EXISTENCE AND STABILITY PROBLEMS OF ECONOMIES OF LABOUR MANAGED FIRMS AND THEIR RELATIONSHIP TO THOSE OF ECONOMIES WITH STRONG UNIONS

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The general equilibrium theory of labour managed economics has recently developed in two directions both of which are related more closely to work done by Vanek (1972) (in their emphasis on the role factor mobility) than to the earlier models of Ward (1958) and Domar (1963). First, Dreze (1973), (1974) has analysed standard general equilibrium models involving no uncertainty, no restrictions on labour mobility and competitively determined prices for fixed factors of production. Defining equilibria as states in which all markets clear and workers have no incentive to move from one co-operative to any other, he shows that the set of equilibria for a labour-managed economy is the same as that of a competitive private ownership twin which is characterized by identical tastes, endowments and production possibilities. A second more detailed approach has been developed by Ichiishi (1977) and Greenberg (1977) among others. They study labour managed economies as species of coalition structure economies in which coalitions of workers form together into firms whose technology is described by defining a production possibility set for every such coalition. Again these studies find that, under fairly general assumptions about technology (i.e., superadditivity of the coalition production sets), possible labour allocations within coalitions and the possibility of dividing amid individuals's labour between several coalitions, with the set of competitive equilibrium allobour management coincides with the set of competitive equilibrium allocations of a private ownership economy with the same tastes, technology and factor endowments. Taken as a whole this work tends to reinforce Vanek's conclusion that, in contrast to Ward's findings, labour managed economies do not have general equilibrium properties that make them substantially inferior to private ownership structures.

Such a conclusion may, however, be prematurely sanguine since both the Dreze and the coalition structure models depend on assumptions which seriously limit their applicability. The most obvious shortcoming of the Dreze model is the certainty assumption, since its implications go far beyond the exclusion from consideration of problems rela-

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ted to risk aversion and risk sharing. For example, in a world of certainty no interference with processes of capital formation arises because workers will leave co-operatives either through quits or retirement without being compenstated for the residual value of their ownership share. Wages can be adjusted in anticipation of such departures so that workers are fully compenstated for any investment undertaken during their working careers. A second problem with the Dreze model is the assumption that the rental payments imposed on co-ops are determined by a process comparable to competitive bidding. In practice, these rents are set by historical accident, tradition and government fiat. Thus, it would be useful to know whether equilibrium market prices exist for any arbitrarily chosen levels of rents. Finally, free labour mobility need not be guaranteed in practice and Nash equilibria may exist for co-operative economies which do not exist in their private ownership twins.[1])

The coalition structure models suffer from the same static structure as the Dreze certainty model and also from the unrealistic extensive possibilities that they assume concerning inter-individual bargaining. Furthermore, and perhaps more seriously, the coalition models operate with a definition of equilibrium similar to that of the Core in game theory. As such an equilibrium allocation places no constraints on the extent of side-payments among members of a coalition and, thus, no constraints on the relative distribution of income within a co-operative. Since matters of equality are critical in a labour-managed economy and constraints on relative wages are invariably present, some equilibria of the coalition models which do not satisfy plausible relative wage constraints, may not be reasonable possibilities.[2]) This might make the set of distributively feasible equilibria far different from those of a private ownership twin.

For all these reasons, it is worthwhile returning to the Ward-type models to examine their general equilibrium properties. In addition, the Ward models describe, as will be shown below, significant situations in which workers exercise indirect control of firms through union negotiations. And, since these situations are not unrealistic, the general equilibrium properties of the Ward-type models are of quite separate interest for what they say about the operation of highly-unionized private ownership economies. This paper is devoted, therefore, to analyzing general equilibria for economies consisting of firms, like those described by Ward, which maximize net earnings per unit of labour.

The paper consists of four sections. The first of these describes precisely the analogy between unionized firms and co-operatives of the Ward type. The second addresses the problem of the existence of equilibrium when firms maximize earnings per worker and, since equilibria do not in general exist for any arbitrary set of rents, it investigates cir-

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[1]) Limit theorems comparable to those derived in market game settings which imply that, as the number of traders becomes large, the set of Nash equilibria converges to the set of competive equilibrium have yet to be proved for labour managed economies, (see for example Shapley and Shubik (197).
[2]) It is, however, likely that allocations corresponding to competitive private ownership equilibria at least satisfy equal pay for equal worker constraints.

cumstances under which equilibria are likely to exist. The third part of the paper deals with the problems of guaranteeing stable price adjustments in non-labour markets and finds that, in contrast to the non-existence problem, a Slutsky-like equation for co-operative firms defines simple simultaneous price-and-rent adjustment which reproduce the stability properties of a labour-managed economy's private ownership twin. And the final section discusses briefly the differences between indirect (i.e., union) and direct labour management, selected problems which are common to both the wider appropriateness of the Ward model for describing the behaviour of highly unionized private ownership economies.

I. *Strong Unions and Labour Managed Co-operatives*

In modelling the behaviour of worker managed firms Ward assumed that they simply maximized output per worker. As a starting point, therefore, we will begin by describing that kind of behaviour formally.

Since we will later be investigating the behaviour of economies with many »firms« we denote a typical »firm« by an index f. We will assume that a »firm« is characterized completely and uniquely by its production set.

(D1) Y_f will denote the production set of a »firm«.

(D2) $Y_f \in Y_f$ will be an $m + 1$ dimensional vector $y_f = (y_{of}, \ldots, y_{nf})$ denoting the production plan of a firm. Elements of y_f which are positive represent the outputs of the firm and negative elements denote inputs. Y_f is, of course, the set of all technologically feasible y_f.

(D3) For convenience y_f will also be written (y_{of}, y_f) where y_{of}, measured negatively is the labour input of the »firm« and y'_f is the n-vector of non-labour elements of the production plan. For the sake of simplicity, we will be assuming that there is only a single labour input.

