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MULTICRITERIA DECISION MAKING MODEL WITH Z-NUMBERS BASED ON FUCOM AND MABAC MODEL

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Abstract. *In the paper is presented a model for selecting a location for a brigade command post during combat operations. Considering that this is a very complex model, which can be approached from several aspects, this paper is limited only to the criteria related to the construction or arrangement of the command post, respectively, the engineering aspect. The selection process is conducted using hybrid FUCOM – Z-number – MABAC model. The FUCOM method is used to define the weight coefficients of criteria based on which the selection is made. The MABAC method, modified by applying Z-number, is used to rank alternatives. The end results indicate that the application of Z-number in decision making includes broader set of uncertainties than standard fuzzy numbers, which is very important for deciding in combat situations.*

Key words: *FUCOM, MABAC, Z-number, fuzzy number, brigade command post.*

1. Introduction – problem description

The Serbian Army performs various combat and non-combat operations. Through the implementation of these, commanders and leaders (from the highest to the lowest level of command) are often in situations in which they have to make more decisions. Most often the end result is a decision made on the basis of previously acquired, mostly theoretical knowledge and on the experience gained by officers during their military service. One example of such an issue is the selection of a brigade command post in a defense operation. Like others, this problem can be solved on the basis of experience and knowledge, but it is much better when the decision is followed by adequate mathematical decision-making model, used as an aid or tool for decision-makers.

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"A command post presents an area, premises or technical means (ship, tank, conveyor, aircraft) in the area of operation of a unit, in which the command is placed with appropriate means during the preparation and conduct of a combat." (Military Lexicon, 1981). There are numerous factors influencing the selection of a command post. These factors (criteria or conditions) can in principle be divided into two groups: 1) the criteria related to the construction of a command post, respectively, performance of works, and 2) the criteria related to the functionality of a command post. The first set of criteria defines the criteria primarily related to fortifying and partly to masking, while in the second set of criteria would be included the criteria related to successful command during combat operations.

Throughout this paper, the authors focused on the first set of criteria, namely, the development of a model to support decision-making when selecting a brigade command post from the perspective of the ability to perform works, respectively, fortifying and masking. The decision-making support model is based on two methods: 1) the FUCOM method - for defining criteria weights, and 2) the MABAC method, which is fuzzified using standard fuzzy numbers and Z numbers - for ranking alternatives.

The FUCOM method was first presented in 2018 (Pamučar et al., 2018). Due to its simple application and reliable results, this method has quickly begun to be applied in other papers (Prentkovskis et al., 2018; Badi & Abdulshahed, 2019; Puška et al., 2019; Cao et al., 2019; Durmić et al., 2019; Stević et al., 2019; Ibrahimović et al., 2019). The most common application of the FUCOM method is found in the process of defining weight coefficients of criteria.

The MABAC method was firstly described in the paper made by Pamučar and Ćirović (2015). After the first publication, large number of authors applied the method (Božanić et al., 2016a; Peng & Yang, 2016; Božanić et al., 2016b; Chatterjee et al., 2017; Gigović et al., 2017; Majchrzycka & Poniszewska, 2018; Ji et al., 2018; Hondro, 2018, Ibrahimović et al., 2019; Luo & Xing, 2019; Wei et. al, 2019). Very soon after the first appearance, the method was applied in fuzzy environment (Roy et al., 2016; Xue et al., 2016; Yu et al., 2017; Sun et al., 2018; Hu et al., 2019; Božanić et al., 2018, Bobar et al., 2020), neutrosophic environment (Peng & Dai, 2018; Pamučar & Božaić, 2019), as well as with the application of rough numbers (Roy et al., 2017; Sharma et al., 2018).

2. Methods applied in the paper

In the following part of the paper, the description of the methods used in the paper is provided.

2.1. FUCOM Method

Considering that basic version of the FUCOM method is used, which is presented in Pamučar et al. (2018), in the further part of the paper, only the steps of the method are listed. More detailed review with the examples is available at Pamučar et al (2018). The FUCOM method consists of three steps:

Step 1. In the first step, the criteria from the predefined set of the evaluation criteria $C = \{C_1, C_2, ..., C_n\}$ are ranked. The ranking is performed according to the significance of the criteria, i.e. starting from the criterion which is expected to have the highest weight coefficient to the criterion of the least significance.

Step 2. In the second step, a comparison of the ranked criteria is carried out and the *comparative priority* ($\varphi_{k/(k+1)}$, $k = 1, 2, ..., n$, where *k* represents the rank of the criteria) of the evaluation criteria is determined. The comparative priority of the evaluation criteria ($\varphi_{k/(k+1)}$) is an advantage of the criterion of the $C_{j(k)}$ rank compared to the criterion of the $C_{j(k+1)}$ rank.

Step 3. In the third step, the final values of the weight coefficients of the evaluation criteria $(w_1, w_2, ..., w_n)^T$ are calculated. The final values of the weight coefficients should satisfy the two conditions. After the verification of the fulfillment of conditions, the weight coefficients of criteria are defined by using the expression (1):

$$\begin{aligned}
&\min \chi \\
&\text{s.t.} \\
&\left| \frac{w_{j(k)}}{w_{j(k+1)}} - \varphi_{k/(k+1)} \right| \le \chi, \ \forall j \\
&\left| \frac{w_{j(k)}}{w_{j(k+2)}} - \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)} \right| \le \chi, \ \forall j \\
&\sum_{j=1}^{n} w_j = 1, \ \forall j \\
&w_j \ge 0, \ \forall j
\end{aligned} \tag{1}$$

2. 2. Z number - MABAC method

The MABAC method is developed by (Pamučar & Ćirović, 2015). It is developed as the method providing crisp values. In this paper is used fuzzified MABAC method by applying Z-numbers. The fuzzification is performed using triangular fuzzy numbers. A general form of triangular fuzzy number is given in the Figure 1.

