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ASSET ALLOCATION WITH MULTI-CRITERIA DECISION MAKING TECHNIQUES

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Abstract: *Determining the weights of assets in a portfolio is one of the fundamental problems of finance. MCDM techniques are employed for asset allocation purposes. However, criterion weights have to be determined before the steps of the techniques are implemented. In this study, the weights of the criteria are determined by four different MCDM weight techniques (CILOS, CRITIC, MEREC, and SECA), and the effect of these weights on the performance of portfolios created with 17 MCDM techniques (ARAS, CoCoSo, CODAS, COPRAS, EDAS, GRA, MABAC, MAIRCA, MARCOS, MOORA, MOOSRA, OCRA, SAW, TODIM, TOPSIS, VIKOR, WASPAS) is examined. 297 criteria (including sectional and cumulative) were calculated using the mean, standard deviation, and correlation, based on historical returns. The returns of the S&P 500 stocks between January 2020 and December 2021 are used as the dataset. Returns for the first 250 trading days are used to determine the weights of the criteria and the stocks in the portfolio. Returns from the following 250 trading days are used for performance evaluation purposes. The experiment was repeated for two more periods. It is found that cumulative criteria have significantly higher weights than sectional criteria. Differences in returns by industry were also examined. The results show that when the MCDM techniques are used to set criterion weights, a higher return is possible.*

Key words: *asset allocation, portfolio optimization, multi-criteria decision making, criteria weights.*

1. Introduction

Portfolio selection and determining the weights of assets in the portfolio problem is one of the most fundamental problems in computational finance (Li & Teo, 2021) and will remain a problem for practitioners, academicians, and society (Alali & Tolga, 2019). The foundations of the problem were proposed by Markowitz (1952) with the classical mean-variance (MV) model. Since the model, which is based on mathematical programming, was introduced to the literature, studies have been carried out to

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develop powerful alternatives to determine the weights of assets in the portfolio to increase the returns or decrease the risks. In stock markets, the prices of assets cannot be predicted with certainty. This uncertainty forms the basis of optimal portfolio creation efforts. If the return on the assets were known with certainty, investors would invest in the alternative that would yield the highest return.

Multi-Criteria Decision Making (MCDM) is a branch of operations research (Sarkar, 2011). MCDM provides a foundation for selecting, sorting, and prioritizing alternatives and helps in the overall assessment (Jahan et al., 2016). A decision matrix where rows indicate the alternatives and columns indicate the criteria is used as an input in a typical MCDM problem. Different alternatives are evaluated against a set of criteria to formulate a comparison of alternatives (Karunathilake et al., 2020). The results can be improved further by assigning weights to different criteria, as the importance can vary extremely from one criterion to another (Karunathilake et al., 2020). MCDM techniques are one of the techniques used in determining the weights of assets in the portfolio. As it is detailed in the literature review section, portfolios have been created with ELECTRE (Vezmelai et al., 2015; Bouri et al., 2002; Xidonas et al., 2010), TODIM (Alali & Tolga, 2019), VIKOR (Fazli & Jafari, 2012; Jerry Ho et al., 2011), COPRAS (Poklepović & Babić, 2014), MAUT (Ehrgott et al., 2004), DEA (Lim et al., 2014), MACBETH (Bana E Costa & Soares, 2004), PROMETHEE (Albadvi et al., 2006; Bouri et al., 2002), TOPSIS (Nguyen & Gordon-Brown, 2012; Unvan, 2019; Kochetov, 2021) and MABAC (Biswas et al., 2019) techniques. Aouni et al. (2018) review the papers that apply multi-criteria decision making (MCDM) tools to portfolio selection. Studies using MCDM in portfolio selection are classified according to their subject, publication years, and journals. The benefits and drawbacks of MCDM techniques are discussed in detail in Kraujaliene (2019) and can be summarized as follows (Alali & Tolga, 2019): (1) It is more appealing to practitioners because no single criterion is considered; (2) it is easier to implement technique calculation steps; and (3) it is relatively easy to observe the change in the system when new criteria are added and the weight set is changed. However, the results of the techniques can be sensitive to the weight set used as input (Kraujaliene, 2019).

When using MCDM techniques, the weights of the criteria should be carefully determined since the weights of the criteria will affect the scores of the alternatives. There are studies in the literature in which criteria weights are determined by techniques such as MACBETH (Bana E Costa & Soares, 2004), BWM (Emamat et al., 2022; Rezaei, 2015), FUCOM (Pamučar et al., 2018), AHP (Saaty, 1977), DEMATEL (Fazli & Jafari, 2012; Jerry Ho et al., 2011). These techniques all need the opinions of an expert to figure out how much weight to give each of the criteria. However, expert assessment may be expensive, biased, take a long time to reach, or access to expert assessment may not be possible. In the literature, methods have also been developed to determine criterion weights objectively such as CILOS (Zavadskas & Podvezko, 2016), CRITIC (Diakoulaki et al., 1995), MEREC (Keshavarz-Ghorabaee et al., 2021) and SECA (Keshavarz-Ghorabaee et al., 2018). These methods do not require expert evaluation and calculate the weights of the criteria by considering the decision matrix. Therefore, in this study the criteria weights are calculated by using the objective weight determination techniques.