(D4) $p = (p_1, \ldots, p_n)$ will be a non-negative vector of non-labour market prices which are determined outside the firm.

(D5) R_f will denote a »rental« payment that must be made by each »firm« regardless of its production decision.
Ignoring labour costs, the profit of a »firm«, after paying rent is;

$$\pi'_f = p \cdot y'_f - R_f.$$

Since y_{of} will in general be negative, the profit per worker is;

$$d_f(y_f) = - \frac{p \cdot y'_f - R_f}{y_{of}}$$

This will be referred to as the »wage« for workers in labour managed co-operatives.

We will now define a labour-managed co-operative in the Ward sence as follows.

(D6) A labour managed co-operative is one which choses $y_f \in y^f$ in order to maximize;

$$d_f = \frac{p \cdot y'_f - R_f}{y_{of}}$$

This definition is limited in several ways. First, it does not explain why a labour- managed firm should attempt to maximize earnings per worker. Second, it embodies no constraint on hiring decision (i.e., y_{of}) such as restrictions on lay-offs or the necessity that d_f be large enough to attract labour from other sectors. And, third, like the Dreze and coalition structure models it is static.

However, the simplifications involved do not, for the most part, alter the implications of the model. Maximizing a function of both wages and other elements of individual satisfaction instead of wages alone would yield a structure that is identical for analytical purposes to that resulting from (D6) at the cost of much added complexity. Maximizing functions of wages and employment are equivalent to maximizing wages subject to appropriate employment constraints. And, such employment constraints may simply be subsumed into the definition of Y_f Only the static nature of the model seriously limits its generality and the effects of this restriction will be discussed at length below.

(D7) $Y^*_f(p,R_f)$ will denote the set of optimal production vectors with prices p and rent R_f for the labour managed co-operative, d^*_f will denote the optimal level of wages and $y^*_f = (y^*_{of}, \ldots, y^*_{nf})$ will denote a member of Y^*_f.

In order to ensure that Y^*_f is non-empty we will assume that;

(A1) Y_f is closed, bounded and convex.

(A2) For every admissable R_f, and every p, there exists $y_f \in y_f$ such that $d_f(y_f) \geq 0$.

(A3) The only y_f of the form $(0, y'_f)$ which are members of Y_f are non-positive (i.e., $y_f \leq 0$). And, there exist no $y_f \in Y_f$ with $y_{of} > 0$.

We will state without proof the following proposition, since the proof itself is simple.

Proposition #1: If (A1) — (A3) are satisfied, then $Y^*_f(p,R_f)$ is non-empty for all $R_f \geq 0$.

The assumptions (A1) to (A3) are admittedly stronger than necessary but even so they do not significantly limit the scope of the model. (A1), except for the boundedness condition, is common to most general equilibrium models and the boundedness assumption is made largely for the sake of simplicity. In most cases, Y_f could be unbounded, but the technology irreversible and, as result as, Y_f could be replaced by a bounded set $\overline{Y}_f$ for which Y^*_f and $\overline{Y}^*_f$ would coincide for any equilibrium levels of p and R_f (see Debreu (1961)).

(A2) merely requires that labour be sufficiently productive and rents sufficiently low so that for some production plan the co-op could pay non-negative wages. If this were not the case, co-operatives would attempt to hire infinitely large amounts of labour to spread net negative rental burdens as thinly as possible. Admittedly, since Y_f is bounded there is an upper limit on the amount of labour a co-op could hire, and, therefore, strictly speaking (A2) is not a necessary condition. However, with an unbounded Y_f the substitution of a bounded $\overline{Y}_f$ described above could not be made as long as $d_f < 0$ were possible. Also, the possibility that co-operative firms as described in (D6) would attempt to hire infinite amounts of the labour is an important problem in assuring the existence of equilibria in economies consisting of such firms.

Assumption (A3) requires that there be not positive net output without some labour input. In reality, it should be satisfied by all existing production sets and to that extent is a weak restriction on Y_f. It is necessary in order to rule out situations in which the maximum level of wages occurs with the smallest positive amount of labour, and, therefore, cases in which co-ops attempt to restrict themselves to an infinitely small amount of labour input.

Having described labour-managed firms, it is now necessary to define, for the purposes of comparison, their private ownership twins.

(D8) A private ownership firm is one which choses $Y_f \in Y_f$ in order to maximize;

$$\pi_f = Y'_f + w \cdot Y_{of}$$

where w is a given price that the firm pays for labour.

In addition to this we will assume that private ownership firms have fixed obligations which they must pay from π_f. These might, for example, be fixed interest payments on a firm's debt or fixed property taxes.

(D9) $C_f \geq 0$ will denote the fixed obligations of a private ownership firm.

A private ownership firm's net earnings are, therefore;

$$E_f = p \cdot y'_f + w \cdot y_{of} - C_f$$

where it should be obvious that maximizing π_f maximizes E_f.

(D10) $Y^*_f(p,w,C_f)$ will denote the set of profit maximizing production plans for a private ownership firm. This could be written $Y^*_f(p,w)$ since y^*_f is independent of C_f. E^*_f will denote the associated maximum value of earnings.

As long as Y_f is compact, $Y^*_f(p,w,C_f)$ is non-empty. Therefore, assumptions (A1) — (A3) are more than sufficient to guarantee that $Y^*_f(p,w,C_f)$ is non-empty.

Next, we will define for convenience the private ownership twin of a labour-managed co-operative.

(D11) Private ownership and labour-managed firms will be referred to as *twins* whenever Y_f is the same for both firms and $C_f = R_f$.

The role of unions in this simple model will be to set wages. A *strong* union will be one which is able to impose any wage it desires on its associated firm and we will assume that it selects the highest possible wage obtainable for its members. In doing so, the union must, however, avoid driving the associated firm into bankruptcy and must always, therefore, choose w so that $E^*_f(w) \geq 0$. Formally;

(D12) A *strong* union is one which is able to determine w and does so in order to maximize w subject to the constraint that $E^*_f(w) \geq 0$. We will denote this maximum wage by w^*, and the corresponding optimal production plan set of the associated firm by $Y^*_f(p,w^*,C_f)$.