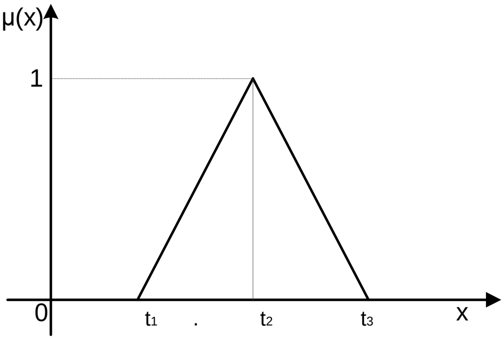


Figure 1. Triangular fuzzy number (Pamučar et al., 2012)

Triangular fuzzy numbers have the form $\tilde{T} = (t_1, t_2, t_3)$ - t_1 - the left distribution of the confidence interval of fuzzy number T, t_2 - fuzzy number membership function has the maximum value - equal to 1, and t_3 - the right distribution of the confidence interval of fuzzy number $\tilde{T}$ (Pamučar et al., 2012).

Z–number presents an extension of classic fuzzy number and provides wider opportunities for considering additional uncertainties following decision making. The concept of Z-number was proposed by Zadeh (2011). In 2012 already Kang et al. (2012a, 2012b) shown in detail the application of Z-numbers in uncertain environment. Later authors consider the application of Z-numbers with different methods of multi-criteria decision making. Sahrom & Dom (2015) present the use of Z-numbers in the hybrid AHP-Z-number-DEA method. Azadeh & Kokabi (2016) use Z-numbers with the DEA method. Azadeh et al. (2013) with the AHP, Yaakob & Gegov (2015) with the TOPSIS method, Aboutorab et al. (2018) with the Best Worst method. Salari et al. (2014) elaborate a novel earned value management model using Z-number.

Z-number represents an ordered pair of fuzzy numbers that appear as Z=($\tilde{A}$, $\tilde{B}$) (Zadeh, 2011). The first component, fuzzy number $\tilde{A}$, represents the fuzzy limit of a particular variable *X*, while the second component fuzzy number $\tilde{B}$ represents, the reliability of the first component ($\tilde{A}$). The appearance of the Z-number with triangular fuzzy numbers is shown in Figure 2 (Zadeh, 2011).

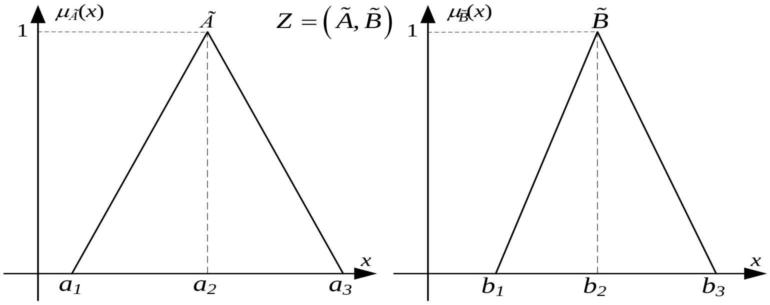


Figure 2. A-Simple Z-number (Kang et al., 2012a)

The general record of triangular Z-numbers can be displayed as

$$\tilde{Z} = \left\{\left(a_1, a_2, a_3; w_{\tilde{A}}\right), (b_1, b_2, b_3; w_{\tilde{B}})\right\} \tag{2}$$

where the values $w_{\tilde{A}}$ i $w_{\tilde{B}}$ represent weight factors of fuzzy numbers $\tilde{A}$ referring to $\tilde{B}$, which for the initial Z-number the majority of authors defines as $w_{\tilde{A}} = w_{\tilde{B}} = 1$, $w_{\tilde{A}}, w_{\tilde{B}} \in [0,1]$ ($w_{\tilde{A}}$ is the height of the generalized fuzzy number and $0 \le w_{\tilde{A}} \le 1$) (Chutia et al., 2013). The transformation of the Z-number into the classical fuzzy number, with the presented evidence, is shown in Kang et al. (2012b). This transformation consists of three steps:

1) Convert the second part ($\tilde{B}$) into a crisp number using the centered method (Kang et al., 2012b):

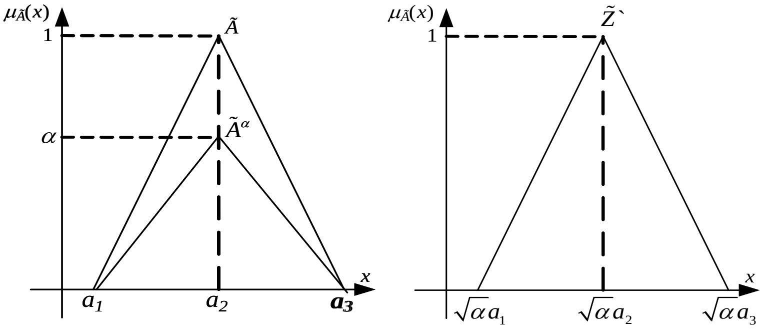
$$\alpha = \frac{a_1 + a_2 + a_3}{3} \tag{3}$$

2) Add the weight of the second part ($\tilde{B}$) to the first part ($\tilde{A}$). The weighted Z-number can be denoted as Kang et al. (2012b)

$$\tilde{Z}^{\alpha} = \left\{\langle x, \mu_{\tilde{A}^{\alpha}}(x)\rangle \middle| \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x)\right\} \tag{4}$$

which can be denoted by the figure 3a. This can be written as (Azadeh et al., 2013):

$$\tilde{Z}^{\alpha} = (a_1, a_2, a_3; \alpha) \tag{5}$$



a) b)

Figure 3. Z-number after multiplying the reliability (a) and the regular fuzzy number transformed from Z-number (b)

3) Convert the weighted Z-number into a regular fuzzy number. The regular fuzzy set can be denoted as Kang et al. (2012b)

$$\tilde{Z}' = \left\{ \langle x, \mu_{\tilde{Z}'}(x) \rangle \middle| \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right) \right\} \tag{6}$$

$$\tilde{Z}' = \sqrt{\alpha} * \tilde{A} = (\sqrt{\alpha} * a_1, \sqrt{\alpha} * a_2, \sqrt{\alpha} * a_3) \tag{7}$$

and it can be present as figure 3b (Kang et al., 2012b).

After describing Z-numbers it is necessary to explain their application in a particular model. These numbers present more comprehensive treatment of uncertainty because when the value of an alternative by a criterion in the form of a standard fuzzy number ($\tilde{A}$) is shown, the degree of certainty of the decision maker or expert ($\tilde{B}$) is also presented. By the above expressions (2-7) is made a transformation of the above fuzzy numbers into a unique fuzzy number. Standard fuzzy MABAC method is further applied. The degree of certainty of the decision makers in the values provided for the evaluation of alternatives by criteria is defined by the expressions presented on the fuzzy linguistic scale, as in the Figure 4.