In this study, the effect of criterion weights on the portfolio return was researched in detail. The weights of the criteria are determined by four different MCDM weight techniques (namely CILOS, CRITIC, MEREC, and SECA), and the effect of these weights on the performance of portfolios created with 17 MCDM techniques (namely ARAS, CoCoSo, CODAS, COPRAS, EDAS, GRA, MABAC, MAIRCA, MARCOS, MOORA, MOOSRA,

OCRA, SAW, TODIM, TOPSIS, VIKOR, WASPAS) is examined. As many techniques as possible were collected to perform a comprehensive performance comparison. The reason for choosing these 17 techniques is that they have similar computational steps and contain few parameters. Apart from these, there are also different MCDM techniques. For example, the preference function type must be determined for the PROMETHEE (Brans & Vincke, 1985) technique.

Alali and Tolga (2019) is employed TODIM technique with 9 inputs for asset allocation purposes. The inputs include short-, mid- and long-term mean, standard deviation, and correlation of the returns. However, the use of shorter-term variables can increase the performance of the portfolio. The motivation behind this study is to increase the inputs by considering sectional and cumulative inputs which are calculated daily. Also, four objective weight determination techniques are utilized to determine the weights of the criteria. This work will reveal which criteria have higher importance weight in portfolio optimization.

The contribution and originality of this study can be summarized as follow:

- The decision matrix is enriched to include daily return as well as sectional and cumulative data.
- Four weight determination techniques and 17 alternative evaluation techniques are utilized in a single study to compare the performance of the methods.
- The theoretical best return concept is utilized to benchmark the performance of the models.

This study tries to find answers to the following research questions:

RQ 1: Do the weights of the different types of inputs (cumulative or sectional) differ from each other?

RQ 2: Do the weights of the different groups of inputs (mean, standard deviation, or correlation) differ from each other?

RQ 3: Does the returns of the MCDM portfolio greater than or equal to equally weighted portfolio?

RQ 4: Does the returns of the MCDM portfolio greater than or equal to the Mean-Variance portfolio?

RQ 5: Does the returns of the MCDM portfolio greater than or equal to the theoretical best return portfolio?

RQ 6: What are the correlation coefficients of the returns of the portfolios created with MCDM techniques?

RQ 7: Which stocks has the highest average weight?

RQ 8: What is the weight distribution of industries in the best portfolio (or any portfolio)?

This paper is organized as follows: In Section 2, studies with MCDM techniques are summarized. In Section 3, the calculation steps of the four techniques that determine the criterion weights are discussed in detail. In section 4, the data set is introduced, and the analysis results are given. Section 5 is devoted to conclusions.

2. Literature Review

Studies published on asset allocation with MCDM techniques can be summarized in chronological order as follow. Dominiak (1997) applied the BIPOLAR technique to select the most attractive stocks. The proposed system is applied to the stocks on the Warsaw Stock Exchange. There are 39 stocks and 7 criteria in the dataset. Both financial ratios and technical indicators are used as criteria. The ranking and ordering of stocks are reported in the study.

Tamiz et al. (1997) compared regression analysis and goal programming. Stocks in the British FTSE 100 index are used in the analysis. Goal programming is used to select a portfolio based on the decision maker's scenarios and preferences. Interest rates and foreign stock market data are used as inputs in the analysis. The model results are compared with the results of the regression analysis model.

Zopounidis et al. (1998) applied the ADELAIS multi-objective linear programming system to portfolio selection. They applied the proposed system to a dataset from the Athens Stock Exchange for the two years of 1989-1990. There were 52 alternatives and 6 criteria in the dataset.

Hurson and Ricci-Xella (2002) employed ELECTRE-TRI and MINORA techniques to create a portfolio evaluation system. Their sample consists of returns from Paris Stock Exchange firms from 1983 to 1991. There were 28 alternatives and 11 criteria in the dataset. In their study, ELECTRE-TRI is used to create a synthesis criterion for common risk. The MINORA technique is used to rank the alternatives. Artificial criteria weights and portfolio ranks are reported.

Bouri et al. (2002) developed an integrated stock evaluation system consisting of ELECTRE, PROMETHEE, and AHP. They employed their proposed system on a dataset from the Tunisian stock market. There were 37 stocks and 5 criteria in the dataset. Optimal proportions for selected stocks are reported.

Bana et al. (2004) employed MACBETH to construct a portfolio. They determined 7 criteria and 50 stocks. MACBETH weights are normalized by dividing them by their sum. Stock weights are reported, and performance is compared with benchmark models.

Ehrgott et al. (2004) extended the Markowitz mean-variance model by integrating the MCDM method, namely the MAUT technique. They employed Standard & Poor's database of stocks and a simulated dataset.

Albadvi et al. (2007) applied PROMETHEE to select the best stocks for investment. They proposed two stages to select the stocks. In the first stage, the industries are ranked, and in the second stage, the companies in those industries are ranked. There were 13 criteria for determining the ranks of industries and 28 criteria for determining the weights of stocks trading on the Tehran Stock Exchange. With the help of expert opinions, the weights of the criteria are determined.

Unvan (2019) employed the dataset of stocks listed in BIST30 (in Turkey) for June 2018. The dataset includes 30 alternatives (stocks) and 6 variables (criteria). The weights of the criteria are determined by path analysis, and these weights are used in TOPSIS analysis. As a result, the weights of 30 stocks after TOPSIS analysis were reported.

Xidonas et al. (2010) employed the ELECTRE-TRI method to select three stocks for an efficient portfolio. They applied their proposed method to a dataset of 66 stocks operating on the Athens Stock Exchange. There were 6 criteria in the decision matrix. The performance of the selected stocks under different scenarios are reported in the study.

