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OPTIMAL CONTROL OF THE LABOUR-MANAGED FIRM

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1. INTRODUCTION

Ever since the seminal work of Ward (1958) the theory of the income-maximizing labour-managed firm has been the subject of intense investigation, and has been developed in a number of directions. In the very short-run context of a fixed membership with variable hours (Sen, 1966; Ireland, 1981; Miyazaki and Neary, 1983), the efficient allocation of labour is the key issue. In the Marshallian short run, with variable membership and fixed capital, the theoretical existence of perverse supply responses has been the key discovery, made by Ward in 1958. In the long run, with variable labour and capital, the supply response becomes indeterminate, but nevertheless the supply function of a labour-managed firm is less elastic than that of a twin profit-maximizing capitalist enterprise (Vanek, 1970; Estrin, 1982). A full general equilibrium analysis of a labour-managed economy with free entry and exit has been presented by Drèze (1985), which indicates the long-run equivalence of the resource allocation in such an economy to that in the benchmark case of competitive capitalism.

However, all these models are set in the essentially limited framework of the comparative statics methodology. Even where the issue of investment is concerned (see Uvalić, 1986 for a review and assessment) the debate has been conducted in terms of comparative statics, and has counterposed the 'underinvestment effect' of Vanek (1977), to the 'horizon effect' of Furubotn (1971) and Pejovich (1973) as two separate analytical approaches, typified by Stephen (1984) as the 'Cornell' and 'Texas' schools of thought. It turns out that this is an essentially false dichotomy induced by reliance on the methodology of comparative statics in a context which has the essential characteristics of a problem in economic dynamics. This issue has been taken up in a series of recent articles by Horvat (1986a, 1986b, 1986c) in which he emphasises the role of time dynamics in economic analysis, and suggests that the neglect of this dimension in the study of the labour-managed firm

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invalidates many of the conclusions which have been established in models of the 'Illyrian' type. In Horvat (1986c) he observes that, at the time of writing, no "elegant theoretical solutions" had appeared which were able to provide a satisfactory account of the dynamic programming problem facing a labour-managed firm engaged in planning future growth. One approach to the problem of the explicit incorporation of time into the analysis of the production process, noted in Horvat (1986a), is the use of the method of calculus of variations. The calculus of variations is, however, only a special case of the more general method of optimal control theory (Seierstad and Sydsaeter, 1987), and in this paper we build on earlier work (Bartlett, 1987), to develop an 'elegant' theoretical approach to the dynamics of the labour-managed firm based on the optimal control approach.

The approach confirms many of the results of the comparative statics method as regards the short-run impact effect, and the long-run asymptotic effect of a price change. However, the more interesting features of the adjustment process which occur along trajectories between equilibria (where most firms are most of the time), are invisible to the static analysis. By investigating the optimal trajectories of employment and capital accumulation this paper provides a link between short-run and long-run analyses which can be used to integrate divergent strands of the existing knowledge about labour-managed firms. For example, it shows how the work of the 'Texas' and 'Cornell' shools can be integrated in a single uniform approach, and provides some useful pointers for applied research on the dynamics of investment and employment growth in the labour-managed firm.

In addition, the analysis helps to clarify the concept of an 'aspiration wage' proposed by Horvat (1976), and highlights the importance of the size of the initial capital stock and the average length of workers' tenure with (or quit rate from) the firm in determining the long-run viability of the labour-managed enterprise.

2 .THE OPTIMAL CONTROL MODEL

In this section we briefly set out the main features of the solution to the optimal control model for a self-financed labour-managed firm. Self-financing is of practical importance for both Western European co-operatives and for Yugoslav self-managed firms, although the latter are more hedged around with regulatory state interventions which would probably best be modelled by the inclusion of some extra constraints on the model's control and state variables. However, as Horvat (1986c) emphasises, the ratio of own to total assets in Yugoslav enterprises is as high as two thirds and, moreover, the provision of external finance is often tied to a contribution of internal finance by the worker managers. Some of the implications of introducing external finance into the model have been discussed in Bartlett (1987), but in this paper we concentrate on the case of pure self-financing, as it provides the neatest way to model the link between the generation of current surplus and the future growth of income.