The several restrictions imposed in (D12) on union objectives are identical to those imposed on the objectives of workers in the labour-managed co-operative. And, as in that case, the restrictions are made largely for expositional convenience. Employment objectives could easily be embodied in restrictions imposed by the union on Y_f and other factors could be accounted for having unions maximize some function of wages and other variables.

The important part of (D12) is the assumption that a union is strong enough to impose any wage it chooses on its associated firm. This eliminates all the bargaining considerations which constitute a significant part of the literature on labour unions. However, there are several possible justifications for making such an assumption. First, appealing to tradition, much of the union literature is based on the idea that unions do indeed set wages and firms respond by setting employment to maximize profits (see Rees (1971) and more recently Warren-Boulton (1977)). Second, many unions have the power, at least in the short run, to bankrupt their associated firms and cause grave wider disruption in the economies of which they are a part. If this is the truth and the threat of bankruptcy is serious enough, then Nash bargaining equilibrium will indeed correspond to outcomes in which union wage levels are imposed on firms. The practical problem with this argument is that it ignores the power of firm managers to either withdraw their capital or refuse new investment. But, the results of such a process would be significant only in the long run and as such these models will be considered later on in this paper. In static models of the type involved here, ignoring threats to withhold investment is not unreasonable. Third, and lastly, if unions able to impose wage levels represent one extreme in a continuum of possible models, their behaviour provides an outer demarcation line for possible types of behaviour within that continuum.

To guarantee that a strongly unionized firm has an optimal set of outputs which is non-empty and characterized by positive wages, we must make the following assumption which is similar to (A2) for the labour managed co-operative.

(A2') C_f and Y_f are set so that for every p there exist $y_f \in Y_f$ such that $p \cdot y'_f \geq C_f$.

This is essentially our guarantee of the possible non-bankruptcy of firms when they ignore labour costs, and should hold true for most viable firms.

The principle result of this section can now be stated as follows;
Proposition #2: Given (A1), (A2') and (A3), $Y^*_f(p,w^*,C_f)$ for a strongly

unionized private ownership firm will equal $Y^*_f(p,R_f)$ for its labour-managed twin for all p.

Proof:

Let $y^{**}_f = (y^{**}_{of}, x^{**}_f)$ be an optimal production vector for the labour-managed twin.

Let $y^*_f = (y^*_{of}, x^*_f)$ be an optimal production vector for the strongly unionized firm.

First, the payments constraint for the unionized firm must be binding.

If $p \cdot x^*_f + x^* \cdot y^*_{of} > C_f$. Then there exists $w^{**} > w^*$ such that $p \cdot x^*_f + w^{**} \cdot y^*_{of} = C_f$. And, since (y^*_{of},x) is not necessarily optimal when the wage is w^{**}, the optimal E^*_f at w^{**} must not be less than C_f. Therefore, $w^{**} > w^*$ is feasible. And, w^* could not be optimal.

Therefore, $p \cdot x^*_f + w^* \cdot y^*_{of} = C_f$. Since $C_f = R_f$;

$$p \cdot x^*_f + w^* \cdot y^*_{of} = R_f.$$

Let d^*_f be the optimal level of dividends for the labour managed co-op, then $y^{**}_f \in Y^*_f(p,R_f)$ implies;

$$p \cdot x^{**}_f + d^*_f \cdot y^{**}_{of} = R_f = C_f$$

We will now show that $d^*_f = w^*$.

If $d^*_f > w^*$, then, except for the trivial case when $y^{**}_f = 0$,

$$p \cdot x^{**}_f + w^* \cdot y^{**}_{of} > C_f = p \cdot x^*_f + w^* \cdot y^*_{of}.$$

If this were true, y^*_f would not be the profit maximizing production plan for the firm. Therefore, d^*_f may not be greater than w^*. If $w^* > d^*_f$, then of y^*_f would exceed d^*_f and y^{**}_f could not be optimal for the labour-managed firm.

Therefore, $d^*_f = w^*$. Thus,

$$Y^*_f(p,w^*,C_f) = (y_f \quad Y_f \mid p \cdot y'_f + d^*_f \cdot y_{of} = R_f) =$$
$$= Y^*_f(p,R_f) \ [Q.E.D.]$$

Proposition #2 shows that, under quite unrestrictive conditions, firms whose labour forces are strongly unionized will respond to changes in market prices as if they are wage maximizing labour co-operatives. But, a more general result could easily have been proved along the same lines. For a wide range of worker objective functions involving either wages or levels and conditions of employment, indirect control of firms through union negotiations will be exactly comparable to direct labour management as long as unions are strong enough to impose their own terms on private ownership firms.

Indeed, stated in this way proposition #2 may seem almost trivial. However, that is not quite the case. It should be clear that the optimal wage for the labour-managed co-operative will be the highest wage that the union of its private ownership twin can obtain. The substantive con-

tent of Proposition #2 is that when confronted with such a wage, the profit-maximizing firm will make production decisions exactly comparable to those of the labour-managed co-operatve.

The importance of Proposition #2 is first of all that it delineates the areas in which the results of indirect control of firm's through union bargaining may differ from direct labour management. Thus, it indicates the grounds on which one or the other system may be preferable. These areas, which include chiefly the dynamic of the bargaining process, differences in objectives between unions and labour managed co-ops, the nature of investment processes and the mechanisms for risk distribution are all discussed in Section V.

The second important aspect of Proposition #2 results from the wage maximization assumption. If unions do indeed seek to maximize wages subject to the constraint that firms meet their fixed charges (including perhaps an appropriate return on past investment), then the peculiar and troublesome supply responses first described by Ward, will be as typical of highly unionized capitalist economies as they are of Ward's Illyria. nder these circumstances the importance of studying the properties economies of Ward-type firms transcends the limits of the theory of traditionally defined labour-managed firms. Thus, the greater part of the remainder of this paper is devoted to just such a study.