Fazli and Jafari (2012) developed a hybrid MCDM-model to select the best securities for investment in the stock market. They combined VIKOR, DEMATEL, and ANP techniques. They used Iran's stock exchange market data from 2006–to 2010. Their model succeeded in selecting the best 2 out of 50 stocks. Financial ratios are used in the decision matrix. Weights of criteria is discussed.

Nguyen and Gordon-Brown (2012) applied SAW and TOPSIS as portfolio allocation processes. They applied trapezoidal fuzzy numbers and a centroid-based defuzzification process. The dataset used in their model is a previously introduced dataset in the literature that consists of nine stocks between the years 1937 and 1954. They used higher-degree moments as inputs to the decision matrix. They report that their proposed methods outperformed the mean-variance portfolio optimization model.

Poklepović and Babić (2014) employed five MCDM methods: OPRAS, linear assignment, PROMETHEE, SAW, and TOPSIS. The weights of the criteria are determined by AHP. The dataset includes the stocks trading on the Croatian capital market between March 2012 and March 2014. The decision matrix has nineteen stocks (alternatives) and nine indicators (criteria). As a result, the weights of the stocks in two different scenarios are presented in the study.

Lim et al. (2014) used actual financial data from 2001 to 2009 to construct a portfolio. The dataset used in their study involves 490-557 firms listed on the Korean exchange. They employed financial indicators as inputs and outputs of DEA. At the end of the study, it was reported that their proposed portfolio selection method was accomplished to yield higher risk-adjusted returns than the benchmark methods.

Vezmelai et al. (2015) employed ELECTRE-III to construct a portfolio. There were 6 criteria (including ratios calculated from firms' financial statements) and 50 alternatives (stocks trading on the Tehran Stock Exchange). The significance of criteria is interpreted. Selected stocks are reported.

Alali and Tolga (2019) adopted the TODIM technique for portfolio allocation. Alternatives are stocks listed in the S&P 500 index. Researchers calculated short-term, mid-term, and long-term returns, standard deviations, and correlation coefficients as input variables. Return is the benefit criteria, while standard deviations and correlation coefficients are assumed to be cost criteria. Several configurations are applied to fine-tune the parameters of the TODIM technique. As a result, it is reported that TODIM can provide a suitable weighting mechanism for the portfolio optimization process.

Biswas et al. (2019) employed a two-staged filtering process to construct a portfolio. Data belonging to 48 funds between September 2015 to June 2018 is used in the study. In the first stage, the performance of each fund is determined with DEA, and in the second stage, the MABAC technique with entropy weights is applied.

Emamat et al. (2022) employed ELECTRE-TRI, Best-Worst Method, and FlowSort methods to select stocks for a portfolio. There were 8 criteria and 50 alternatives (stocks on the Tehran Stock Exchange) in the decision matrix. Criteria include both returns and ratios obtained from the financial statements of firms. Selected stocks are presented under different scenarios.

This article reviews studies in which MCDM techniques are used for stock selection or asset allocation in a portfolio. The review showed that MCDM techniques were successfully used in portfolio allocation on datasets in stock markets of different countries. Numerous MCDM techniques are available, and new techniques continue to be developed. This study aimed to comprehensively compare the performance of techniques on the same dataset. The weights of criteria were determined with objective techniques, and it was investigated to which types of criteria these techniques assigned higher weights.

3. Methodology

The calculation steps of four MCDM methods that are employed to determine the criteria weights objectively are summarized in this section.

3.1. MCDM Techniques for determining the Criteria Weights

3.1.1. CILOS Method

CILOS (Criterion Impact Loss) method was developed by Zavadskas and Podvezko in 2016. It provides an objective method to determine the criteria weights in multi-criteria decision-making problems.

Suppose that there are m alternatives and n criteria. The decision matrix will be defined with Equation 1 as follow:

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \tag{1}$$

where x_{ij} represents the value of ith alternative ($i \in \{1,2,\dots,m\}$) on jth criterion ($j \in \{1,2,\dots,n\}$).

In this method, the elements in the cost-criteria set are transformed into the benefit ones by using the following Equation 2.

$$x_{ij}^N = \frac{\min_i x_{ij}}{x_{ij}} \tag{2}$$

The new matrix will be $\boldsymbol{X} = \left[x_{ij}^N\right]$. The values x_{ij}^N corresponding to the maximum values of ith criteria are taken from matrix $\boldsymbol{X}$ with k_i rows to form a square matrix $\boldsymbol{A} = \left[a_{ij}\right], a_{ij} = x_i, a_{ij} = x_{k_i j}$ $(i,j = 1,2,\dots,n)$, which implies that the highest values of all criteria will be located in the principal diagonal of the matrix. The ith row of matrix $\boldsymbol{A}$ contains the elements of the row k_i of matrix $\boldsymbol{X}$. The matrix of the relative loss $\boldsymbol{P} = \left[p_{ij}\right]$ is formed by following Equation 3.

$$p_{ij} = \frac{x_j - a_{ij}}{x_j} = \frac{a_{ii} - a_{ij}}{a_{ii}}, \qquad (p_{ii} = 0; i,j = 1,2,\dots,n) \tag{3}$$

The elements p_{ij} in the CILOS matrix $\boldsymbol{P}$ show the relative loss of the jth criterion, if the ith criterion is selected to be the best.

