^n - 1}{M\Gamma_2 - 1} + A_2 \frac{(M\Gamma_2)^n - 1}{M\Gamma_2 - 1} = \frac{(M\Gamma_2)^n - 1}{M\Gamma_2 - 1}$$

The rate of gross investment is the same in both sectors and so investment balance appears to be

$$rK_1^* + rK_2^* = (M\Gamma_2)^n = X_2^*, \quad K_j^* = A_j \frac{(M\Gamma_2)^n - 1}{M\Gamma_2 - 1},$$

$$r = \frac{1}{e\nu} + (M\Gamma_2 - 1)$$

Define technical coefficients

$$\lambda_j^* = \frac{L_j}{X_j^*}, \quad \kappa_j^* = A_j \frac{(M\Gamma_2)^n - 1}{M\Gamma_2 - 1} / X_j^*$$

where asterisks denote averages over all vintages. The usual vertical summation of the components of value balances produces the required price equations:

$$\begin{aligned} rp_2^* \kappa_1^* + w\lambda_1^* &= p_1 \\ rp_2^* \kappa_2 + w\lambda_2^* &= p_2^* \end{aligned} \qquad (1.22)$$

The equations have the following characteristics:

(1) λ^*_j are not technical coefficients of any specific vintage of plants; they represent averages calculated with respect to all vintages of plants in existance.

(2) The number of plants constructed annually increases by M, their efficiency by Γ_2. Since both factors are constant, the stock must increase equally as gross investment. Consequently, capital coefficient in the second sector does not change

$$\kappa_2^* = \kappa_2 = \frac{K_2}{X_2} = \text{const.}$$

and the asterisk is not necessary. In the first sector output increases by $M\Gamma_2$ and the number of plants by M so that K_1 decreases by Γ_2. In efficiency units K_1 = const. For $K_1^* = K_1$, technological progress would be Neutral I. Technological progress implied here is such that one machine of any vintage produces the same number of machines of the next vintage but successive vintages of machines produce increasing number of baskets.

(3) The number of plants is measured in efficiency units and so p_2^* is the price of the plant having efficiency of the vintage t—n but produced by the capital stock of the last *n* vintages.

(4) By definition, standard baskets do not change and so p_1 represents the price of the actual basket of wage goods.

(5) More recent vintages of plants have λ_j and $p_2\kappa_j$ smaller then are those in (1.22e) and will, therefore, earn extra profits which will be used to cover depreciation charges. Since prices are falling and technology of any vintage remains constant, depreciation must be concentrated in early years. We are still left with the task of finding the price of the plant of the current vintage. The number of plants constructed currently is $X_2^* = (M\Gamma_2)^n$ in efficiency units and $X_2 = M^n$ in physical units. — The value of these plants remains the same however are they counted, $p_2^*X_2^* = p_2X_2$. Consequently

$$p_2 = \frac{p_2^*X_2^*}{X_2} = p_2^*\Gamma_2^n$$

If also changes in λ_j are known, we may say more about price equations. Suppose technological progress is Neutral I: capital coefficients remain unchanged, while labour conffficients decrease by Γ_1. With respect to the foregoing pattern of TP, that implies $\Gamma_2 = 1$, $M = \Gamma_1$. From one vintage to the next the number of plants increases by Γ_1 and so do outputs X_1 and X_2. Since labour productivity also increases by Γ_1, the number of workers employed remains constant. Thus, labour is equally distributed among vintages, L/n workers operating machines of each vintage. It follows

$$\lambda_j^* = \frac{L_j}{X_j^*} = \frac{nLj/n}{x_j\,(\Gamma_1^{1-n} + \ldots + \Gamma_1^{-1} + 1)} = \lambda_j(t)\,\frac{n\,(\Gamma_1^{-1} - 1)}{\Gamma_1^{-n} - 1}$$