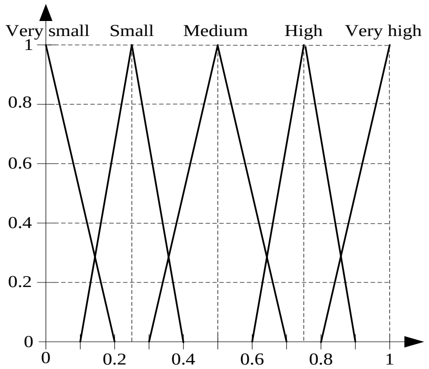


Figure 4. Fuzzy linguistic descriptors for evaluating the degree of conviction of experts (Bobar et al., 2020)

Hybrid model Z number – MABAC is taken from Bobar et al. (2020). The Fuzzy MABAC method consists of 7 steps (Božanić et al., 2018, Bobar et al., 2020):

Step 1. Forming of the initial decision matrix ($\tilde{X}$). Matrix is formed with a grade of alternatives based on criteria $A_i = \left(\tilde{x}_{i1}, \tilde{x}_{i2} ..., \tilde{x}_{in}\right)$, where $\tilde{x}_{ij}$ represents the value of i-th alternative (*i*=1,2,...m), based on j-th criteria (*j*=1,2,...,n)

$$\tilde{X} = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} K_1 & K_2 & \dots & K_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{11} & \tilde{x}_{22} & & \tilde{x}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{x}_{1m} & \tilde{x}_{2m} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{array} \tag{8}$$

Step 2. Converting Z-numbers to regular fuzzy number. This process is performed by applying the expressions (2) to (7). The output provides new initial fuzzy decision-making matrix ($\tilde{P}$)

Step 3. Normalization of new initial decision-making matrix ($\tilde{P}$).

The calculation of the elements of normalized matrix ($\tilde{N}$) depends on the type of criteria. For beneficial criteria this calculation is executed according to the expression:

$$\tilde{t}_{ij} = \frac{x_{ij} - x_i^-}{x_i^+ - x_i^-} \tag{9}$$

For detriment criteria the calculation is executed according to the expression:

$$\tilde{t}_{ij} = \frac{x_{ij} - x_i^+}{x_i^- - x_i^+} \tag{10}$$

Values x_{ij}, x_i^+, x_i^- represent elements of the initial matrix of decision-making ($\tilde{X}$). The values x_i^+, x_i^- are defined as explained bellow

- $x_i^+ = \max(x_{1r}, x_{2r}, ..., x_{mr})$ - represent maximal values of the right distribution of fuzzy numbers of the observed criteria alternatives
- $x_i^- = \min(x_{1l}, x_{2l}, ..., x_{ml})$ - represent minimal values of the left distribution of fuzzy numbers of the observed criteria alternatives.

Consequently, the normalized matrix $(\tilde{N})$ is calculated

$$\tilde{N} = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} K_1 & K_2 & \dots & K_n \end{array} \\ \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \dots & \tilde{t}_{1n} \\ \tilde{t}_{11} & \tilde{t}_{22} & & \tilde{t}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{t}_{1m} & \tilde{t}_{2m} & \dots & \tilde{t}_{mn} \end{bmatrix} \end{array} \tag{11}$$

Step 4. Calculation of the weighted matrix ($\tilde{V}$) elements.

Elements of this matrix are calculated based on the following expression:

$$\tilde{v}_{ij} = w_i \bullet \tilde{t}_{ij} + w_i \tag{12}$$

In the previous expression $\tilde{t}_{ij}$ represents elements of the normalized matrix($\tilde{N}$), whereas w_i presents weight coefficients of the criteria. Weighted matrix ($\tilde{V}$) is visualized in the following way

$$\tilde{V}=\begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \cdots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \cdots & \tilde{v}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \cdots & \tilde{v}_{mn} \end{bmatrix} \tag{13}$$

Step 5. Determination of the approximate border area matrix ($\tilde{G}$). The border approximate area for each criteria is determined based on the expression:

$$\tilde{g}_i=\left(\prod_{j=1}^{m}\tilde{v}_{ij}\right)^{1/m} \tag{14}$$

The matrix of approximate area $(\tilde{G})$ has a format n x 1, where n presents overall sum of criteria number and is represented in the following way

$$\begin{array}{cc} & \begin{array}{cccc} K_1 & K_2 & \ldots & K_n \end{array} \\ \tilde{G}= & \begin{bmatrix} \tilde{g}_1 & \tilde{g}_2 & \ldots & \tilde{g}_n \end{bmatrix} \end{array} \tag{15}$$

Step 6. Calculation of the matrix elements of alternatives distance from the border approximate area ($\tilde{Q}$). The distance of alternatives from the border approximate area ($\tilde{q}_{ij}$) is defined with the expression:

$$\tilde{Q}=\tilde{V}-\tilde{G} \tag{16}$$

Afterwards the matrix is calculated $\tilde{Q}$

$$\tilde{Q}=\begin{bmatrix} \tilde{q}_{11} & \tilde{q}_{12} & \cdots & \tilde{q}_{1n} \\ \tilde{q}_{21} & \tilde{q}_{22} & & \tilde{q}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{q}_{m1} & \tilde{q}_{m2} & \cdots & \tilde{q}_{mn} \end{bmatrix} \tag{17}$$

Step 7. Ranking of alternatives. The value estimation of criteria functions of alternatives is gained from the sum of the distance of alternatives from the border approximate areas ($\tilde{q}_i$). The ultimate values of criteria functions of alternatives are gained from the sum of elements of the matrix $\tilde{Q}$ in rows:

$$\tilde{S}_i=\sum_{j=1}^{n}\tilde{q}_{ij},\ j=1,2,\ldots,n,\ i=1,2,\ldots,m \tag{18}$$