The main diagonal of the $\boldsymbol{P}$ matrix is replaced with the sum of the rows of $\boldsymbol{P}$ matrix to obtain the $\boldsymbol{F}$ matrix (Equation 4).

$$F = \begin{bmatrix} -\sum_{(i=1)}^{n} p_{i1} & p_{12} & \cdots & p_{1n} \\ p_{21} & -\sum_{(i=1)}^{n} p_{i2} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{m1} & p_{m2} & \cdots & -\sum_{(i=1)}^{n} p_{in} \end{bmatrix} \quad (4)$$

The $\boldsymbol{F}$ matrix is used in the following systems of equation:

$$\boldsymbol{Fw} = 0 \quad (5)$$

Since a homogenous system of m equations and m unknowns have an infinite number of solutions, the following line is added to the equation system of equation:

$$w_1 + w_2 + \cdots + w_n = 1 \quad (6)$$

3.1.2. CRITIC Method

CRITIC (Criteria Importance Through Intercriteria Correlation) method was developed by Diakoulaki and colleagues in 1995. Let the decision matrix be as in Equation 1. The matrix will be normalized by using the following Equation 7.

$$x_{ij}^{N} = \begin{cases} \dfrac{x_{ij} - \min\limits_{k} x_{kj}}{\max\limits_{k} x_{kj} - \min\limits_{k} x_{kj}}, & if\ j \in BC \\ \dfrac{\max\limits_{k} x_{kj} - x_{ij}}{\max\limits_{k} x_{kj} - \min\limits_{k} x_{kj}}, & if\ j \in CC \end{cases} \quad (7)$$

where BC and CC represents benefit and cost criteria respectively. Let $V_j = \left[x_{ij}^{N}\right]_{m\times1}$ denotes the vector of jth ($j \in \{1,2,\ldots,m\}$) criterion. Let ρ_{jk} denotes the correlation between jth and kth vectors (j and $k \in \{1,2,\ldots,m\}$). The correlation coefficient is calculated using the following Equation 8:

$$\rho_{jk} = \frac{\sum_{i=1}^{m}(x_{ij}^{N} - \overline{x_j^{N}})(x_{ik}^{N} - \overline{x_k^{N}})}{\sqrt{\sum_{i=1}^{m}\left(x_{ij}^{N} - \overline{x_j^{N}}\right)^2 \sum_{i=1}^{m}\left(x_{ik}^{N} - \overline{x_k^{N}}\right)^2}} \quad (8)$$

All of the possible correlation coefficients are represented in a matrix as following

$$\rho_{ij} = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1m} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{m1} & \rho_{m2} & \cdots & \rho_{mm} \end{bmatrix} \quad (9)$$

$\rho_{11}, \rho_{22}, \ldots, \rho_{mm}$ will be equal to 1.

The standard deviation of the elements of each vector (σ_j) is calculated with the following Equation 10:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij}^N - \overline{x_j^N}\right)^2}{m-1}} \tag{10}$$

$$\sigma_j = [\sigma_1 \quad \sigma_2 \quad \dots \quad \sigma_n]$$

Criteria scores are calculated by using the following Equation 11:

$$c_j = \sigma_j \sum_{k=1}^{m}\left(1 - r_{kj}\right) \tag{11}$$

Finally, all of the criteria scores are normalized by using the following equation to obtain the criteria of the weights in the interval [0,1] which satisfies $\sum w = 1$.

$$w_j = \frac{c_j}{\sum_{k=1}^{n} c_k} \tag{12}$$

3.1.3. MEREC Method

MEREC (Method based on the Removal Effects of Criteria) was developed by Keshavarz-Ghorabaee and colleagues (2021). This method also aims to determine the criteria weights objectively.

Suppose there are m alternatives and n criteria. The decision matrix will be as in Equation 1. Normalization of the matrix will be ensured by applying the following equation.

$$x_{ij}^N = \begin{cases} \dfrac{\min\limits_k x_{kj}}{x_{ij}}, & if\ j \in BC \\ \dfrac{x_{ij}}{\max\limits_k x_{kj}}, & if\ j \in CC \end{cases} \tag{13}$$

Where BC and CC represents benefit and cost criteria set respectively. By applying following Equation 14, a score is assigned to all alternatives

$$S_i = \ln\left(1 + \left(\frac{1}{n}\sum_j \left|\ln\left(x_{ij}^N\right)\right|\right)\right) \tag{14}$$

Another performance score is calculated by removing each criterion with the following Equation 15:

$$S'_{ij} = \ln\left(1 + \left(\frac{1}{n}\sum_{k,k\neq j} \left|\ln\left(x_{ij}^N\right)\right|\right)\right) \tag{15}$$

Let E_j denote the effect of removing jth criterion. E_j can be calculated by using the following Equation 16:

$$E_j = \sum_i \left| S'_{ij} - S_i \right| \tag{16}$$

To obtain the final weights, removal effects (E_j) is normalized by using the following Equation 17:

$$w_j = \frac{E_j}{\sum_k E_k} \tag{17}$$

3.1.4. SECA Method

SECA (Simultaneous Evaluation of Criteria and Alternatives) technique is developed by Keshavarz-Ghorabaee and colleagues in 2018. It aims to determine both the weights of the criteria and the scores of the alternatives in a single method.

Suppose that there are m alternatives and n criteria. The decision matrix will be defined as in Equation 1. All of the elements of decision matrix must be greater than zero. The decision matrix must be normalized with the following Equation 18:

$$X_{ij}^N = \begin{cases} \dfrac{x_{ij}}{\max\limits_k x_{kj}}, & if\ j \in BC \\ \dfrac{\min\limits_k x_{kj}}{x_{ij}}, & if\ j \in CC \end{cases} \tag{18}$$

where BC and CC represents benefit and cost criteria set respectively. Applying Equation 18 to the decision matrix ensures that the dataset will be dimensionless.