II. EXISTENCE OF EQUILIBRIA

The principal problem with market economies of Ward-type co-ops is not that, in a planning environment, their peculiar supply responses make them unstable but rather that for an arbitrarily chosen set of rents equilibria may well not exist. This occurs because there is a fundamental discontinuity in the way that co-ops, in contrast to conventional private ownership firms, shut down.

Figure 1 illustrates what takes places for a firm which produces a single output using only labour as an input. If the co-op is subject to a rent R and faces a price p for its output, then the wage it pays measured in units of its own output is equal to the slope of a line from a height R/p on the vertical axis to the point on the production possibility curve 0—0' where the firm operates.[3] The optimum scale of operation occurs, therefore, where a ray from R/p is just tangent to 0—0'. As p falls, R/p rises along the vertical axis and the optimum level of output rises while the maximum level of wages falls. If there is a minimum real wage necessary to attract or retain workers in the co-op which is equal to the slope of the line B—B', then, when p falls far enough so that R/p rises above B', the equilibrium level of output returns discontinuously to zero. The discontinuous aggregate excess demand curves which result may well not generate market clearing prices.

This possibility can occur in a general equilibrium context even in very simple models.

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[3] For point A in Fig. 1, the height OA' is the total output and, thus, OA' minus R/p is the output available for distribution among the labour force whose size is OL. Net output per worker, which is the level of wages, is then (OA'-R/p)/OL. But this in turn is just the slope of the line from (R/p) to A.
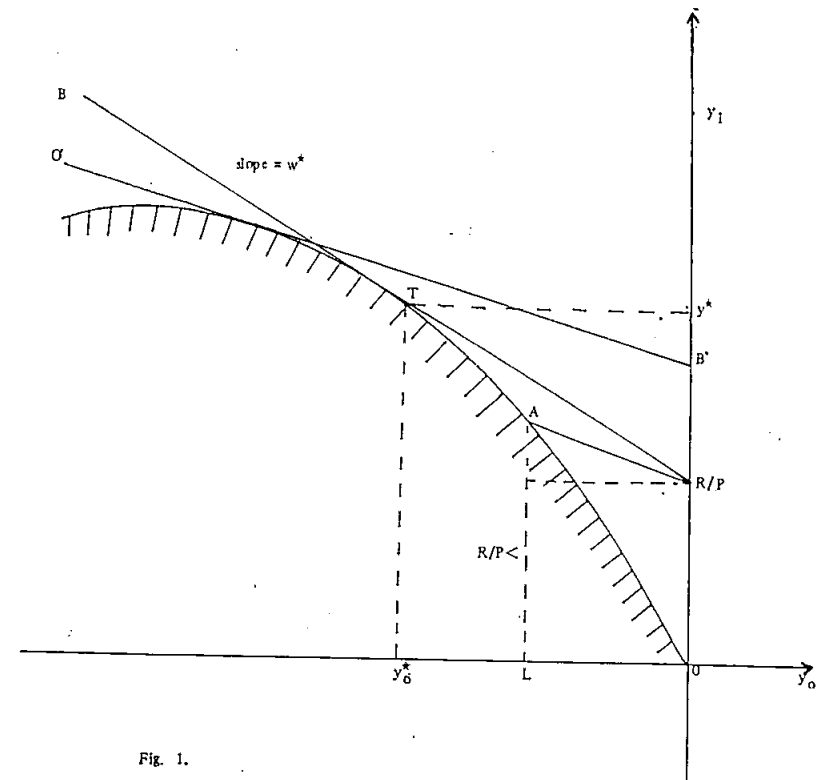
Fig. 1.

EQUILIBRIUM FOR THE LABOR-MANAGED FIRM

Consider an economy with two sectors and only labour as an input.

One sector, which might correspond in reality to subsistence agriculture or to small business, will be assumed to produce its output in individual establishments whose common production function is characterized by constant returns to scale. Denoting its output appropriately, the production relationship for that sector can be written;[4]

$$y_2 = y_{02} \tag{2—1}$$

In using the output of the other sector as numeraire, wages in the individual sector are;

$$w_2 = p_2 \tag{2—2}$$

where p_2 is the relative price of output in the individual sector.

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[4] Remember that inputs are measured negatively.

The second sector of the economy, which might correspond to an industrial sector, will be assumed to consist of a single co-op producing an output which is numeraire. We will assume that its production function takes the form;

$$y_1 = (-y_{01})^b \quad \text{where } 1 > b > 0 \tag{2—3}$$

The rent imposed on the co-op will be assumed to be set at a level R — measured in terms of the numeraire output. Then, the desired output, labour demand and wage of the co-op are respectively;

$$y^*_1 = \frac{R}{1-b} \tag{2—4}$$

$$y^*_{01} = - \left(\frac{R}{1-b}\right)^{1/b} \tag{2—5}$$

and

$$w^*_1 = \frac{y^*_1 - R}{-y^*_{01}} = b \left(\frac{R}{1-b}\right)^{(1-1/b)} \tag{2—6}$$

The total supply of labour will be assumed to be fixed at y_0. Its assignment to industries will depend upon wages. We will assume that;

$$\text{if } w^*_1 \geq w_2 \text{ then } y_{01} = y^*_{01} \tag{2—7}$$

$$\text{and if } w^*_1 < w_2 \text{ then } y_{01} = 0.$$

Labour that is not employed in the co-operative sector will be assumed to find a place in the individual sector.[5] Such a labour assignment scheme is not, of course, strictly comparable to what takes place in a market economy. However, it does capture the spirit of Ward's model and seems to describe fairly accurately what would happen in an economy with co-operative firms in which employment may be especially desirable. If earnings are higher in the co-operative sector then jobs in that sector are rationed to the level of the co-op's demand. If wages are higher in the individual sector, then workers may not be coerced into the co-operative sector.