The problem facing the labour-managed firm in an intertemporal setting is assumed to be the maximization of the integral of expected, disposable income per head of the average worker over time, suitably discounted[1]. The firm produces output Q(t) by means of labour, L(t), and capital, K(t), where we are careful to meet Horvat's requirement that all the relevant variables should be seen to be indexed by time, t. We allow for the existence of increasing returns to scale by assuming an 'S' — shaped production technology, with initial increasing returns, but eventually decreasing returns. We shall see later that it is the region of increasing returns which locates an equilibrium for the labour-managed firm and so Horvat's worry that an unrealistic assumption of decreasing returns drives the results of theory (1986b) need not be of great concern here. We further specify the production technology by the conditions:[2]

$$Q_{K(t)}, Q_{L(t)}, Q_{KL(t)} = Q_{LK(t)} > 0; \; Q_{KK(t)}, Q_{LL(t)} < 0$$

Output is sold on a competitive market at a price p(t), which we allow to be parametrically constant, so that $p(t) = p$. Total income per worker is then given by $y(t) = p \cdot Q(t)/L(t)$, and disposable income per worker is $w(t) = (1 - s(t)) \cdot y(t)$, where s is the rate of savings out of revenue. The idea of expected income requires a little more attention. Clearly it will vary with the worker's expected tenure with the firm. If a worker expects to remain with the firm for fifty years, then expected income will be greater than for a worker who expects to leave the firm next week. One way of coping with this problem would be to impose a finite time horizon, T, on the firm's planning problem equal to the expected duration of tenure of the average worker, as in the work of Furubotn and Pejovich. However, this entails some analytical inconvenience as well as being inherently implausible. The firm's planners know very well that the actual date of the planning horizon will be continually shifting through time, even with a fixed average expectation of tenure, and the firm will (hopefully) continue to exist after date T actually arrives. Therefore an alternative approach is adopted here. We allow the firm to plan over an infinite horizon, but take account of the 'horizon effect' associated with finite expectation of tenure, by incorporating directly into the discount factor a variable measuring the instantaneous probability that the average worker will quit the firm. This probability, 'π', is actually a parameter of the hazard function for the duration of tenure (Kiefer, 1988). The discount factor, 'r', can then be represented as

$$r = \pi + i$$

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[1] The accords with Horvat's stated preference (1986c, p. 413) that: "I find the Wardian objective function — maximization of worker income — useful and reasonable".

[2] Clearly in the present context, the 'marginal product' of a factor is well defined with respect to time. Changes in factor inputs take place over time, and have well defined effects on output flows over time. Technical progress could be incorporated through shift factors A(t), as in $Q(t) = A(t) \cdot F(K(t), L(t))$ etc, but for ease of exposition we set $dA/dt = 0$.

where i is the market rate of interest, or the opportunity cost of internally invested funds.

In this model there are two control variables, s(t) and L(t); K(t) the capital stock is the state variable which evolves through time according to the dynamic relation:

$$\dot{K} = s(t) \cdot y(t) \cdot L(t) - \delta \cdot K(t) \tag{1}$$

where δ is the constant rate of depreciation of the capital stock. It is assumed that labour is costlessly variable. Horvat objects that in the short run labour is fixed, and that the labour-managed firm would not reduce its labour force. Later on we will see that a 'no-firing rule' makes some intuitive sense in smoothing out adjustments between optimal trajectories following a parameter change. However, expansion should hold no difficulty of adaptation, especially where there is available surplus labour, and in a long-run model, the assumption of a fixed labour force is in any case not particularly helpful.[3]

The problem thus becomes:

$$\max_{L(t),\, s(t)} V(t) = \int_{t=0}^{t=\infty} w(t) \cdot e^{-rt} \cdot dt$$

subject to: dynamic relation (1) and $K(0) = K_0$, $0 < s(t) < 1$; $L(t) > 0$.

The solution to the problem now requires maximization of the current value Hamiltonian, H, where:

$$H = (1 - s(t)) \cdot y(t) + m(t)\,[s(t) \cdot y(t) \cdot L(t) - \delta \cdot K(t)] \tag{2}$$

with m(t) being the current value adjoint variable.