Let $V_j = \left[x_{ij}^N\right]_{m\times 1}$ denotes the vector of jth ($j \in \{1,2,\dots,m\}$) criterion. To capture the within-criterion variation information, the standard deviation of the elements of each vector (σ_j) is calculated (Equation 10). Let ρ_{jk} denotes the correlation between jth and kth vectors (j and $k \in \{1,2,\dots,m\}$). It is possible to calculate the degree of conflict between the jth criterion and the other criteria by following Equation 19:

$$\pi_j = \sum_{k=1}^{m} (1 - \rho_{jk}) \tag{19}$$

both σ_j and π_j vectors must be normalized with the following equations:

$$\sigma_j^N = \frac{\sigma_j}{\sum_{k=1}^{n} \sigma_k} \tag{20}$$

$$\pi_j^N = \frac{\pi_j}{\sum_{k=1}^{n} \pi_k} \tag{21}$$

Developers of the model proposed a multi-objective non-linear programming model. And later transformed that model to a simpler one as follows:

$$\max Z = \lambda_a - \beta(\lambda_b + \lambda_c)$$

$$\lambda_b = \sum_{k=1}^{n} \left(w_j - \sigma_j^N\right)^2 \tag{22}$$

$$\lambda_c = \sum_{k=1}^{N} \left(w_j - \pi_j^N\right)^2$$

Subject to

$$\lambda_a \leq \mathrm{S_i}, \qquad \forall i \in \{1,2,\ldots,m\}$$

$$S_i = \sum_{k=1}^{n} w_j x_{ij}^N, \qquad \forall i \in \{1,2,\ldots,m\}$$

$$w_j \leq 1, \qquad \forall j \in \{1,2,\ldots,n\}$$

$$w_j \geq \epsilon, \qquad \forall j \in \{1,2,\ldots,n\}$$

$$\sum_{k=1}^{n} w_j = 1$$

ϵ is the lower bound of the criteria weights. In other words, weights of the criteria will be in $[\epsilon, 1]$. The β coefficient will be greater than 0. In this study, ϵ is fixed at 0.001 and the β coefficient at 1. One of the major advantages of the SECA method is that it objectively determines the weights of the criteria.

3.2. MCDM Techniques Employed for Stock Weight Determination

In the study, 17 MCDM techniques were used to determine the weights of stocks in the portfolio. Studies that introduced these techniques to the literature are summarized in Table 1.

Table 1. MCDM techniques.

No	Abbreviation	Publication Year	MCDM Name	Study introducing the technique to the literature
1	ARAS	2010	Additive Ratio Assessment	(Zavadskas et al., 2010)
2	COCOSO	2019	Combined Compromise Solution	(Yazdani et al., 2019)
3	CODAS	2016	Combinative distance-based Assessment	(Keshavarz Ghorabaee et al., 2016)
4	COPRAS	1994	Complex Proportional Assessment	(Zavadskas et al., 1994)
5	EDAS	2015	Evaluation Based on Distance from Average Solution	(Ghorabaee et al., 2015)
6	GRA	1982	Grey Relational Analysis	(Ju-Long, 1982)
7	MABAC	2015	Multi-Attributive Border Approximation area Comparison	(Pamučar & Ćirović, 2015)
8	MAIRCA	2014	Multi-Attribute Ideal-Real Comperative Analysis	(Pamučar et al., 2014)
9	MARCOS	2020	Measurement of Alternatives and Ranking According to Compromise Solution	(Stević et al., 2020)
10	MOORA	2006	Multi-Objective Optimization on basis of Ratio Analysis	(Brauers & Zavadskas, 2006)
11	MOOSRA	2012	Multi-Objective Optimization on the Basis of Simple Ratio Analysis	(Das et al., 2012)
12	OCRA	1994	Operational Competitiveness Rating	(Parkan, 1994)
13	SAW	1954	Simple Additive Weighting	(Churchman & Ackoff, 1954)

No	Abbreviation	Publication Year	MCDM Name	Study introducing the technique to the literature
14	TODIM	1991	Tomada de Decision Inerative Multicritero (in Portuguese), Interactive and Multi-criteria decision making	(Gomes & Lima, 1991)
15	TOPSIS	1981	Technique for Order Preference by Similarity to Ideal Solution	(Hwang & Yoon, 1981)
16	VIKOR	1998	Vise Kriterijumska Optimizacija I Kompromisno Resenje	(Opricovic, 1998)
17	WASPAS	2012	Weighted Aggregated Sum Product Assessment	(Zavadskas et al., 2012)

4. Analysis

In the context of this paper, a portfolio refers only to a combination of shares with different weights in the S&P500 stock list. The return of a portfolio for a future period can be calculated using the following Equation 23:

$$r_{port} = \sum_{i=1}^{n} w_i r_i \tag{23}$$

where r_{port} is the return of the portfolio, w_i is the weight of asset, r_i is the return of the asset i and n is the number of asset. The aim is to assign to each stock a weight (w_i) and observe the return of the portfolio in the future period.