By defuzzification of the values obtained, final rank of the alternatives is obtained. Defuzzification can be performed by applying the expressions (Seiford, 1996; Liou and Wang, 1992):

$$\mathrm{A}=((\mathrm{t}_3-\mathrm{t}_1)+(\mathrm{t}_2-\mathrm{t}_1))/3+\mathrm{t}_1 \tag{19}$$

$$\mathrm{A}=\left[\lambda\mathrm{t}_3+\mathrm{t}_2+(1-\lambda)\mathrm{t}_1\right]/2 \tag{20}$$

3. Description of criteria and calculation of weight coefficients

The selection of a location for a command post is made on the basis of five criteria, obtained by analyzing available literature. Basic criteria for selecting a location of a brigade command post are shown from the most significant (C1) to the least significant (C5), respectively, C1> C2> C3> C4> C5. The criteria on which depends the location of a command post are as follows:

- **C1 - Time required for engineering works**. This criterion implies the total time required for preparatory, main and final works on the engineering arrangement of a command post. (Hristov, 1978). Through this criterion, various elements such as the influence of land to the selection of the type of object to be constructed, geological composition of the soil, *etc*., are indirectly evaluated.
- **C2 - Deposits of building materials**. Various materials are used in the construction of fortification structures, such as: timber, steel and concrete elements and stone. Through this criterion, the existence of material deposits in the vicinity of the area of works, the quantities and types of materials, as well as the possibility of its incorporation into facilities in its existing form or after processing are evaluated.
- **C3 - Masking conditions**. Masking conditions include the possibility of concealing preparations for the execution of works, centralized processing of certain elements (timber, reinforced concrete elements, *etc*.) and direct works on the fortification.
- **C4 - Influence of the enemy**. This criterion implies the ability of the enemy to detect the preparation and execution of works and the possibility of direct action from the ground and from the air. (Šećković, 1972).
- **C5 - Possibilities of use of workshops, technical means and tools**. In the areas of a potential command posts, it is desirable to have the possibility of using local plants (workshops, quarries, sawmills, *etc*.), tools (pickaxes, shovels, crowbars, *etc*.) and technical means suitable for fortification (dozers, loaders, diggers, etc.), in order to economize the forces, resources and time required to perform the works.

The set of criteria from C1 to C5 consists of two subsets:

- the "C +" is a set of criteria of the benefit type, which means that the higher value of criteria is more favorable (the criteria C2, C3 and C5), and
- the"C -" is a set of criteria of the cost type, which means that the lower value of criteria is more favorable (the criteria C1 and C4).

The criterion C1 is presented as numerical, while the other criteria are presented as linguistic.

The weight coefficients of the criteria are obtained using the FUCOM method. Criteria ranks are calculated based on the data on their mutual comparison, as in the Table 1.

Table 1. Importance of criteria

Criteria	C1	C2	C3	C4	C5
Importance ($\varpi_{C_{j(k)}}$)	1	2	3.5	5	6

The values of the calculated weight coefficients are provided in the Table 2.

Table 2. Weight coefficient of criteria

Criteria	C1	C2	C3	C4	C5
w_j	0.465	0.232	0.133	0.093	0.077

4. Model testing

Ten alternatives were defined to test the model. Prior to the process of selecting the best alternative from the set of offered ones, a scale for evaluating linguistic criteria had been defined, as in the Figure 5

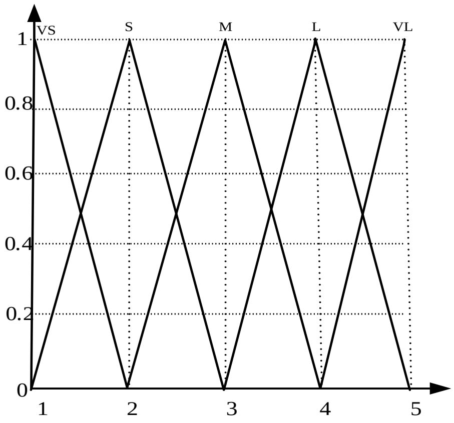


Figure 5. Graphic display of fuzzy linguistic descriptors (Božanić et al., 2016b)

Linguistic criterion can be described with five values: very small (VS), small (S), medium (M), large (L), very large (VL).

The initial decision-making matrix is shown in the Table 3.

Table 3. Initial decision making matrix

Alternative index	C1		C2		C3		C4		C5	
	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$
A_1	(3,4,6)	M	VS	VS	VS	H	L	H	VL	S
A_2	(2,3,4)	VS	S	VH	M	M	VL	VS	M	M
A_3	(4,5,7)	H	L	VS	S	VH	VS	M	L	H
A_4	(3,6,7)	S	M	M	VS	VS	M	S	VL	VS
A_5	(4,8,8)	VH	VL	S	S	H	VL	VH	VS	VH
A_6	(3,5,6)	VS	L	H	VL	M	L	M	S	M
A_7	(4,6,7)	M	VS	S	L	VS	VS	VS	L	S
A_8	(5,8,9)	S	M	H	L	S	VL	H	M	VS
A_9	(6,6,8)	H	S	VH	M	VH	S	S	S	H
A_{10}	(4,6,9)	VH	VL	M	VL	S	M	VH	VS	VH

In the next step, the quantification of linguistic descriptors is performed, as shown in the Table 4.

Table 4. Quantification of linguistic descriptors

Alternative index,	C1		C2			C5	
	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	...	$\tilde{A}$	$\tilde{B}$
A_1	(3,4,6)	(0.8,1,1)	(1,1,2)	(0,0,0.2)	...	(4,5,5)	(0.1,0.25,0.4)
A_2	(2,3,4)	(0,0,0.2)	(1,2,3)	(0.8,1,1)	...	(2,3,4)	(0.3,0.5,0.7)
A_3	(4,5,7)	(0.55,0.75,0.95)	(3,4,5)	(0,0,0.2)	...	(3,4,5)	(0.55,0.75,0.95)
A_4	(3,6,7)	(0.1,0.25,0.4)	(2,3,4)	(0.3,0.5,0.7)	...	(4,5,5)	(0,0,0.2)
A_5	(4,8,8)	(0.8,1,1)	(4,5,5)	(0.1,0.25,0.4)	...	(1,1,2)	(0.8,1,1)
A_6	(3,5,6)	(0,0,0.2)	(3,4,5)	(0.55,0.75,0.95)	...	(1,2,3)	(0.3,0.5,0.7)
A_7	(4,6,7)	(0.55,0.75,0.95)	(1,1,2)	(0.1,0.25,0.4)	...	(3,4,5)	(0.1,0.25,0.4)
A_8	(5,8,9)	(0.1,0.25,0.4)	(2,3,4)	(0.55,0.75,0.95)	...	(2,3,4)	(0,0,0.2)
A_9	(6,6,8)	(0.55,0.75,0.95)	(1,2,3)	(0.8,1,1)	...	(1,2,3)	(0.55,0.75,0.95)
A_{10}	(4,6,9)	(0.8,1,1)	(4,5,5)	(0.3,0.5,0.7)	...	(1,1,2)	(0.8,1,1)

In the second step, Z-numbers were converted to regular fuzzy numbers. After converting into a regular fuzzy number, new initial decision-making matrix was obtained, as in the Table 5.