Setting $H_s = H_L = 0$ for an interior solution gives:

$$s: \quad -y(t) + m(t) \cdot y(t) \cdot L(t) = 0 \tag{3}$$

$$L: \quad y_L(t) \cdot L(t) + s(t) \cdot y(t) = 0 \tag{4}$$

Now (3) and (4) combine to give:

$$p \cdot Q_L(K(t), L^*(t)) = (1 - s^*(t)) \cdot y(K(t), L^*(t)) = w^*(t) \tag{5}$$

Thus along the optimal path, disposable income is set equal to the marginal product of labour (indexed by time).

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[3] In any case, costly labour force adjustment is a secondary issue. Some implications for dynamics have already been provided by Feichtinger (1984) in a model with fixed capital.

This relation gives a formal justification of Horvat's claim (1986b, p 26) that even if profits are positive "that does not imply that in a worker managed [firm] profit will be entirely distributed in wages. Behaviour is determined by the distributed — not total — income per worker". In addition, it lends force to his notion of an "aspiration wage" (Horvat, 1986a). Once the optimal controls s*(t) and L*(t) are determined, the aspiration wage is given by $w^*(t) = (1 - s^*(t)) \cdot y(K(t), L^*(t))$. This is set equal to the instantaneous marginal product of labour, as if to maximize total profits, given the 'aspiration wage', for all t. In other words the aspiration wage set, w*(t), embodies within it the optimal controls s*(t) and L*(t), and can be viewed as a summary measure of those controls for accounting purposes.

The actual optimal path of the controls is now easily determined. (In the following we drop the time subscript for notational convenience). Pontrygin's Maximum Principle tells us that along an optimal trajectory the adjoint variable evolves according to $\dot{m} = mr - H_K$. In the present context this gives:

$$\dot{m} = -m\,[p \cdot Q_K - (r + \delta)]$$

but from (3) we can transform this into:

$$\dot{L} = L[p \cdot Q_K - (r + \delta)] = \varnothing_1(K, L)$$

and from the accumulation relation (1) and from the optimal condition (4) we obtain:

$$\dot{K} = p \cdot Q - L \cdot p \cdot Q_L - \delta \cdot K = \varnothing_2(K, L)$$

We now have a coupled dynamic system in K and L, given by $\varnothing_1$ and $\varnothing_2$. At an equilibrium, $\dot{L} = \dot{K} = 0$, and so:

$$\epsilon(K^*, L^*) = 1 + r \cdot v(K^*, L^*)$$

where ϵ is the sum of the output elasticities of capital and labour inputs, and v is the capital-output ratio.[4] Thus an equilibrium is located in the region of increasing returns to scale. This feature of the equilibrium solution was discovered by Vanek (1977). In contrast to Vanek's treatment, however, the present model has been developed consistently within the methodology of economic dynamics, and whereas Vanek assumed the equilibrium to be globally stable, it is shown below that the equilibrium is a 'saddle point' with interesting dynamic properties. In addition the equilibrium solution incorporates the 'horizon effect' of Furubotn and Pejovich through its treatment of the quit rate which enters directly into the discount factor, r. Equilibrium output approaches the constant returns to scale region with $\epsilon = 1$ the longer the time horizon (since r falls directly with a lower the quit rate

[4] For more details see Bartlett (1987).

associated with a longer time horizon). Since the methodological approach adopted here incorporates the time dimension in a systematic way, we are now in a position to analyse the stability of the system and make rapid progress in pushing forward the analysis of the intertemporal behaviour of the labour-managed firm.