Given (2—7), the supply function for the co-operative sector as a function of p_2 is;

$$y^*_1 = \left(\frac{R}{1-b}\right) \quad \text{if } w^*_1 \geq p_2$$

$$\tag{2—8}$$

$$y^*_1 = 0 \quad \text{if } p_2 > w^*_1$$

where w^*_1 is defined by (2—6).

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[5] This assumes that R is set so that $Y_0 + Y^*_{01} > 0$.

The supply function of the individual sector is;

$$y^*_2 = y_0 + y^*_{01} \quad \text{if } p_2 \leq w^*_1$$

and   (2—9)

$$y^*_2 = y_2 \quad \text{if } p_2 > w^*_1$$

We will assume that all workers have identical demand functions arising from utility functions of the form;

$$u = y_1^\mu \, y_2^{1-\mu}$$

The resulting aggregate demands for goods one and two as a function of p_2 are;
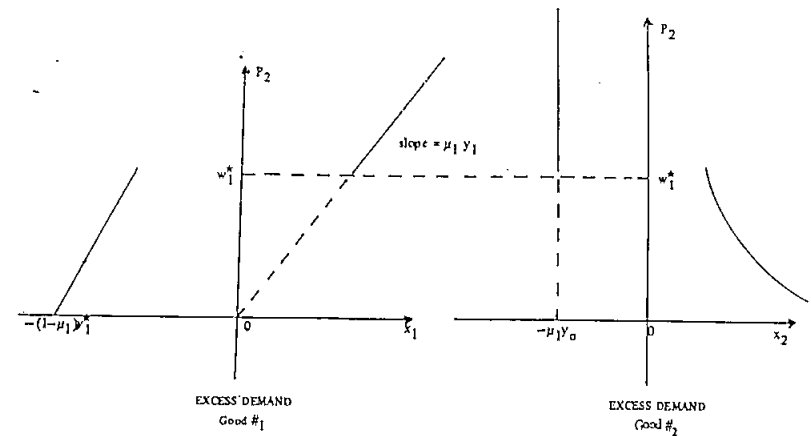
$$d_1 = \begin{cases} \mu_1 y_0 p_2 & \text{if } p_2 > w^*_1 \\ \mu_1 y^*_1 + \mu_1 p_2 (y_0 + y^*_{01}) & \text{if } p_2 \leq w^*_1 \end{cases} \tag{2—10}$$

and

$$d_2 = \begin{cases} (1 - \mu_1) y_0 & \text{if } p_2 > w^*_1 \\ ((1 - \mu_1)/p_2)(y^*_1 + p_2(y_0 + y^*_1)) & \text{if } p_2 \leq w^*_1 \end{cases} \tag{2—11}$$

Combining equations (2—8) and (2—10), the excess demand function for the output of the co-operative sector is;

$$x_1 = \begin{cases} \mu_1 p_2 y_0 & \text{if } p_2 > w^*_1 \\ \mu_1 p_2 (y_0 + y^*_{01}) - (1 - \mu_1) y^*_1 & \text{if } p_2 \leq w^*_1 \end{cases}$$

EXCESS DEMAND   EXCESS DEMAND
Good #1   Good #2

EXCESS DEMANDS WHEN
$\mu_1 w^*_1 (y_0 + y^*_{01}) - (1 - \mu_1) y^*_1 < 0.$

Fig 2.

If $\mu_1 p_2(y_0 + y^*_{01}) - (1 - \mu_1)y^*_1$ is negative, then, as shown in Fg. 2, there is no level of p_2 which will allow markets clear. Thus, in even this simple economy, no equilibrium may exist.

One possible remedy for this difficulty is to adjust rents continuously with prices to eliminate the possibility of discontinous shutdowns and one way of doing so, as shown by Dreze, is to set rents equal to the maximum profits that each individual co-op could achieve in a competitive environment. However, such an approach merely ignores the operational problem of what happens if rents are fixed at levels which are not equal to their appropriate competitive values. A. 1, since using any less precise guidelines for setting rents does not guarantee the existence of equilibrium, non-existence is a potentially serious problem for either strongly unionized or labour-managed economies.

III. STABILITY

In contrast, if for an initial level of rents an equilibrium exists, then adjusting the output of firms by simultaneous price and rent adjustment is no more difficult in a labour-managed economy. Domar (1963) suggested the possibility of using simultaneous price and wage adjustments to control the productive sector. However, he concluded pessimistically that calculating the necessary rent adjustments would be beyond the capacity of most planning organizations. In fact, the necessary adjustments are almost self-evident and emerge immediately from the derivation of a Sluttsky equation analysis of the comparative static responses of a labour-managed co-operative.

To perform that analysis we must begin by assuming that the supply responses of a labour-managed firm are differentiable and, for convenience, that there is a unique optimal output for every level of prices and rents. Also, we will make the model slightly more general than that introduced in Section I above by assuming that there are several types of labour. Formally this can be done by assuming that the production plan of a typical firm is described by;

$$y^f = (y^f_o, x^f) = (y^f_{o1}, \ldots, y^f_{om}, x^f_1, \ldots, x^f_n)$$

where y^f_o is an m-dimensional vector of labour inputs and x^f is an n-dimensional vector of non-labour inputs and outputs (with inputs being measured negatively). Labour inputs may be differentiated by either skill type or the dates at which they are applied.