Table 5. New initial decision-making matrix

Alternative index	C1	C2	...	C5
A_1	(2.12,2.83,4.24)	(0.26.0.26,0.52)	...	(2,2.50,2,50)
A_2	(0.52,0.77,1.03)	(0.97,1.93,2.90)	...	(1.41,2.12,2.83)
A_3	(3.46,4.33,6.06)	(0.77,1.03,1.29)	...	(2.6,3.46,4.33)
A_4	(1.5,3,3.5)	(1.41,2.12,2.83)	...	(1.03,1.29,1.29)
A_5	(3.86,7.73,7.73)	(2,2.5,2.5)	...	(0.97,0.97,1.93)
A_6	(0.77,1.29,1.55)	(2.6,3.46,4.33)	...	(0.71,1.41,2.12)
A_7	(2.83,4.24,4.95)	(0.5,0.5,1)	...	(1.5,2,2.5)
A_8	(2.5,4,4.5)	(1.73,2.6,3.46)	...	(0.52,0.77,1.03)
A_9	(5.2,5.2,6.93)	(0.97,1.93,2.9)	...	(0.87,1.73,2.6)
A_{10}	(3.86,5.8,8.69)	(2.83,3.54,3.54)	...	(0.97,0.97,1.93)

In the third step, the normalization of the new initial decision-making matrix was performed, using the expressions 9 and 10 respectively, as in the Table 6.

Table 6. Normalized initial decision-making matrix

Alternative index	C1	C2	...	C5
A_1	(0.54,0.72,0.8)	(0,0,0.06)	...	(0.39,0.52,0.52)
A_2	(0.94,0.97,1)	(0.17,0.41,0.65)	...	(0.24,0.42,0.61)
A_3	(0.32,0.53,0.64)	(0.13,0.19,0.25)	...	(0.55,0.77,1)
A_4	(0.64,0.7,0.88)	(0.28,0.46,0.63)	...	(0.14,0.2,0.2)
A_5	(0.12,0.12,0.59)	(0.43,0.55,0.55)	...	(0.12,0.12,0.37)
A_6	(0.87,0.91,0.97)	(0.57,0.79,1)	...	(0.05,0.24,0.42)

Alternative index	C1	C2	...	C5
A_7	(0.46,0.54,0.72)	(0.06,0.06,0.18)	...	(0.26,0.39,0.52)
A_8	(0.51,0.57,0.76)	(0.36,0.57,0.79)	...	(0,0.7,0.14)
A_9	(0.22,0.43,0.43)	(0.17,0.41,0.65)	...	(0.09,0.32,0.55)
A_{10}	(0,0.35,0.59)	(0.63,0.8,0.8)	...	(0.12,0.12,0.37)

By applying the expression (12) in the following step is obtained the weighted matrix ($\tilde{V}$), as in the Table 7.

Table 7. Weighted matrix

Alternative index	C1	C2	...	C5
A_1	(0.72,0.8,0.84)	(0.23,0.23,0.25)	...	(0.11,0.12,0.12)
A_2	(0.4,0.92,0.93)	(0.27,0.33,0.38)	...	(0.1,0.22,0.12)
A_3	(0.61,0.71,0.76)	(0.26,0.28,0.29)	...	(0.12,0.14,0.15)
A_4	(0.76,0.79,0.87)	(0.30,0.34,0.38)	...	(0.09,0.09,0.09)
A_5	(0.52,0.52,0.74)	(0.33,0.36,0.36)	...	(0.09,0.09,0.11)
A_6	(0.87,0.89,0.92)	(0.37,0.41,0.46)	...	(0.08,0.1,0.11)
A_7	(0.68,0.72,0.8)	(0.06,0.06,0.18)	...	(0.1,0.11,0.12)
A_8	(0.7,0.73,0.82)	(0.25,0.25,0.27)	...	(0.08,0.08,0.09)
A_9	(0.57,0.66,0.66)	(0.32,0.37,0.41)	...	(0.08,0.1,0.12)
A_{10}	(0.47,0.63,0.74)	(0.27,0.33,0.38)	...	(0.09,0.09,0.11)

In the fifth step is obtained the approximate border area matrix ($\tilde{G}$), by applying the expression (14), as in the Table 8.

Table 8. Approximate border area matrix

Alternative index	C1	C2	...	C5
A_1	(0.67,0.73,0.8)	(0.29,0.32,0.35)	...	(0.09,0.1,0.11)

In the sixth step, using the expression (16), the distance of the alternatives from the border approximate area was obtained, as in the Table 9.

Table 9. Matrix of the distance of alternatives from border approximate areas

Alternative index	C1	C2	...	C5
A_1	(-0.09,0.07,0.07)	(-0.12,-0.09,-0.05)	...	(0,0.02,0.03)
A_2	(0.1,0.19,0.26)	(-0.08,0,0.09)	...	(-0.02,0.01,0.03)
A_3	(-0.19,-0.01,0.1)	(-0.09,-0.05,0)	...	(0.01,0.04,0.06)
A_4	(-0.04,0.06,0.21)	(-0.06,0.01,0.08)	...	(-0.02,-0.01,0)
A_5	(-0.28,-0.21,0.07)	(-0.02,0.04,0.07)	...	(-0.03,-0.01,0.01)
A_6	(0.07,0.16,0.25)	(0.01,0.09,0.17)	...	(-0.03,-0.01,0.02)
A_7	(-0.13,-0.01,0.13)	(-0.11,-0.08,-0.02)	...	(-0.02,0.01,0.03)

Alternative index	C1	C2	...	C5
A_8	(-0.1,0,0.15)	(-0.04,0.04,0.12)	...	(-0.03,-0.02,0)
A_9	(-0.24,-0.06,0)	(-0.08,0,0.09)	...	(-0.03,0,0.03)
A_{10}	(-0.34,-0.1,0.07)	(0.02,0.09,0.13)	...	(-0.03,-0.01,0.01)

The final values of the criteria functions with the rank of alternatives are provided in the Table 10.