$$\lambda_j^* = \lambda_j\,(t)\,\frac{-\,n\gamma_1}{\Gamma_1\,(\Gamma_1^{-n} - 1)}, \qquad \lambda_j\,(t) = \frac{nx_j}{L_j}$$

where x_j is the output of the last vintage which employs L_j/n workers and λ_j (t) is labour coefficient of the last vintage. Price equations (22e) for the last vintage under Neutral I TP are

$$rp_2\kappa_1 + w\lambda_1\,\frac{n\gamma_1}{\Gamma_1\,(1 - \Gamma_1^{-n})} = p_1$$

$$rp_2\kappa_2 + w\lambda_2\,\frac{n\gamma_1}{\Gamma_1\,(1 - \Gamma_1^{-n})} = p_2 \qquad \text{(1.22e I)}$$

In these equations $w\lambda_j$ represent actual wages, while

$w\lambda_j\left(\frac{n\gamma_1}{\Gamma_1\,(1 - \Gamma_1^{-n})} - 1\right)$ indicates how much average replacement cost $1/e_\upsilon$ must be augumented to provide an appropriate depreciatiton. For older vintages replacement will have to be reduced to avoid losses and keep wages a $w\lambda_j$ for the respective vintage *j*.

With unembodied technological progress no such complications arise. Plants of all vintages are equally efficient, and κ_j are ordinary capital coefficients: the number of physical plants per unit of output.

$$rp_i\kappa_j + w\lambda_j = p_j, \qquad r = 1/\upsilon + (GM\Gamma_2 - 1), \qquad j = 1,2 \qquad (1.22u)$$

The interpretation is as follows:

(1) Under UTP all plants and, of course, all workers are equally efficient at the last year level. Consequently, κ_j and λ_j represent ratios of physical quantaties. In the current year their efficiency increases further and that affects the necessary volume of investment.

(2) The profit rate $\pi = GM\Gamma_2$ is the same for both types of TP.

(3) The replacement ratio is different for each type of TP as shown in (1.21). Under ETP, ν is a function of efficiency factor too, while under UTP it is not.

This difference is quantitatively negligible. For $M = \Gamma_2 = 1.03$ and $G = 1.01$, the respective replacement ratios are $1/e\nu = 0.010$, $1/u\nu = 0.018$. Since technological progress is both embodied and unembodied, the actual difference is somewhere between the two figures. Note that $M\Gamma_2 - 1 = gx_1$ ($GM\Gamma_2$ if labour also expands) is the rate of growth of output (1.18). The same rate of growth appears in (1.21a). Consequently, if output rate of growth is used in calculating replacement ratio, that implies assuming embodied technological progress.

Under ETP the average ν for the economy is constant (as long as $GM\Gamma_2$ = const.) in time, but depreciation rates are different for different vintages at any given t and change for a given vintage over time. Under UTP, ν = const. over time and across vintages.

Obsolescence

Older technology is less efficient and, therefore, less profitable. In order to avoid losses, technical life-span of plants might have to be truncated and the obsolete machines scrapped at an earlier time.

Whatever the pattern of technological progress, production will become unprofitable when prices — which are the same for all vintages —will no longer cover wage costs

$$p_j < w\lambda_j^{\tau}$$

where $\lambda\tau$ is labour coefficient of vintage τ. If labour coefficients decrease by $\Gamma\lambda$ (as in all eight patterns of TP considered in Table 1.1), and λ_j belongs to latest vintage, the inequality

$$p_j < w\lambda_j\Gamma_{\lambda}^{n}$$

determnies the life-span of plants n. The faster technological progress — the larger $\Gamma\lambda$ — the more likely it is that technical n will have to be truncated.

Technologically determined n is likely to be truncated if (a) technological progress is fast (large $\Gamma\lambda$) and (b) production is labour intensive (w_j large relatively to $p_2\kappa_j$).