To compute earnings for each type of labour, we will assume that there is a fixed vector of weights, $\alpha = (\alpha_1, \ldots, \alpha_m)$ which determines the relative amounts of labour contributed by each type worker. The earnings per unit of labour are, therefore;

$$d_f = \frac{p \cdot x^f - R_f}{-\alpha \cdot y^f_o} \tag{3—1}$$

and the payment for a unit of labour the jth type is;

$$w_{jf} = c_j d_f. \tag{3—2}$$

Next, we will write the production constraint in a conventional production function form, namely;

$$H^f(y^f) = 0. \tag{3—3}$$

Labour-managed firms will still be assumed to maximize dividends for all types of labour, which since α is fixed, can be done by;

$$max\ d(y)\ s.\ t.\ H(y) = 0.$$

The first order optimum conditions derived by Domar (1963) among others take the form;

$$\alpha_j(d/p_1) = H_{1j}/H_1$$
$$j = 1, \ldots, m \text{ for labour inputs}$$

and $p_i/p_1 = H_i/H_1$
$i = 1, \ldots, m \text{ for non-labour elements} \tag{3—4}$

where H_{1j} is the partial of H with respect to the jth kind of labour and H_i is the partial of H with respect to the ith element of x.

Equations (3—4) can be simplified by assuming that good one is numeraire (i. e., $p_1 = 1$) and writing H_k/H_1 as F_k. They then take the form;

$$\alpha_j d = F_{1j}$$
$$j = 1, \ldots, m \text{ for labour inputs}$$

$$p_i = F_i \tag{3—4a}$$
$i = 2, \ldots, m \text{ for non-labour elements}$

Differentiating (3—4a) with respect to p and R and substituting from (3—4a) yields the comparative static equations;

$$\begin{bmatrix} M_F \end{bmatrix} \begin{bmatrix} \dfrac{dx}{dy_0} \end{bmatrix} = -\begin{bmatrix} I_{n-1} & 0 \\ \frac{1}{L}A & -\frac{1}{L}\alpha \\ 0 & 0 \end{bmatrix} \begin{bmatrix} dp \\ dR \end{bmatrix} \tag{3—5}$$

where M_F is an $(n + m) \times (n + m)$ matrix of the form;

$$M_F = \begin{bmatrix} F_i & F_{1j} \\ F_{ki} & F_{k1j} \end{bmatrix} \tag{3—6}$$

where F_{ki} is the partial of F_i with respect to the kth element of y, F_{k1j} is the partial of F_{1j} with respect to the kth element of y and k run over

all types of labour and all not labour inputs but the 1st good.

A is an $m \times (n-1)$ matrix of the form;

$$A = \alpha Z' = \begin{bmatrix} \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_m \end{bmatrix} \quad [x_2, \ldots, x_n] \qquad (3\text{—}7)$$

and z is an $(n-1)$ — vector consisting of the last $(n-1)$ elements of $\underline{x}$.

L is the total weighted labour supply used by the co-op, namely;

$$L = -\alpha' y_o = \overset{m}{\underset{1}{\Sigma}} \alpha_j(-y_{oj}) \qquad (3\text{—}8)$$

Finally, $dp = (dp_2, \ldots, dp_n)$ is an $(n-1)$ — vector of changes in the non-numeraire prices. Solving (3—5);

$$\frac{dy}{dp_i} = -M_F^{-1} \begin{bmatrix} I_{n-1} & 0 \\ \frac{1}{L}A & 0 \\ 0 & 0 \end{bmatrix} \{e_i\} \qquad (3\text{—}9)$$

where $dy = \begin{bmatrix} dx \\ dy_0 \end{bmatrix}$ and e_i is an $(n-1)$ — dimensional unit vector

with a one in the $(i-1)^{th}$ — location. Also;

$$\frac{dy}{dR} = -M_F^{-1} \begin{bmatrix} 0 \\ -\frac{1}{L}\alpha \\ 0 \end{bmatrix} \qquad (3\text{—}10)$$

$$\frac{dy}{dp_i} = -M_F^{-1} \begin{bmatrix} 0 \\ \frac{1}{L}Ae_i \\ 0 \end{bmatrix} - M_F^{-1} \begin{bmatrix} 0 \\ e_i \\ 0 \end{bmatrix} \qquad (3\text{—}11)$$

But, $Ae_i = \alpha z' e_i = \alpha \cdot x_i$. Therefore, substituting from (3—10);

$$\frac{dy}{dp_i} = -x_i \frac{dy}{dR} - M_F^{-1} \begin{bmatrix} 0 \\ e_i \\ 0 \end{bmatrix} \qquad (3\text{—}12)$$

To analyse the second term from the righthand side of (3—12), consider the private ownership twin of the labour-managed co-operative and assu-

me that it faces wages $(w_1, \ldots, w_m)$ for the various types of labour. If $w_j = \alpha_j d$, then initial equilibrium for the private ownership firm is identical to that for the co-op. It can then be shown that the effect of a change in the price of the ith good, holding wages and all other prices constant, is;

$$\frac{dy}{dp_i}\Big|_{p0} = -M_F^{-1} \begin{bmatrix} e_i \\ 0 \\ 0 \end{bmatrix} \qquad (3\text{—}13)$$

Therefore, the comparative static response of a labour-managed co-op of the Ward-type can be written in the Slutsky-like form;

$$\frac{dy}{dp_i} = -x_i \frac{dy}{dR} + \frac{dy}{dp_i}\Big|_{p0} \qquad (3\text{—}14)$$

where the first term could be described as a pure rent effect and the second term as a substitution effect.

Equation (3—14) leads immediately to Proposition #3. *Proposition #3:* A price change dp accompanied by a simultaneous rent change;

$$dR = x \cdot dp$$

will induce the same comparative static response form a wage maximizing labour-managed co-operative as the price change alone would induce from a twin private ownership firm.

Proof: Let $\dfrac{dy}{dp} = \left[\dfrac{dy}{dp_2}, \ldots, \dfrac{dy}{dp_n} \right]$ which is an $(n+m) \times (n-1)$ dimensional matrix.