Table 10. Ranking of alternatives

Alte-rna-tive index	Z-number MABAC method			fuzzy MABAC method			Classic MABAC method	
	$\tilde{S}_i$	S_i	Rank	$\tilde{S}_i$	S_i	Rank	S_i	Rank
A_1	(-0.28,-0.06,0.14)	-0.07	9	(-0.38,-0.02,0.3)	-0.03	7	0.04	5
A_2	(0.01,0.24,0.44)	0.23	2	(-0.26,0.11,0.45)	0.1	3	0.19	3
A_3	(-0.28,0.01,0.22)	-0.01	5	(-0.28,0.17,0.49)	0.13	2	0.2	1
A_4	(-0.17,0.03,0.27)	0.04	3	(-0.37,-0.01,0.44)	0.02	5	-0.01	7
A_5	(-0.41,-0.24,0.14)	-0.17	10	(-0.4,-0.12,0.36)	-0.05	8	-0.17	9
A_6	(0.07,0.3,0.5)	0.29	1	(-0.23,0.17,0.54)	0.16	1	0.19	2
A_7	(-0.24,-0.06,0.15)	-0.05	8	(-0.33,0,0.38)	0.02	6	0	6
A_8	(-0.22,0,0.26)	0.01	4	(-0.5,-0.13,0.34)	-0.09	9	-0.18	10
A_9	(-0.32,0.01,0.22)	-0.03	6	(-0.5,-0.04,0.24)	-0.1	10	0.04	8
A_{10}	(-0.36,0,0.24)	-0.04	7	(-0.34,0.16,0.47)	0.1	4	0.16	4

In addition to the rank of alternatives obtained by applying Z-number MABAC model, in the Table 10 are also provided the ranks of alternatives obtained by applying classic MABAC method and by applying fuzzy MABAC method (excluding Z-number). Comparative ranking of alternatives provides significant differences in ranking. The alternative A_6, the second-ranked in the application of classic MABAC method, appears as the first-ranked in the rest of the cases. The alternative A_3, which is the first-ranked when applied the MABAC method, is the second-ranked when fuzzy MABAC method is applied, and even the fifth-ranked when Z –number MABAC model is applied. This difference clearly indicates the need to mathematically examine rank correlation. Considering that these are different models, rank differences can be expected, but they should not be significantly different. In this sense, rank correlation control is performed using the Spearman's coefficient

$$S = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2-1)} \tag{21}$$

where is:

- S - the value of the Spearman's coefficient,
- Di - the difference in the rank of the given element in the vector w and the rank of the correspondent element in the reference vector,
- n - number of ranked elements.

Spearman's coefficient takes values from the interval -1,1. When the ranks of the elements completely coincide, the Spearman's coefficient is 1 ("ideal positive correlation"). When the ranks are completely opposite, the Spearman's coefficient is -1 ("ideal negative correlation"), that is, when S = 0 the ranks are unregulated.

The rank correlation of alternatives using Spearman's coefficient is provided in the Table 11.

Table 11. Spearman's coefficient values using different models

	Z-number MABAC method	fuzzy MABAC method	Classic MABAC method
fuzzy Z number MABAC method	1	0.923	0.895
fuzzy MABAC method		1	0.984
Classic MABAC method			1

Table 11 shows that the rank correlation is extremely high, suggesting that new model is performing well, considering two new uncertainties which are not considered by classic MABAC method (uncertainty about the evaluation of alternatives by criteria, as well as the degree of certainty in assigned values of alternatives by criteria).

5. Sensitivity analysis

Logically, the last step in model evaluation is sensitivity analysis. Sensitivity analysis is performed by applying different scenarios changing the weight coefficients of criteria, where different criterion was favored in each scenario. (Pamučar et. al. 2017). The display of weight coefficients according to the scenarios is given in the Table 12.

Table 12. Weight coefficient in different scenario

Criterion	S-0	S-1	S-2	S-3	S-4	S-5
C1	0.465	0.4	0.15	0.15	0.15	0.15
C2	0.232	0.15	0.4	0.15	0.15	0.15
C3	0.133	0.15	0.15	0.4	0.15	0.15
C4	0.093	0.15	0.15	0.15	0.4	0.15
C5	0.077	0.15	0.15	0.15	0.15	0.4

In the Table 13 is provided the rank of alternatives using different scenarios.

Table 13. Ranking of alternatives by applying different scenarios

Alternative index	S-1		S-2		S-3		S-4		S-5	
	S_i	rank	S_i	rank	S_i	rank	S_i	rank	S_i	rank
A_1	-0.05	8	-0.17	10	-0.12	10	-0.13	8	-0.04	7
A_2	0.23	2	0.13	2	0.15	2	0.20	1	0.16	2
A_3	0.06	3	0.03	5	0.09	4	0.16	2	0.19	1
A_4	0.02	4	0.00	6	-0.12	8	0.03	6	-0.05	8
A_5	-0.19	10	-0.09	9	-0.12	9	-0.23	10	-0.14	10
A_6	0.23	1	0.24	1	0.25	1	0.12	4	0.13	3
A_7	0.00	6	-0.07	8	-0.05	7	0.12	5	0.02	5
A_8	-0.04	7	-0.01	7	-0.04	6	-0.14	9	-0.11	9
A_9	0.01	5	0.07	4	0.14	3	0.14	3	0.06	4
A_{10}	-0.06	9	0.09	3	0.04	5	-0.02	7	-0.02	6

Table 13 shows different ranks of alternatives for different scenarios, indicating that the model produced is sensitive to changes in the criteria weights. Regardless of the different ranks, it is noted that the alternatives A_6 and A_2 are at the top in all scenarios, while the alternatives A_1 and A_5 are ranked as the worst in most scenarios. The next step in sensitivity analysis is the application of the Spearman's coefficient, to establish and analyze rank correlations when applying different scenarios, as in the Table 14.