2. EVALUATION OF INVESTMENT PROJECTS

A) Preliminaries

Conceptual issues again

Before we proceed, same clarification is necessary. What does it mean when we say that capital coefficients have been reduced? One possible interpretation is that machines are still physically identical so that we can measure capital stock by simply counting the machines, old as well as new. Then an increased productivity of machines, with unchanged productivity of labour, implies a decrease in technical composition of resources, more workers per machine

$$\frac{(1+\gamma)^{-1}\kappa^{o}}{\lambda^{o}} = k < k^{o}, \qquad \gamma > 0$$

Such an effect is perfectly possible if, due to improved repair services, machine runs more hours per year while the crew of operators is enlarged by including repairmen. But the possibilities of such improvements must soon be exhausted. An opposite change, improvement in labour productivity with κ's constant, implies more machines per worker. Although specialization, division of labour and a better organization of work may make it possible that one worker operates two machines instead of one, such possibilities are also soon exhausted and after every worker will have been equipped with three, or five or ten machines further technological progress will have to come to a standstill. Thus, a continuous reduction of κ's is only slightly less unrealistic than a continuous improvement of λ's.

The two changes can be simultaneous: both labour and capital coefficients may decrease. If they decrease at the same proportional rate, machine-labour ratio will not change but output will increase. The proportional decrease of λ's will increase outputs and reduce prices in the same proportion, leaving relative outputs and relative prices unchanged. If a proportional decrease of κ's is superimposed, X_1 will increase further still—and X_2 will be somewhat reduced. Both will decrease even more with the price of the more capital intensive good being reduced more than the other. However, it is again unrealistic to expect that workers equipped with the same physical machines can indefinitely produce more and more output. Thus, eventually ve must assume that machines change as well and that an unembodied technological progress is supplemented by technological progress embodied in new machines.

If machines change, we can still continue our physical count due to synchrony principles, but the comparison of most of the characteristic coefficients before and after the change loses economic meaning. This raises the problem of a definition of a new unit o measurement. The unit must be invariant itself. I shall take the basket of consumer goods in the base period as such a standard. This will be the *standard basket* and we shall measure changing machines in terms of the standard basket. When consumer goods change, we are in trouble again. To price statisticians the problem is known as the problem of new products and of quality change. I shall assume that at the time new products are introduced, both old and new products are produced by the same machines and labour. Thus, cost ratios will determine price ratios, and new baskets will be expressed in terms of standard baskets. Long-run equilibrium is assumed throughout, and so extra profits do not arise. It is always possible to select as a numéraire a consumer good — such as milk — whose quality can be standardized and which is not likely to disappear from the market.

As for quality changes, we leave the evaluation to the consumer who will equalize the marginal utility of dinar (labour day) spent in any line of consumption.

In order to measure the quality of machines (and embodied labour) in terms of standard baskets, all we have to do is to put $\overline{p_1} = 1$ in our price equations and then evaluate everything in new prices. Since nothing else changes, quantities of goods and resources and technical coefficients remain unchanged. Consequently, relative prices remain the same and only the absolute price level changes.

Basket and Standard prices

If the wage rate is taken as a numéraire $w = 1$, absolute prices are exact labour-time prices. If the basket of consumer goods is taken as a numéraire, $p_1 = 1$, prices are expressed in terms of baskets (p_j/p_1) and nominal wage is transformed into real wage. Baskets have a standard size and composition of commodities. The size changes in time, the composition does not. The standard basket is the wage basket of some base period when one wage buys one basket and so $w = p_1$. In this period the number of baskets produced is equal to the number of workers, $X_1 = L$. In later periods L remains constant, but X_1 increases and so labour productivity increases. Similarly, the nominal wage remains constant, $w = 1$, but the real wage increases.