The total change in y is;

$$dy = \left[\frac{dy}{dp} \right] [dp] + \left[\frac{dy}{dR} \right] dR$$

Substituting from (3—14);

$$dy = \left[\frac{dy}{dp} \right]_{p0} [dp] - (x \cdot dp) \left[\frac{dy}{dR} \right] + \left[\frac{dy}{dR} \right] \cdot dR$$

But, $dR = x \cdot dp$. Therefore,

$$\frac{dy}{dp} = \left[\frac{dy}{dp} \right]_{p0}.$$

[Q.E.D.]

Given the straightforward and easily calculated form of the rent adjustments in Proposition #3, it should be clear that, in a planning context, stable adjustments, once an initial equilibrium has been found, are no more difficult to achieve in a labour-managed economy than in a classical private ownership economy. The central problem seems, therefore, to be that of managing the smooth shutdown of firms in order to guarantee that an equilibrium will exist.

IV. DIRECT AND INDIRECT CONTROL — INVESTMENT, RELATIVE WAGES AND RISK SHARING

As noted above, the models developed so far in this paper are basically static in nature. An important remaining question is, therefore, how far the problems cited already exist when reasonable processes of investment and disinvestment are taken into account. Unfortunately, the answer to that question depends critically on the assumptions made about the nature of the investment process. And, since there is no obviously most reasonable process, it is not worthwhile presenting at length explicit models which may have limited applicability. This section will, therefore, be devoted in part to analyzing the common qualitative implications that seem to emerge from such models. In addition, it contains brief discussions of several issues — including relative wage levels for various types of labour and the problem of risk-spreading — which have been cited as possible difficulties in labour-managed economies.

The easiest investment assumption to make is that firms rent perfectly malleable capital at an exogenously given interest rate. Capital inputs under these circumstances are already embodied in the model as elements of y_f. Unfortunately, this creates a problem in the definition of C_f, since realistically the most important element of C_f is contractual interest payments. However, if the production functions of unionized firms embody first increasing and then decreasing returns to scale, it is possible to define C_f as consisting of the overhead expenditures necessary to achieve the increasing returns. In this case, therefore, the introduction of investment does not substantially alter the nature of the model.

In practice, however, investment is long-lived and capital stocks can adjust only slowly. Investment is, therefore, paid for at a moment of installation and produces benefits of the course of an extended subsequent lifetime. Furobotn (1974) and others have claimed that this will lead to chronic underinvestment, since workers, who are not able to sell their shares of a co-op's physical capital stock, will enjoy the benefits investment over only part of its useful life. If this is a problem, however, it is a practical rather than a theoretical one. It can easily be shown that appropriate borrowing and repayment schemes to finance investment can match benefits for workers exactly to the costs that they bear.[*] Then, investment returns to individual workers will be exactly comparable to those which would be earned by investors who sub-

[*] This is but one of the implications of the results from Dreze (1974).

sequently sell the assets in private ownership economies. Indeed, it is possible to specify financing rules for co-ops that will lead to chronic overinvestment (e. g., a simple pathological example is a case in which two dollars are borrowed and distributed to workers for every dollar invested). It may well be that risk considerations and capital market failings would prevent compensatory financing of the sort involved here. But, then the villain is as much the failure of capital structures as it is of the underlying form of labour management.

Moreover, investment problems are likely to be far worse in privately owned firms with strong unions. Unions can only effectively hire new capital by accepting wages which allow the investment. This process has the disadvantage first of all of forcing investors (or at least those with equity interests) to infer future returns from past union behaviour rather than contractual arrangements. Such an arrangement should almost inevitably increase the risk of investment, raise the cost of capital and reduce the level of investment. Second, a bargaining process, in which the reluctance of investors to commit their funds will usually elicit higher rates of return, is one which creates strong possibilities for underinvestment whenever the supply of funds is less than perfectly elastic. Finally, the fact that in reality, because of rigidities in the bargaining process, wage and financing decisions are made independently, means that balancing borrowing and investment policies to match costs and benefits is much more likely to be feasible for labour managed firms than for unionized ones. Thus, the kinds of problems that distrub Furobotn are more likely to be characteristic of highly unionized economies than labour-managed ones.

Risk-spreading is also an area in which there is no inherent weakness and at least one strength in a labour managed structure. If contingent commodity markets exist, then diversifiable risks can in theory be spread as well in a labour-managed economy as in a private ownership one. And, to the extent that fixed costs make bankruptcy risks non-diversifiable, labour-managed firms, in which a far greater proportion of contracted costs are done on a contingency basis (i. e., labour plus some capital costs vis. equity capital costs), are considerably safer than private ownership ones.

However, one special difficulty in labour-managed firms, beyond the problem of non-existence is the theoretical difficulty of setting the dividend weights α for various types of labour. If the wage of the jth type of worker is defined as;

$$w^i_j = \alpha_{ij} d_i \qquad (4—1)$$

Then;

$$\frac{d\alpha_{ij}}{dw^i_j} = \alpha_j d_j \left(\frac{y^i_{oi}}{L_i} \right) \qquad (4—2)$$

where $L = \Sigma \alpha^t_k y^t_{ok} \equiv$ total weighted labour force. Since this is always positive, workers always benefit from reducing the weights assigned to

the labour of workers of other types, and thus, in principle there are no equilibrium levels of relative wages. Dreze (1974) gets around this difficulty by assuming effectively that the existence of alternative "wages" for each type of labour constrain the values that co-ops can assign to α without losing their labour forces. However, if rents are set too low, so that there are excess supplies of labour to the labour-managed sector, these constraints disappear and there are again no equilibrium levels of internal wages.

On balance, however, if the appropriate alternative to labour-management is not idealized private-ownership but is instead strongly or even partially unionized private ownership, then even in traditional Ward—Domar terms labour management may do very well in comparison.