Table 14. Spearman's coefficient values obtained using different sensitivity analysis scenarios

	S-0	S-1	S-2	S-3	S-4	S-5
S-0	1	0.976	0.960	0.954	0.927	0.911
S-1		1	0.945	0.956	0.972	0.960
S-2			1	0.988	0.945	0.943
S-3				1	0.960	0.962
S-4					1	0.990
S-5						1

As observed from the Table 14, the values of the Spearman's coefficient are extremely high and very close to the ideal positive correlation. This indicates a stable and sensitive enough model

6. Conclusions

By the FUCOM – Z-number – MABAC model presented have successfully been evaluated the locations for a command post selection in military combat operations. With a parallel presentation of the application of classic MABAC method and its modifications by the use of fuzzy numbers, respectively, Z-number, it can be noted that the modification of the MABAC method using Z-number provides broader range of possibilities for considering uncertainty. It is difficult to cover large number of uncertainties following combat operations through conventional multi-criteria decision-making methods, which is why it is important to include at least a part of those uncertainties in the decision-making process. Fuzzy MABAC model includes some uncertainties, while Z-number – MABAC model increases the number of uncertainties treated. This indicates that the application of Z-number is extremely useful in the processes in which is not possible to predict all the elements that influence decision making, because, contrary to uncertainty, it can include other factors that are not fully measurable but can influence the final outcome.

The introduction of a model for the selection of a command post location in combat operations significantly advances this process: it helps decision makers understand the factors that influence the selection more comprehensively, and provides less experienced decision makers with the support in decision making based on their predecessor's experience. Considering that there is a number of decisions made during combat operations and followed by a high degree of uncertainty, undoubtedly, the model presented can significantly facilitate making decision on selection of a command post.

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References

Aboutorab, H., Saberi, M., Asadabadi, M.R., Hussain, O. & Chang, E. (2018). ZBWM: The Z-number extension of Best Worst Method and its application for supplier development, Expert Systems With Applications, 107, 115-125.

Azadeh, A. & Kokabi, R. (2016). Z-number DEA: A new possibilistic DEA in the context of Z-numbers. Advanced Engineering Informatics, 30, 604–617.

Azadeh, A., Saberi, M. & Pazhoheshfar, P. (2013). Z-AHP: A Z-number Extension of Fuzzy Analytical Hierarchy Process. 7th IEEE International Conference on Digital Ecosystems and Technologies (DEST), 141-147.

Badi, I. & Abdulshahed, A. (2019). Ranking the Libyan Airlines by using Full Consistency Method (FUCOM) and Analytical Hierarchy Process (AHP). Operational Research in Engineering Sciences: Theory and Applications, 2(1), 1-14.

Bobar, Z., Božanić, D., Djurić, K.A. & Pamučar, D. (2020). Ranking and Assessment of the Efficieny of Social Media using the Fuzzy AHP-Z Number Model – Fuzzy MABAC. Acta Polytechnica Hungarika, 17(3), 43-70.

Božanić, D., Kočić, J. & Tešić, D. (2018). Selecting location for construction of single-span Bailey Bridge by applying fuzzy MABAC method. Procedings of The 2nd International Conference on Management, Engineering and Environment, Obrenovac, Serbia, 407-416.

Božanić, D., Pamučar, D. & Karovic, S. (2016a). Use of the fuzzy AHP - MABAC hybrid model in ranking potential locations for preparing laying-up positions. Military Technical Courier/Vojnotehnički glasnik, 64, 705-729.

Božanić, D., Pamučar, D. & Karović, S. (2016b). Application the MABAC method in support of decision-making on the use of force in a defensive operation. Tehnika, 71(1), 129-137.

Cao, Q., Esangbedo, M.O., Bai, S. & Esangbedo, C.O. (2019). Grey SWARA-FUCOM Weighting Method for Contractor Selection MCDM Problem: A Case Study of Floating Solar Panel Energy System Installation. Energies, 12, 2481.

Chatterjee, P., Mondal, S., Boral, S., Banerjee, A. & Chakraborty, S. (2017) Novel hybrid method for non-traditional machining process selection using factor relationship and Multi-Attributive Border Approximation Method. Facta Universitatis, Series: Mechanical Engineering, 15(3), 439-456.

Chutia, R., Mahanta, S. & Datta, D. (2013). Linear equations of generalized triangular fuzzy numbers. Annals of Fuzzy Mathematics and Informatics, 6(2), 371-376.

Durmić, E., Tomašević, M., Vasiljević, M., Stević, Ž. & Chatterjee, P. (2019). An integrated FUCOM-rough SAW model for sustainable supplier evaluation. Procedings of the Seventh international conference Transport and Logistics, Niš, Serbia, 21-27.

Gigović, LJ., Pamučar, D., Božanić, D. & Ljubojević, S. (2017). Application of the GIS-DANP-MABAC multi-criteria model forselecting the location of wind farms: A case study of Vojvodina, Serbia. Renewable Energy, 103, 501-521.

Hondro, R.K. (2018). MABAC: Pemilihan Penerima Bantuan Rastra Menggunakan Metode Multi- Attributive Border Approximation Area Comparison. Jurnal Mahajana Informasi, 3 (1), 41-52.

Hristov, S. (1978). Organization of engineering work (Only in Serbian: Organizacija inžinjerijskih radova). Belgrade: Military publishing institute/Vojnoizdavački zavod.

Hu, J., Chen, P. & Yang, Y. (2019). An Interval Type-2 Fuzzy Similarity-Based MABAC Approach for Patient-Centered Care. Mathematics, 7, 140.

Ibrahimović, F.I., Kojić, S.Lj., Stević, Ž.R. & Erceg, Ž.J. (2019). Making an investment decision in a transportation company using an integrated FUCOM-MABAC model. Tehnika, 69(4), 577-584.

Ji, P., Zhang, H.-Y. & Wang, J. (2018). Selecting an outsourcing provider based on the combined MABAC–ELECTRE method using single-valued neutrosophic linguistic sets. Computers & Industrial Engineering, 120, 429-441.

Kang, B., Wei, D., Li, Y. & Deng, Y. (2012a). Decision Making Using Z-numbers under Uncertain Environment. Journal of Computational Information Systems, 8, 2807-2814.