A change in labour productivity may be expressed in three different ways:

$$\Gamma_1 = \frac{p_1(0)\,X_1(t)}{p_1(t)\,X_1(t)} = \frac{p_1(0)}{p_1(t)} \qquad (2.1a)$$

$$\Gamma_1 = \frac{p_1(0)\,X_1(t)}{p_1(0)\,X_1(0)} = \frac{X_1(t)\,/L}{X_1(0)/L} = \frac{\bar{w}(t)}{\bar{w}(0)} \tag{2.1b}$$

$$\Gamma_1 = \frac{p_1(0)\,X_1(t)}{p_1(t)\,X_1(t)} = \frac{p_1(0)\,X_1(t)}{wL} = \frac{X^1(t)}{L},\quad p_1(0) = w = 1 \tag{2.1c}$$

Γ_1 in (2.1a) is a Paasche index of prices, and in (2.1b) is its dual, a Laspeyres index of quantities. The latter is also a ratio of real wages. If the wage rate is used as a numériaire of the system, $w = 1$, and in the base period a basket is calibrated so that one wage buys one basket, then $\Gamma_1(0) = 1$ and so $\Gamma_1(t) = X_1(t)/L$ represents the current labour productivity in terms of baskets, and also an increase of labour productivity over the base period. Γ_1 is defined so as to measure changes in labour productivity in the *consumer good sector*, i.e., it measures changes in direct and indirect labour expended in the production of one basket. At the same time it also measures *systemic changes* in labour productivity, i.e., it measures changes in the volume of live labour (employed in both sectors) per unit of net output.

If the standard basket is used as a numériaire of the system, we have an invariant unit of account. Prices in terms of baskets are

$$\bar{w} = \frac{w}{p_1} = \frac{X_1}{L} = \frac{1 - r\kappa_2}{\lambda_2 r\kappa_1 + \lambda_1(1 - r\kappa_2)} \tag{2.2a}$$

$$\bar{p}_1 = \bar{w}p_1 = 1 \tag{2.2b}$$

$$\bar{p}_2 = \frac{p_2}{p_1} = \bar{w}p_2 = \frac{\lambda_2}{\lambda_2 r\kappa_1 + \lambda_1(1 - r\kappa_2)} \tag{2.2c}$$

The real wage $\bar{w}$ represents at the same time current productivity of labour. It indicates how many baskets can be produced by expanding one worker-year. The basket price of a plant, $\bar{p}_2$, shows the number of baskets that can be produced by the labour embodied in a plant. The machines may change, but plants still can be counted in terms of labour embodied in baskets.

The remarkable property of prices in terms of wage goods baskets is that they can be derived as products of ordinary labour-time prices and the real wage, $\bar{p}_i = \bar{w}p_i$. If labour productivity increases equally in both sectors, as under Neutral I and Capital Saving TP, relative prices will not change and basket-prices will remain constant,

$$\bar{p}_i(t) = \bar{w}(t)p_i(t) = \bar{w}(0)\Gamma^t p_i(0)\Gamma^{-t} = \bar{w}(0)p_i(0) = \bar{p}_i(0)$$

The basket price of a plant, $\bar{p}_2$, depends on all technical coefficients and not only on the state of technology in the machine sector as under labour-time. A proportional increase in labour productivity (Neutral I TP), $\lambda_j = \mathring{\lambda}_j \Gamma^{-1}$, leaves p_2 unchanged. A decrease of κ_1 increases relative efficiency of machines, and $\bar{p}_2$ increases. A decrease of κ_2, reduces p_2 which means that one physical machine now contains a smaller number of equivalent (in terms of labour) baskets. A neutral TP of type II increases $\bar{p}_2$, while Mixed Neutral TP reduces $\bar{p}_2$. In general, changes in $\bar{p}_2$ indicate how does TP change the basket content of machines under new productivity of labour.