The outline of the study is summarized in Figure 1. The first step is to retrieve roughly two years' worth of historical stock price data. It is assumed that stocks are traded for 250 days each year. The data for the first 250 days is used for portfolio optimization (determining the weight of the securities in the portfolio), and the data for the following 250 days is used to examine the portfolio's performance. The data set reserved for portfolio optimization is used with five separate weight sets and 17 MCDM techniques. The performance of the resulting 85 portfolios is examined both statistically and financially by comparing them with benchmark models. This process was repeated for two different periods. All of the calculations are performed on the MATLAB platform. For each MCDM technique, a function is created to perform the necessary calculation steps.

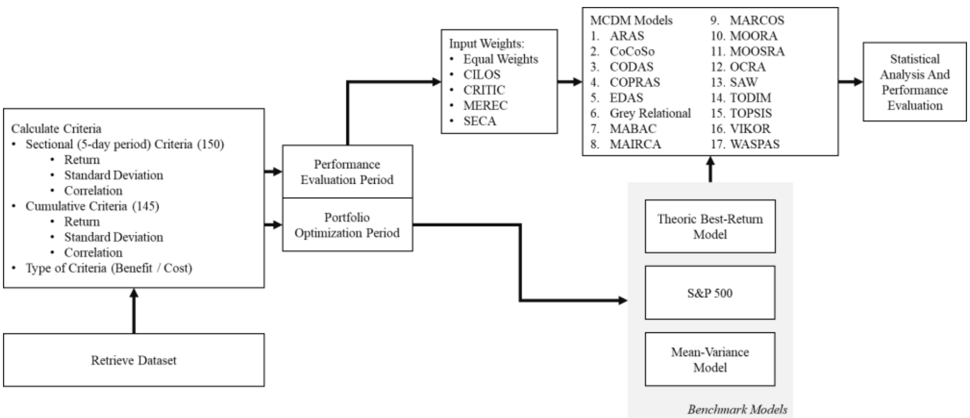


Figure 1. Outline of the study

4.1. Dataset Description

Historical closing price data for S&P 500 stocks between January 07, 2020, and December 31, 2020, is used for portfolio construction. There are 250 trading days within this period. Historical closing price data between January 04, 2021, and December 29, 2021 (250 trading days) is used for performance evaluation. Data is retrieved from finance.yahoo.com.

The closing price of each stock is transformed to return with the following Equation 24:

$$r_t = \frac{p_{t+1} - p_t}{p_t} \tag{24}$$

where p_t represents the closing price and r_t represents the return value for day t. Descriptive statistics for the S&P500 index and some of the stocks are represented in Table 2. Values in the table cover the overall 500 trading days. Jarque-Berra tests run for the return series of each stock as well as the return series for the S&P 500 index, and significant evidence is found against the null hypothesis stating normal distribution.

Table 2. Summary of descriptive statistics.

Stock	S&P500	MMM	AOS	ABT	…	ZBH	ZION	ZTS
Industry		Industrials	Industrials	Health Care		Health Care	Financials	Health Care
Min	-0.12	-0.09	-0.09	-0.10	…	-0.11	-0.12	-0.15
Max	0.09	0.13	0.10	0.11	…	0.16	0.24	0.12
Mean	0.00	0.00	0.00	0.00	…	0.00	0.00	0.00
Median	0.00	0.00	0.00	0.00	…	-0.00	-0.00	0.00
Std. Dev.	0.02	0.02	0.02	0.02	…	0.03	0.03	0.02
Skewness	-0.67	0.02	0.08	0.07	…	0.52	0.91	-0.40
Kurtosis	16.54	10.93	5.57	9.45	…	10.07	10.54	15.11
Jarque-Berra (p-value)	0	0	0	0	…	0	0	0

S&P 500 price and return data is plotted in Figure 2 (a). In the figure, it is determined that there were fluctuations in the price and rate of return at the beginning of the period, after which the price series increased steadily, and the return series remained relatively stable. In the (b) part of the figure, there is a scatterplot with risk values (standard deviation in return) on the horizontal axis and return values on the vertical axis. The chart also shows the risk and return values for the S&P 500 index. As expected, the risk value of the S&P 500 index is lower than most stocks.

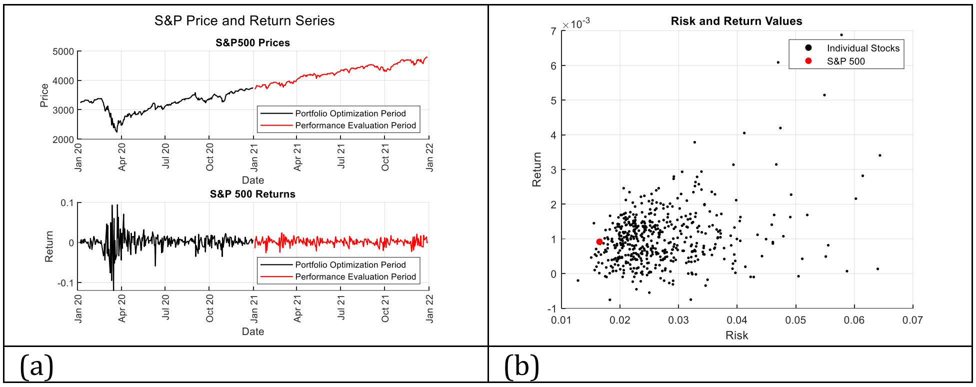


Figure 2. S&P500 price and return series (a) and scatter diagram of returns (b)

The Global Industry Classification Standard (GICS) distribution is presented in Table 3. There are 11 sectors. The sector with the most significant number of firms (75 firms) was information technology, while the sector with the least number of firms (21 firms) was determined as energy. Jarque-Berra tests are run for the return series of each stock, and significant evidence is found against the null hypothesis stating normal distribution for the industries; consumer discretionary, health care, information technology, and materials. In other industries, the mean return is not distributed normally.