3. INTERTEMPORAL BEHAVIOUR AND MULTIPLE EQUILIBRIA

This system has some interesting dynamics as can be seen from the phase diagram in Fig. 1. The slope of the isoclines, $\varnothing_1 = \varnothing_2 = 0$, is given by:

$$\varnothing_1\text{:}\quad dK/dL\,\big|_{\dot{L}=0} = -Q_{KL}/Q_{KK} > 0$$

$$\varnothing_2\text{:}\quad dK/dL\,\big|_{\dot{K}=0} = L \cdot p \cdot Q_{LL}/(p \cdot Q_K - \delta - L \cdot p \cdot Q_{KL}) >/< 0$$

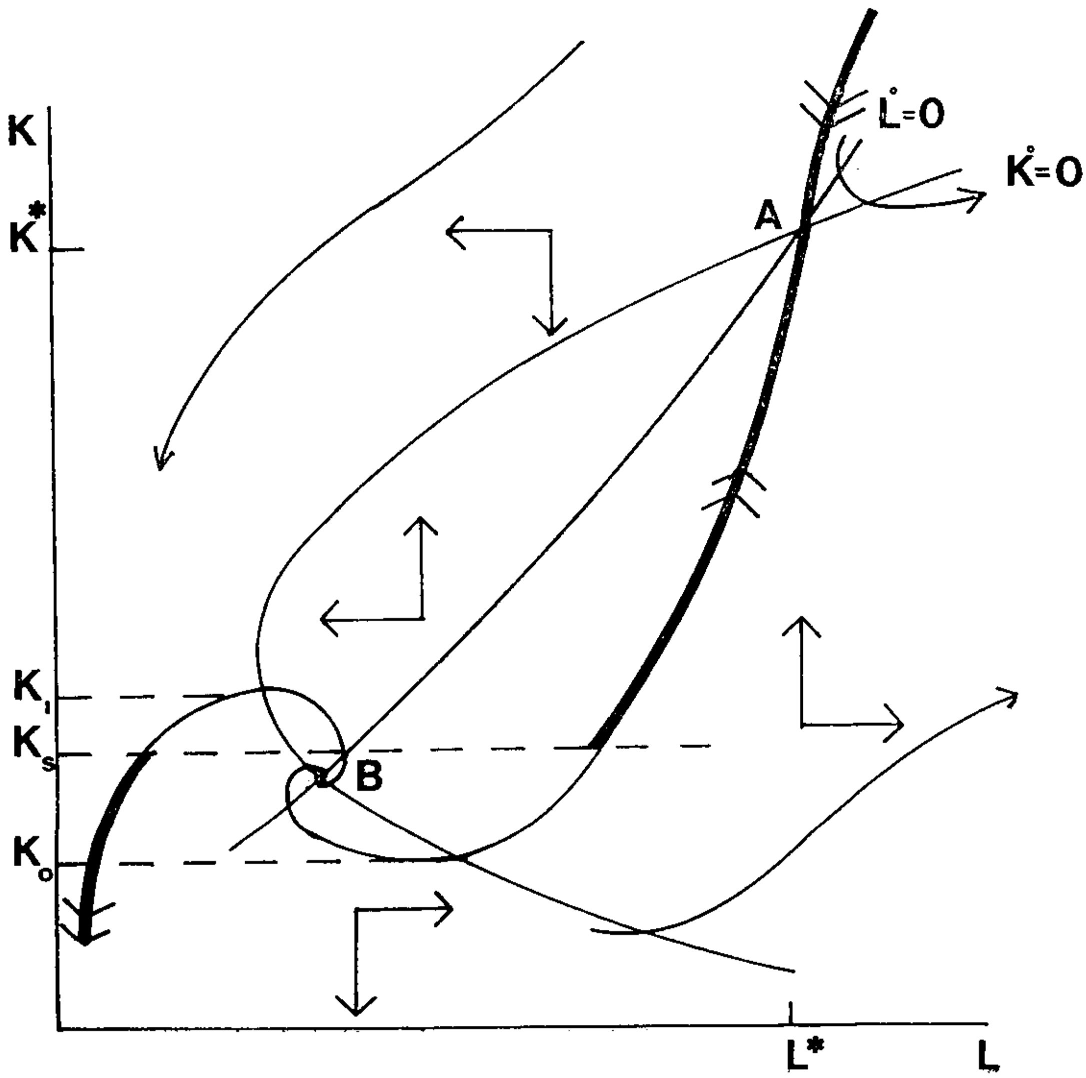

Fig 1

Clearly, $\varnothing_1$ is upward sloping in K—L space. However the slope of $\varnothing_2$ is a little more complicated. This varies with the sign of $(p \cdot Q_K - \delta - L \cdot p \cdot Q_{KL}) = G(K, L)$. At an equilibrium, with $\varnothing_1 = 0$, we know that $(p \cdot Q_K - \delta) = r$. The simplest assumption on the third derivative of the production function is that $Q_{KLL} = Q_{KLK} = 0$. Then the sign of $G(K, L)$ at (K^*, L^*) varies directly with L: at some large L it is positive, at some small L it is negative.