If we have the following relation between two stocks of machines produced in current and base years

$$\bar{p}_2 K = A \bar{\mathring{p}}_2 \mathring{K}$$

then the quantity of machines in terms of baskets in the current period is A times larger than the stock of the base period. $K/\mathring{K}$ represents a change in the actual number of machines, but that has no economic meaning because machines of different efficiency are not commensurable. Only if $\bar{p}_2 = \mathring{p}_2$ — when labour productivity increases equally in both sectors — the relation between physically counted machines may have an economic meaning, namely $K = A\mathring{K}$.

We may standardize basket prices in terms of real wage of some base period productivity

$$\bar{\bar{p}}_j = \frac{\bar{p}_j}{\bar{w}^o} = \frac{\bar{w}}{\bar{w}^o} p_j = \frac{\bar{p}_j}{p_1 \overline{w^o}}$$

Such prices will prove to have some interesting properties. They will be called *standard prices.*

Present Values

When individual investment projects are evaluated, inputs and outputs are dated and the rule has to be discovered how to aggregate them diachronically.

In a stationary economy no problem arises because time does not matter — although even here alternative techniques with the same labour inputs but shorter production periods are preferable.

Presumably a rational consumer prefers more consumption forever to less consumption forever and the use of techniques involving dated inputs is determined by the composition of demand. It is important to realize that stationariness is analytically timeless because neither of the two dimensions of labour — the number of workers and their productivity — is subject to change. As soon as the number of workers changes — either positively or negatively — time dimension is activated directly: the available labour *time* changes. When productivity changes, time dimension is implied since change means an effect per unit of *time*. It is, therefore, obvious on *a priori* grounds that the two time effects must be properly accounted for if two diachronically different streams of inputs and outputs are to be compared.

The conventional textbook rule for calculating the present value of a stream of past inputs says that they should be accumulated at the ruling interest rate. But we are never given precise instructions on how to find the appropriate interest rate nor at what prices inputs ought to be evaluated. As to the prices, presumably they must be constant prices since otherwise inputs are not commensurable. Yet, how do we establish a constant price for machines whose costs and productivities vary?

Prices in terms of baskets

Supprose a long-term social plan envisages the rate of full employment expansion of consumer goods output to be π. By the end of the planning horizon the present output of baskets will have increased to

$$X_I(t) = X_I(0)\,(1 + \pi)^t \tag{2.3}$$

In this system the two dated batches of consumer goods, $X_I(0)$ and $X_I(t)$, are equivalent only if (2.3) is satisfied. The expression (2.3) describes the state of the system within the planning period, it identifies the system.

In describing the system we simply apply the synchrony rule by reducing future (t) and past (—t) outputs to present time in order to make them comparable. The weights used are $(1 + \pi)^t$. A future output will be equivalent to the present output if the following relation is satisfied

$$X_I(0) = X_I(t)\,(1 + \pi)^{-t} \tag{2.4a}$$

and similary for a past output

$$X_I(0) = X_I(-t)\,(1 + \pi)^t \tag{2.4b}$$

We call the former procedure discounting, and the latter accumulating. The discount and the accumulation rate is the same (π).

A system may also change unevenly. In that case it will be identified by applying dated discount rates $\pi(t)$. We shall mostly consider regularly growing systems with constant discount rates.

There are two possible causes of growth: technological progress and the growth of labour force. Thus the discount factor will be structured

$$1 + \pi = G\Gamma, \quad G = 1 + g, \quad \Gamma = 1 + \gamma$$

γ is positive by definition, while g may also be negative. It is, therefore, possible that $G\Gamma < 1$ which makes π negative, $\pi < 0$. It is possible, but not likely empirically.