Kang, B., Wei, D., Li, Y. & Deng, Y. (2012b). A Method of Converting Z-number to Classical Fuzzy Number. Journal of Information & Computational Science, 9(3), 703-709.

Liou, T.S. & Wang, M.J. (1992). Ranking fuzzy numbers with integral value. Fuzzy Sets and Systems, 50, 247-256.

Luo, S.Z. & Xing, L.N. (2019). A Hybrid Decision Making Framework for Personnel Selection Using BMW, MABAC, PROMETHEE. Interantional Journal, of Fuzzy Systems, 21(8), 2421-2424.

Majchrzycka A. & Poniszewska-Maranda A. (2018) Control Operation Flow for Mobile Access Control with the Use of MABAC Model. In: Kosiuczenko P., Madeyski L. (eds) Towards a Synergistic Combination of Research and Practice in Software Engineering. Studies in Computational Intelligence, vol 733. Springer, Cham.

Military Lexicon (Only in Serbian: Vojni leksikon) (1981). Belgrade: Military publishing institute/Vojnoizdavački zavod.

Pamučar, D. & Božanic, D. (2019). Selection of a location for the development of multimodal logistics center: application of single – valued neutrosophic MABAC model. Operational Research in Engineering Sciences: Theory and Applications, 2(2), 55-71.

Pamučar, D., & Ćirović, G (2015). The selection of transport and handling resources in logistics centres using Multi-Attributive Border Approximation area Comparison (MABAC). Expert systems with applications, 2015, 42(6), 3016-3028

Pamučar, D., Božanić, D. & Ranđelović, A. (2017). Multi-criteria decision making: An example of sensitivity analysis. Serbian Journal of Management, 12(1), 1-27.

Pamučar, D., Đorović, B., Božanić, D. & Ćirović, G. (2012). Modification of the dynamic scale of marks in analytic hierarchy process (ahp) and analytic network approach (anp) through application of fuzzy approach. Scientific Research and Essays, 7(1), 24-37.

Pamučar, D., Stević, Ž. & Sremac, S. (2018). A New Model for Determining Weight Coefficients of Criteria in MCDM Models: Full Consistency Method (FUCOM), Symmetry, 10, 393.

Peng, X. & Dai, J. (2018). Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function. Neural Computing and Applications, 29(10), 939–954.

Peng, X. & Yang, Y. (2016). Pythagorean Fuzzy Choquet Integral Based MABAC Method for Multiple Attribute Group Decision Making. International Journal of Intelligent Systems, 31, 989-1020.

Prentkovskis, O., Erceg, Ž., Stević, Ž., Tanackov, I., Vasiljević, M. & Gavranović, M. (2018). A New Methodology for Improving Service Quality Measurement: Delphi-FUCOM-SERVQUAL Model. Simmetry, 10, 757.

Puška, A., Stojanović, I. & Maksimović, A. (2019). Evaluation of sustainable rural tourism potential in Brcko district of Bosnia and Herzegovina using multi-criteria analysis. Operational Research in Engineering Sciences: Theory and Applications, 2(2), 40-54.

Roy, A., Ranjan, A., Debnath, A. & Kar, S. (2016). An extended Multi Attributive Border Approximation area Comparison using interval type-2 trapezoidal fuzzy numbers. ArXiv ID: 1607.01254.

Roy, J., Chatterjee, K., Bandhopadhyay, A. & Kar, S. (2017). Evaluation and selection of Medical Tourism sostes: A rough Analytic Hierarchy Process based Multi Attributive Border Approximation area Comparison approach. Expert Systems, e12232, https://doi.org/10.1111/exsy.12232.

Sahrom, N.A. & Dom, R.M. (2015). A Z-number extension of the hybrid Analytic Hierarchy Process-Fuzzy Data Envelopment Analysis for risk assessment. Proceedings of the 7th International Conference on Research and Education in Mathematics: Empowering Mathematical Sciences through Research and Education (ICREM7), Kuala Lumpur, Malaysia, 19-24.

Salari, M., Bagherpur, M. & Wang, J. (2014). A novel earned value management model using Z-number. Interational Joural of Applied Decision Sciences, 7(1), 97-118.

Šećković, V. (1972). Engineering reconnaissance (Only in Serbian: Inžinjerijsko izviđanje) Belgrade: Military printing company/Војна штампарија.

Seiford, L.M. (1996). The evolution of the state-of-art (1978-1995). Journal of Productivity Analysis, 7, 99-137.

Sharma, H.K., Roy, J., Kar, S. & Prentkovskis, O. (2018). Multi criteria evaluation framework for prioritizing indian railway stations using modified rough ahp-mabac method. Transport and Telecommunication, 19(2), 113-127.

Stević, Ž., Pamučar, D. & Sremac, S. (2019). An integrated FUCOM-EDAS model for decision making in transportation of dangerous goods. Procedings of the XV International May Conference on Strategic Management, 15(1), Bor, Serbia, 17-25.

Sun, R., Hu, J., Zhou, J. & Chen, X. (2018). A Hesitant Fuzzy Linguistic Projection – Based MABAC Method for Patients' Priorizitation. International Journal of Fuzzy Systems, 20 (7), 2144-2160.

Wei, G., Wei, C., Wu, J. & Wang, H. (2019). Supplier Selektion of Medical Products with a Probabilistik Linguistic MABAC method. Environmental Research and Public Health, 16, 5082.

Xue, Y.X., You, J.X., Lai, X.D. & Liu, H.C. (2016). An interval-valued intuitionistic fuzzy MABAC approach for materila selection with incomplete weight information. Applied Soft Computing, 38, 703-713.

Yaakob, A.M. & Gegov. A. (2015). Fuzzy Rule-Based Approach with Z-Numbers for Selection of Alternatives using TOPSIS. IEEE International Conference on Fuzzy Systems At Istanbul, Turkey, 1-8.

Yu, S-M., Wang, J. & Wang, J-Q. (2017) An Interval Type-2 Fuzzy Likelihood-Based MABAC Approach and Its Aplication in Selecting Hotels on a Tourism Website. International Journal of Fuzzy Systems, 19(1), 47-61.

Zadeh, L.A. (2011). A note on Z-number. Information Sciences, 181, 2923-2932.