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UPOREDNA ANALIZA PROBLEMA POSTOJANJA I STABILNOSTI RAVNOTEŽE U PRIVREDI U KOJOJ POSTOJE PREDUZEĆA SA RADNIČKIM SAMOUPRAVLJANJEM I PRIVREDI SA JAKIM SINDIKATIMA

Bruce G. GREENWALD

Rezime

Istraživan je problem postojanja ravnoteže u privredama koje čine samoupravna preduzeća, a koje karakteriše odsustvo i tržišta i kvazitržišta neradnih (materijalnih) resursa. Uz to, ispitivani su problemi

stabilnosti pri čemu se za »kooperative« Ward-ovog tipa došlo do jednačine koja podseća na Slutskog i koja, preko »rente«, generiše stabilnu putanju ka ravnoteži. Najzad, u radu je utvrđena analogija između samoupravnih preduzeća i preduzeća sa jakim sindikatima što vodi zaključku da mnoga svojstva samoupravne privrede mogu biti karakteristična i za ekonomije sa jakim sindikatima. U tom kontekstu, ispitivan je još i problem investiranja snošenja rizika u samoupravnim kooperativama.

U prvom odeljku rada »moćni sindikat« definisan je kao onaj koji je u stanju da firmi nametne nadnicu određenog nivoa. Zatim je pokazano da se takva firma, u pogledu varijacija u obimu ponude, ponaša kao Ward-ova kooperativa — pod pretpostavkom da sindikat traži maksimalnu nadnicu, koja firmi dopušta samo da preživi, da ne bankrotira, i pod pretpostavkom da firma maksimizira profit. To implicira zaključak da ekonomije sa jakim sindikatima karakteriše isto patološko ponašanje koje je Ward pripisivao privredama čija preduzeća maksmiziraju plate svojih radnika.

U drugom odeljku je pokazano da, pod prilično ubedljivim pretpostavkama, nema ravnotežnog stanja — ni u »Ward-ovoj privredi«, ni u privredi koju karakterišu snažni sindikati. Uzrok nepostojanja ravnoteže je u izvesnom diskontinuitetu u procesu odumiranja preduzeća. Nasuprot idealnim konkurentskim firmama koje tiho odumiru, smanjujući proizvodnju kad cene padaju, Ward-ova kooperativa ekspandira proizvodnju, u istim uslovima, do maksimuma — koji upravo prethodi propasti. Taj fenomen sličan je po svojoj prirodi (ali je još gori po svom intenzitetu) fenomenu rastućih, a zatim opadajućih prinosa u preduzeću.

U trećem odeljku je pokazano da — nasuprot problemu samog postojanja ravnoteže — problem stabilnosti (značajan jer ponuda kooperativa pada kad cene rastu) može biti savladan, ako izvestan centralni organ kontinuelno prilagođava i nameće određeni iznos »rente« kao obavezu koju kooperative moraju ispuniti. Domar (1963) je mišljenja da privredama, ali zaključuje da bi samo izračunjavanje pojedinih veličina u tom procesu prilagođavanja moglo prevazići mogućnosti mnogih planskih organa. U stvari, izvođenje jednačine tipa Slutskog za Ward-ove kooperative pokazuje da se proces prilagođavanja »renti«, neophodan za postizanje klasične elastičnosta ponude u odnosu na promene cena — $d_p = (d_{p1}, \ldots d_{pn})$ — može predstaviti sa $dR^j = x^j \cdot d_p$, gde je $x^j = (x_1^j \ldots, x_n^j)$ inicijalni vektor proizvodnje za j-tu kooperativu a dR^j je promena rente koja joj se nameće. Proizilazi zaključak da, dok u ekonomijama sa jakim sindikatima, pri čemu nema usmeravanja proizvodnje, može biti ozbiljnih problema sa stabilnošću — u kooperativnoj privredi, sa centralnim planskim organom, ti problemi ne moraju doći do izražaja.

U poslednjem odeljku ispitivan je problem investicija i snošenja rizika u »sindikalizovanoj« i samoupravnoj firmi. Analiza pokazuje da, u principu, direktno samoupravljanje verovatno daje bolje rezultate, odnosno ispoljava veću efikasnost, u tom domenu nego sindikalistički model. Tako se ispostavlja da ako se kao standard za upoređenje ne uz-

me idealizovana kapitalistička kompetetivna firma, nego kapitalistička firma koja je pod snažnim uticajem sindikata — onda je privreda sa Wardovim kooperativama daleko aktraktivnija alternativa, nego što to pokazuje prvobitna tehnička analiza.

WORKER MANAGEMENT AND WORKER-OWNER SHARING IN THE HIERARCHICAL WORK ORGANIZATION*

Masahiko AOKI**

1. INTRODUCTION

The aim of paper is the microscopic analysis of the implications of worker management in the context of a model of a firm facing growth opportunities. In particular, it attempts to do the following:

(1) Provide an explicit treatment of the internal organization of the firm (section 2). In most of the models considered by earlier authors, the aim of the worker-managed firm is identified with the maximization of (life-time) income per worker, treating all the workers associated with the firm as homogeneous.[1] Sometimes the firm is assumed to be interested in its growth, which supposedly reflects the preference of the manager (see Atkinson [2]). But the managerial growth maximization hypothesis seems to me to be made *ad hoc* to explain an apparent tendency of worker-managed firms to grow. By assuming the seniority principle of the internal organization of the firm, the workers' preference for the growth of the firm is deducible from the more fundamental axiom of their life-time income maximization.

(2) Examine the way in which the financial structure of the firm is chosen in conjunction with the growth planning of the firm (section 3). The firm is supposed to have two options to finance capital formation. It can either plough back income or borrow from the outside under specified terms. Atkinson's model [2] allows for the possibility of internal financing, but the fraction of internal financing in the total cost of capital formation is treated as a parameter of the model, not as a variable.

(3) Compare the short-run and long-run market policy of the worker-managed firm with that of the state-owned, decentralized firm (section 4.). The difference between the two types of firms lies in that, for the former, the total income of the firm is disposable at the discre-

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[1] An exception which I have come across is a paper by Meade [4] discussing the behaviour of inegalitarian cooperatives in Vanek[5], ch. 11: their approaches are, however, quite different from that adopted here.