An investment project consists of two parts, of investment outlays and of a stream of net outputs. The two streams may partly overlap. In our system investment outlays consist of plants constructed and net outputs consist of consumer baskets. The two composite commodities cannot be compared directly, and so we compare their values whereby plants are measured in standard baskets. A plant is worth while constructing if its cost in terms of baskets is no greater than its output of equivalent baskets over its lifetime. The construction of the plant may take more than one year. To simplify matters at this stage of analysis, I shall assume that the plant was constructed in one year, $X_2(0) = 1$, which will be considered the base year. Then our investment rule says that the project will be eligible if the present value ot the stream of future net value is not greater than investment outlays

$$\overline{p}_2(0)X_2(0) \leqq \sum_t X_1(t)\,(1 + \pi)^{-t} \tag{2.5a}$$

The rule is quite orthodox, only the garb is somewhat novel. The prices are constant prices in terms of standard baskets of wage goods and the discount rate is not an anonymous market rate of interest, but a very definite $\pi = G\Gamma - 1$.

In which way is all this related to the labour theory of prices?

Labour and standard prices

Nothing is changed in (2.4a) if the same price is used to evaluate both outputs. We only assume that $g = 0$ and so $\pi = \gamma$ Consequently,

$$p_1(0)X_1(0) = p_1(0)X_1(t)\,(1 + \gamma)^{-t} \tag{2.6}$$

We know that the labour value of net output is equal to live labour and so does not change when labour does not change

$$p_1(0)X_1(0) = p_1(t)X_1(t) = L \tag{2.7}$$

It follows from (2.6) and (2.7)

$$p_1(t) = p_1(0)\,(1 + \gamma)^{-t} \tag{2.8}$$

and we can derive the first theorem.

Theorem I. If employment does not change, $L = \text{const.}$, while labour productivity increases at the rate γ, this rate functions also as a discount rate and the present value of future consumer baskets is equal to their historical costs,

$$p_l(0)X_l(t)\,(1+\gamma)^{-t} = p_l(t)X_l(t).$$

Similar reasoning applies to the investment rule as well. Multiply both sides of (2.5) by $p_l(0)$

$$p_l(0)\bar{p}_2(0)X_2(0) = p_l(0)\,[\bar{w}(0)p_2(0)]\,X_2(0) \leqq \cdot\, p_l(0)\sum_t X_l(t)\,(1+\gamma)^{-t}$$

Since $p_l(0)\,\bar{w}(0) = 1$, it follows

$$p_2(0)\,X_2(0) \leqq \sum_t p_l(t)\,X_l(t) \qquad (2.9)$$

which means that labour expanded on the construction of a plant must at least be recuperated in its output.

If the plant will be constructed over τ wears, the present value of future investment outlays in standard prices is

$$\sum_\tau \bar{p}_2(\tau)\,X_2(\tau)\,(1+\gamma)^{-\tau} = \sum_\tau \bar{w}(\tau)\,p_2(\tau)X_2(\tau)\,(1+\gamma)^{-\tau} =$$

$$= \bar{w}(0)\sum_\tau p_2(\tau)\,X_2(\tau)$$

and similarly for baskets

$$\sum_t \bar{p}_l(t)\,X_l(t)\,(1+\gamma)^{-t} = \sum_t \bar{w}(t)\,p_l(t)\,X_l(t)\,(1+\gamma)^{-t} =$$

$$= \bar{w}(0)\sum_t p_l(t)\,X(t)$$

and the general investment rule is simply

$$\sum_\tau p_2(\tau)\,X_2(\tau) \leqq \sum p_l(t)\,X_l(t) \qquad (2.9)$$

which leads to the next theorem.

Theorem II. If employment does not change, while labour productivity increases, investment outlays and net products are made comparable by being evaluated in historical labour prices. Alternatively, any two dated products are equivalent if their labour values are equal.

For an investment project to be eligible, the present value of Marshallian quasi rents must be at least equal to the present value of investment. This criterion yields the following

Corollary I. Labour time expended in investment activities must at least be recuperated in net output.