Table 3. GICS sector distribution of firms and descriptive statistics of daily returns.

	n	Freq (%)	Min	Max	Mean	Median	Std	Skew.	Kurt.	JB (p)
Communication Services	27	5.35	-0.001	0.002	0.001	0.001	0.001	0.09	2.21	0.50
Consumer Discretionary	61	12.08	-0.000	0.010	0.001	0.001	0.002	2.81	11.85	0.00
Consumer Staples	32	6.34	-0.001	0.002	0.000	0.001	0.001	-0.54	2.88	0.28
Energy	21	4.16	-0.002	0.001	-0.000	-0.001	0.001	0.16	1.81	0.28
Financials	67	13.27	-0.002	0.003	0.001	0.000	0.001	0.37	3.64	0.16
Health Care	64	12.67	-0.001	0.009	0.001	0.001	0.001	3.58	21.94	0.00
Industrials	73	14.46	-0.002	0.004	0.001	0.001	0.001	0.14	3.95	0.14
Information Technology	75	14.85	-0.001	0.009	0.002	0.002	0.001	22.37	12.14	0.00
Materials	28	5.55	-0.000	0.004	0.001	0.001	0.001	1.54	6.30	0.00
Real Estate	29	5.74	-0.001	0.001	0.000	0.000	0.001	-0.19	2.18	0.46
Utilities	28	5.55	-0.001	0.001	0.000	0.000	0.001	0.28	3.63	0.50
Total	500	100.00								

4.2. Calculation of Inputs

The input values used in this study were developed based on the input values used in the study of Alali and Tolga (2019). The authors employed 9 criteria in their study, namely short-term return, mid-term return, long-term return, short-term standard deviation, mid-term standard deviation, long-term standard deviation, short-term correlation, mid-term correlation, and long-term correlation. Calculating the average correlation for each equity in the investment universe is done with the following equation, 25

$$\gamma_i = \frac{\left[\left(\sum_{p=1}^{m} \rho_{i,p}\right) - 1\right]}{m-1} \quad (25)$$

where γ_i is the average correlation for each equity, m is the number of equities in the investment universe, and $\rho_{i,p}$ is the sample correlation between equity i and p, calculated with Equation 8.

In this study, the data set has been enriched in two aspects (Figure 3):

- Sectional Inputs: The mean, standard deviation, and correlation values were calculated to cover a longer period. They were calculated in 5-day intervals for the past 250 days. There are 50 (=250/5) 5-day intervals in the data set covering 250 trading days. Since the mean (Equation 26), standard deviation (Equation 27), and correlation coefficients (Equation 28) are calculated for each interval, there are 50x3 = 150 input values in this first step. These criteria are called "sectional type criteria." The equations used to calculate the sectional criteria are as follows:

$$\mu = \frac{1}{5}\sum_{i=k}^{k+5-1} r_i\,, \qquad k = 1, 6, 11, \dots, 236, 241, 246 \tag{26}$$

$$s = \sqrt{\frac{\sum_{i=k}^{k+5-1}\left(r_i - \frac{1}{5}\sum_{j=k}^{k+5-1} r_j\right)^2}{5-1}}, \qquad k = 1, 6, 11, \dots, 236, 241, 246 \tag{27}$$

$$\rho_{ab} = \frac{\sum_{t=1}^{p}\left(a_t - \frac{1}{5}\sum_{i=k}^{k+5-1} a_{ti}\right)\left(b_t - \frac{1}{5}\sum_{i=k}^{k+5-1} b_{ti}\right)}{\sqrt{\sum_{t=1}^{p}\left(a_t - \frac{1}{5}\sum_{i=k}^{k+5-1} a_{ti}\right)^2}\sqrt{\sum_{t=1}^{p}\left(b_t - \frac{1}{5}\sum_{i=k}^{k+5-1} b_{ti}\right)^2}}, \quad k = 1, 6, 11, \dots, 236, 241, 246 \tag{28}$$

where, r_i stands for the asset return in day i, calculated with the equation. p represent the number of days in the period. ρ_{ab} represents the correlation coefficient calculated between stocks a and b. a_{ti} and b_{ti} represents the return of stock a and b, respectively.

- Cumulative Inputs: The mean (Equation 29), standard deviation (Equation 30), and correlation coefficient (Equation 31) values were calculated using the data set for the past 10 days (the most recent 10 days), the past 15 days,…., the past 245 days, and the past 250 days. These types of criteria are called "cumulative criteria." The 250-day dataset is divided into 5-day cumulative intervals. However, the calculations for the most recent five days were already performed in the previous step. Therefore, there are 49 (=(250/5)-1) cumulative intervals. Since three criteria are calculated for each cumulative interval, 49x3 = 147 input variables are added to the decision matrix from this aspect. The equations used in calculating the cumulative type criteria are as follows:

$$\mu_t = \frac{1}{250-k+1}\sum_{i=k}^{250} r_i\,, \qquad k = 241, 236, 231, \dots, 11, 6, 1 \tag{29}$$

$$s_t = \sqrt{\frac{\sum_{i=k}^{250}\left(r_i - \frac{1}{250-k+1}\sum_{i=k}^{250} r_i\right)^2}{250-k+1-1}}, \qquad k = 241, 236, 231, \dots, 11, 6, 1 \tag{30}$$