The dynamic motion in the K—L plane is given by the relations:

$$\delta \dot{L}/\delta L = L \cdot p \cdot Q_{KL} > 0$$

$$\delta \dot{K}/\delta K = (p \cdot Q_K - L \cdot p \cdot Q_{KL} - \delta) >/< \text{ as } dK/dL \;\Big|_{\dot{K}=0} </> 0$$

Thus, there are three long-run equilibria, at A, B, C with the equilibria at C being an extinction equilibrium (0, 0). The equilibrium at B is unstable, and that at A is a stable long-run equilibrium. Associated with these equilibria are two trajectories which are candidates for optimality. Which one is optimal depends upon initial conditions. For $K(0) > K_0$, trajectory I, leading to K^*, L^* at C is optimal, while for $K(0) < K_0$, trajectory II, leading to the extinction equilibrium C is optimal. In between these two levels of initial endowment of capital there is a level, Ks, which can be called the 'Skiba point' (Skiba, 1977),[5] which provides a critical cut-off point which separates trajectory I from trajectory II as optimal paths.

The implication is, clearly, that there is a 'critical minimum effort' condition for the viability of the labour-managed firm. If initial capital stock is not sufficiently high, then there will be no incentive for the workforce to sacrifice current income today for the sake of higher future income tomorrow. There will be no incentive to accumulate, and initial capital will be 'eaten up'. On the other hand, given a sufficient initial endowment of capital to ensure viability, the labour-managed firm can be expected to expand through a program of capital accumulation which will involve the continued expansion of employment, up to the position of long-run equilibrium. Even though the existing workers take into account the impact on their incomes of admitting new workers (Madžar, 1986), it turns out that they will be willing to employ them, since the extra capital accumulation, which the extra output they produce allows, means that the incomes of all workers, both old and new, can be raised along the optimal path towards equilibrium.

The existence of the 'critical minimum effort' hurdle on the initial capital structure of the firm which our analysis has uncovered implies some role for industrial policy in relation to the labour-managed firm. Such firms should be subsidised in their initial start-up capital, or a form of 'Investment Guarantee Fund' as recently proposed by the European Parliament for cooperative firms within the EEC,

[5] See also Davidson and Harris (1981).

should be established. The purpose of the Guarantee Fund would be to ensure access to the required amount of start-up capital to enable the firm to achieve viable and self-sustaining growth.

4. COMPARATIVE DYNAMICS

Of particular concern in the traditional literature on the labour-managed firm has been the issue of price responsiveness (for a survey see Bartlett and Uvalić, 1986). The solution of the optimal control problem allows us to examine this issue in an integrated way, comparing short- and long-run responsiveness within the framework of the phase diagram. The impact of a price change on the qualitative behaviour of the system is found by considering the induced shifts of the isocline functions $\varnothing_1 = \varnothing_2 = 0$. First we examine the sign of dK/dp along $\varnothing_1 = 0$, (by the implicit function theorem), holding L constant:

$$dK/dp \mid_{\varnothing_1=0} = -Q_K/p \cdot Q_{KK} > 0$$

consequently, this isocline shifts upwards in the phase plane, following an increase in price.

For the $\varnothing_2 = 0$ isocline, holding K constant, we find that:

$$dL/dp \mid_{\varnothing_2=0} = (Q - L \cdot Q_L) / (L \cdot p \cdot Q_{LL}) < 0$$

consequently this isocline shifts upwards (in the neighbourhood of A), and to the left (globally) for a price increase. The situation is as shown in figure 2. The change in the position of the long-run equilibrium might go in either direction. For a high value of Q_{KL} ($= Q_{LK}$), it will tend to move in the direction illustrated in the diagram. A high value of Q_{KL} implies a large cross-effect of capital accumulation on labour productivity, which may dominate the short-run tendency of Q_L to increase faster than y. In this case both long-run equilibrium capital stock and employment increase. Otherwise, either employment decreases and capital stock increases, or they both decrease. Thus long-run equilibrium employment will rise following a price increase only if the associated capital accumulation raises labour productivity sufficiently to make the employment of more labour attractive.