Yet another consequence follows from Theorem II. Suppose projects are evaluated in standard prices, which are clearly also constant prices. Then the following is true

$$\bar{\bar{p}}_j(t)\, \Gamma^{-t} = \frac{\bar{p}_j(t)}{\bar{\bar{w}}(0)}\, \Gamma^{-t} = \frac{\bar{w}(t)}{\bar{\bar{w}}(0)}\, p(t)\Gamma^{-t} = p(t)$$

Corollary II. Discounted standard values are equal to historical labour values. The rate of growth of labour productivity is the discount rate. Since $\bar{\bar{p}}_j(0) = p(0)$, the own interest rate of the composite commodity represented by the standard basket of wage goods is the universal rate of discount if labour does not expand ($g = 0$).

That ought to be obvious. Instead of measuring output in terms of baskets, output is measured in labour time of equal productivity. Effective labour time expands at the rate γ — the rate of expansion of output — and must be discounted at the same rate in order to be comparable with the present size of labour time.

If labour force also increases, physical labour time increases at the rate g, and global labour time — labour of equal productivity — at the combined rate $\pi = \Gamma G - 1$ which is also the discount rate.

Theorem III. If physical labour expands by factor G and effective labour by factor Γ, global labour of equal productivity will expand by the combined factor $1 + \pi = G\Gamma$. In order to achieve comparability with the presently available labour, the synchrony rule requires that global labour be compressed into present size. In order to reduce global labour to the size of presently available labour, discount factor $G\Gamma$ must be applied. If values are expressed in standard prices, the discount factor is $G\Gamma$, if they are expressed in labour prices the discount factor is G.

C) Comparisons of Investment Projects

A simple investment criterion emerges: labour expended must be recuperated.

If investment outlays are treated as negative outputs, the present value of an investment project must be non-negative, $V \geqslant 0$, if the project is to be acceptable. Let a project have a life-span of *n* years and involve *s* different investment and consumption goods, then the present value of the projects is given as follows:

$$\text{Standard prices: } V_s = \sum_{t=1}^{n} (1+\pi)^{-t} \sum_{j=1}^{s} \bar{\bar{p}}_j(t)\, X_j(t) \tag{2.10a}$$

$$\text{Labour prices: } V_L = \sum_{t=1}^{n} (1+g)^{-t} \sum_{j=1}^{s} p_j(t)\, X_j(t) \tag{2.10b}$$

$$V_s = V_L, \quad (1+\pi) = (1+g)(1+\gamma)$$

If γ and g change over time, $(1+\pi)^{-t}$ ought to be replaced by $\prod_t [1+\pi(t)]^{-1}$ and $(1+g)^{-t}$ by $\prod_t [1+g(t)]^{-1}$. Investment projects are compared by means of V: the higher is the present value of a project the more is it profitable.

Before we conclude, a curious case may be mentioned again. If labour shrinks by the same factor by which labour productivity expands, $G = \Gamma$, net output will be stagnant, $G^{-1}\Gamma = 1$, but per capita net output (= real wage) will grow at the rate γ. Real wage will always increase at the rate of technological progress (regardless of what happents to G), because technological progress is the only source of — economic progress. Stationariness means that there is no discounting because effective labour does not change.

In general there are four possible states of economy which are described by four different discount rates:

1. *Stationary conditions:* $\pi = g = \gamma = 0$ or $\pi = (1-g)(1+\gamma) - 1 = 0$. The value of an investment project is expressed in labour or in standard prices. Global labour time either does not change, or the changes cancel out.

2. *Constant labour time and technological progress:* $g = 0$, $\gamma > 0$, $\pi = \gamma$. The value of an investment project is expressed in historical labour prices (diachronic labour time).

3. *Changes in labour time and constant technology:* $g \gtreqless 0$, $\gamma = 0$, $\pi = g$. The value of an investment project is expressed in synchronic labour prices. i.e., in labour prices corrected for the change in the socially available labour time (labour force).

4. *Changes in labour time and technological progress:* $g \gtreqless 0$, $\gamma > 0$, $1 + \pi = (1 + g)(1 + \gamma)$. Historical labour time is increased or reduced for the change in the socially available labour time.

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