It is of some interest that the explicit incorporation of time into the dynamic optimisation problem for the labour-managed firm has not actually much affected the results of traditional comparative static analysis, as developed in, for example, Vanek (1970). In fact, what was perhaps most disturbing about that analysis was the short-run response of employment to a price increase. Even when the long-run employment response is positive, there will be a short-run 'jump' in the control variable, L, in a negative direction, as in Fig. 2. (That the new optimal path to A' can cross the old optimal path to A is shown in the Appendix). However, even when the long-run response of employment is negative, as in Fig. 3, it only requires a fairly reasonable

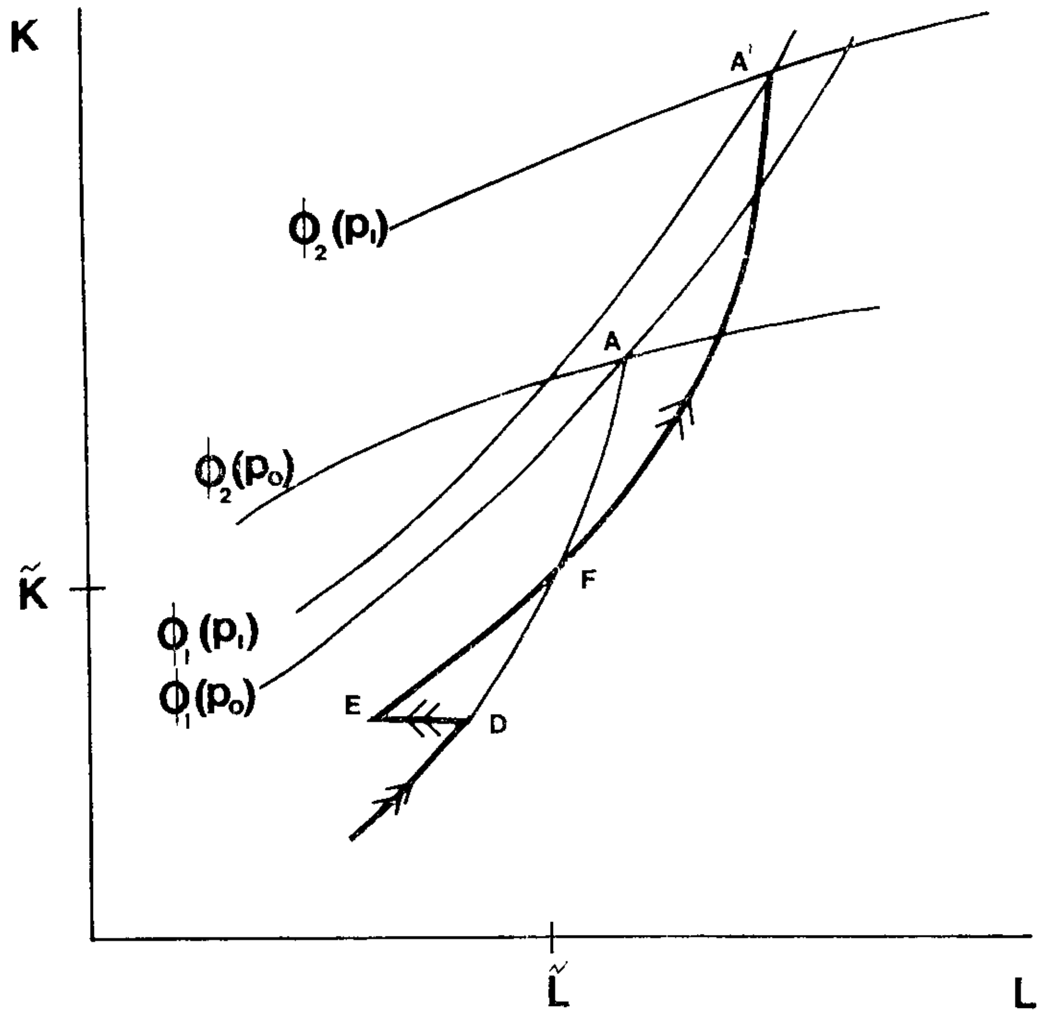

Fig. 2

restriction on the domain of that control variable, for this anomalous behaviour to disappear. Thus we may impose the restriction $L \geq 0$, as suggested by Horvat, (1986c) ("workers do not fire their colleagues"). Then the adjustment to a price rise takes the form of a suspension of new employment engagements until the new optimal trajectory III is attained. Initial adjustment to a price increase involves an accelerated accumulation of capital, until new hiring of labour once again becomes profitable for the existing workers, at which point employment expansion picks up once again and accumulation proceeds at a normal level along the optimal path. Thus the optimal path would follow DE'A', rather than DEA' as a 'naive' comparative dynamics (or statics) approach would suggest.

Further comparative dynamics results, for changes in the discount rate and the depreciation rate can be developed in a similar fashion. An increase in the discount rate, for example, shifts the $\varnothing_1 = 0$ locus to the right whilst leaving $\varnothing_2 = 0$ unaffected, (since $dK/dr \mid_{\varnothing_2 = 0} = 1/p \cdot Q_{KK} < 0$). This implies an unambiguous increase in long-

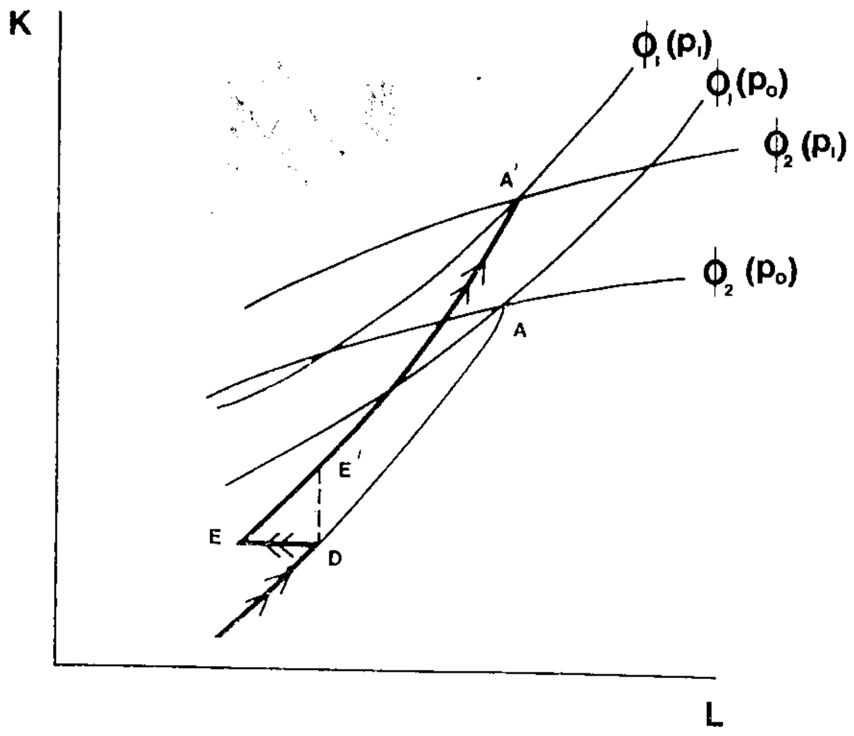

*Fig. 3**

run optimal capital stock and employment.[6] It is significant that this result could come about either through an increase in the average quit probability, perhaps due to a worsening of industrial relations environment within the firm or an improvement in relative outside opportunities, or through an increase in nominal interest rates on outside bank deposits.

[6] This resembles the conventional comparative statics solution in which an increase in fixed costs raises long run employment and output. See also Smith and Ye (1988), who set up a two-state-variable problem using the calculus of variations to analyse the dynamics of a labour-managed firm with costly labour force adjustment. However, partly due no doubt to the intrinsic difficulty of analysing dynamic systems with two state variables, they are unable to solve for the dynamic path of employment; nor can they analyse comparative dynamics. They therefore make little progress over the conventional static formulations of the problem.

* enclosed.

5. CONCLUSION

In this paper we have set out, and solved, an optimal control problem for a self-financed labour-managed firm, and shown how multiple equilibria occur and are related to each other.

The existence of multiple equilibria implies that there is a 'critical minimum effort' condition which determines the viability of the labour-managed firm, such that where the initial endowment of capital is too small in relation to the critical level, then there will be little incentive to further capital accumulation. The firm, in this case, follows a trajectory towards an extinction equilibrium. If initial capital is sufficient, then the optimal trajectory is one along which capital accumulates and the labour force expands. New workers are taken on, even after the full impact of their engagement on income distribution is fully taken into account, since in this case expansion is sufficiently productive to allow a continuous increase in the workforce up to the equilibrium size of the firm.

Appropriate industrial policy to assist the new entrant labour-managed firm achieve viability should therefore be found in the introduction of initial capital subsidies, or in the creation of a 'Loan Guarantee Fund', as recently suggested by the European Parliament, for cooperative enterprises within the EEC.

The concept of an 'aspiration wage' first suggested by Branko Horvat in 1967, has been interpreted in terms of the implied valuations of the control variables for the model, the savings rate and the level of employment, along the optimal trajectory. In this connection, it has been shown how disposable income per head rather than gross income per head is the relevant variable of interest to the enterprise in its decision to hire new labour. The difference between the two provides a surplus for accumulation, with the rate of savings being chosen along with the level of employment so as to maximise the intertemporal functional defined on expected income per head of the average worker, taking into account the average length of tenure with the firm.

Finally, comparative dynamics of a price change were examined, and it was found, in conformity with comparative static results, that the long-run responsiveness of employment to a price change (at the long-run equilibrium) was indeterminate, but that it was more likely to be positive where there were powerful cross-effects between accumulation and labour productivity. As in conventional comparative statics results, short-run perversity of employment adjustment to a price rise would always be negative and given by a 'jump' in the control variable. However, even where the long-run response was negative, short-run perversity could easily be expunged from the model through the imposition of a reasonable no-firing constraint, which led to an adjustment between optimal trajectories in which hiring was suspended for an interval during which accelerated accumulation located the firm on a new, higher, optimal path towards long-run equilibrium size.

APPENDIX

We show that the 'new' optimal path can cross the 'old' optimal path following a price increase from p (0) to p (1) with p (1) > p (0) (as for example in Fig. 2). Following a suggestion of Kamien and Schwartz (1981) we inspect the sign of dK/dL along the optimal paths at the crossing point $\tilde{K}, \tilde{L}$ and consider whether the condition that the slope of the new path with p = p (1) should be less than that of the old path with p = p (0) is contradicted by the condition that p (1) > p (0). The slope is given by the ratio of $\varnothing_2$ to $\varnothing_1$:

$$dK/dL = [p(Q - L \cdot Q_L) - \delta \cdot K] / [L(p \cdot Q_K - (r + \delta))]$$

At $\tilde{K}, \tilde{L}$, we require $dK/dL\,|_{p(0)} > dK/dL\,|_{p(1)}$

$$\text{or:} \quad \frac{p(0) \cdot (Q - L \cdot Q_L) - \delta \cdot K}{p(0) \cdot Q_K - (r + \delta)} > \frac{p(1) \cdot (Q - L \cdot Q_L) - \delta \cdot K}{p(1) \cdot Q_K - (r + \delta)}$$

which reduces to: $(Q - L \cdot Q_L) \cdot (r + \delta) / (Q_K \cdot \delta \cdot K) > 0$

and the condition is satisfied so long as $(Q - L \cdot Q_L) > 0$.

Since equation (5) holds along an optimal path, this inequality holds, and the condition is satisfied. (Since, from (5), Q_L $(1 - s) \cdot Q/L$). Therefore the paths can cross as proposed.

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