$$\rho_{ab} = \frac{\sum_{t=1}^{p}\left(a_t - \frac{1}{250-k+1}\sum_{i=k}^{250} a_{ti}\right)\left(b_t - \frac{1}{250-k+1}\sum_{i=k}^{250} b_{ti}\right)}{\sqrt{\sum_{t=1}^{p}\left(a_t - \frac{1}{250-k+1}\sum_{i=k}^{250} a_{ti}\right)^2}\sqrt{\sum_{t=1}^{p}\left(b_t - \frac{1}{250-k+1}\sum_{i=k}^{250} b_{ti}\right)^2}}, \quad k = 241, 236, 231, \ldots, 11, 6, 1 \qquad (31)$$

where p represents the number of trading days in a period. r_i represents the return for day i, calculated with the equation. a_{ti} and b_{ti} represents the return of stock a and b, respectively.

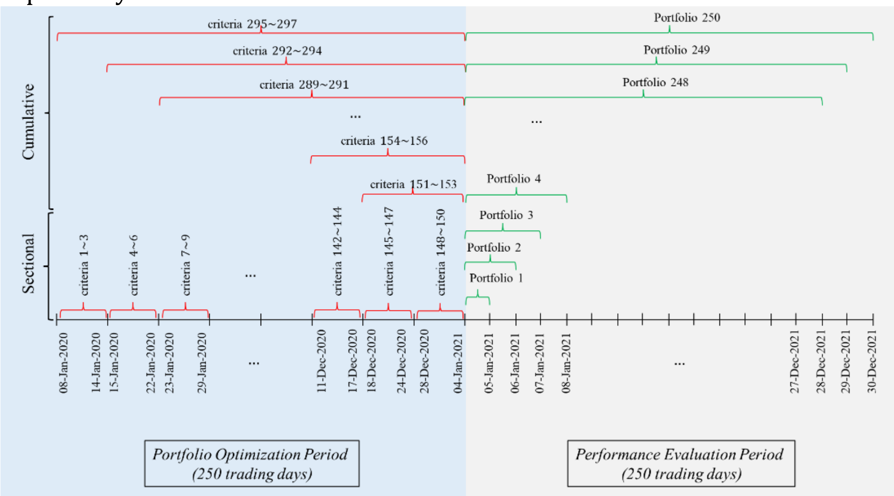


Figure 3. Portfolio optimization and performance evaluation periods

Thus, input values calculated at sectional intervals (150 criteria) and cumulative intervals (147 criteria) were collected together, and a decision matrix with 500 alternatives and 297 criteria was constructed. The decision matrix is summarized in Table 4. A Jarque-Berra test is performed for each criterion, and significant evidence is found against the null hypothesis stating normal distribution.

Table 4. Summary of the decision matrix.

Cr	SD	ED	IT	F	CT	Min	Max	Mean	Med	Std	Skew.	Kurt.
1	08/01	14/01/20	S	M	B	-0.02	0.05	0.00	0.00	0.01	1.34	14.04
2	08/01	14/01/20	S	S	C	0.00	0.12	0.01	0.01	0.01	7.49	98.92
3	08/01	14/01/20	S	C	C	-0.39	0.43	0.19	0.24	0.19	-0.83	2.89
4	15/01	22/01/20	S	M	B	-0.02	0.02	0.00	0.00	0.01	-0.36	5.04
5	15/01	22/01/20	S	S	C	0.00	0.04	0.01	0.01	0.01	1.46	5.57
6	15/01	22/01/20	S	C	C	-0.38	0.39	0.15	0.19	0.17	-0.83	3.06
...	...	...	...	...	...	...	...	...	...	...	...	...
145	18/12	24/12/20	S	M	B	-0.02	0.03	0.00	-0.00	0.01	0.40	4.47
146	18/12	24/12/20	S	S	C	0.00	0.06	0.02	0.01	0.01	1.81	7.96
147	18/12	24/12/20	S	C	C	-0.38	0.41	0.17	0.21	0.16	-0.89	3.24
148	28/12	04/01/21	S	M	B	-0.04	0.02	0.00	0.00	0.01	-1.35	16.22
149	28/12	04/01/21	S	S	C	0.00	0.06	0.01	0.01	0.01	2.50	14.63
150	28/12	04/01/21	S	C	C	-0.45	0.48	0.24	0.27	0.18	-0.90	3.42
151	18/12	04/01/21	C	M	B	-0.03	0.02	0.00	0.00	0.01	-0.75	9.52
152	18/12	04/01/21	C	S	C	0.00	0.05	0.01	0.01	0.01	2.09	9.73
153	18/12	04/01/21	C	C	C	-0.20	0.39	0.18	0.20	0.12	-0.73	3.25
154	11/12	04/01/21	C	M	B	-0.03	0.02	0.00	0.00	0.00	-0.13	11.30
155	11/12	04/01/21	C	S	C	0.01	0.05	0.01	0.01	0.07	2.04	8.93

156	11/12	04/01/21	C	C	C	-0.19	0.42	0.21	0.22	0.12	-0.69	3.09
...	...	...	C	...	...	...	...	...	...	...	...	...
292	15/01	04/01/21	C	M	B	-0.00	0.01	0.00	0.00	0.00	2.52	16.20
293	15/01	04/01/21	C	S	C	0.02	0.08	0.03	0.03	0.01	1.70	6.85
294	15/01	04/01/21	C	C	C	-0.01	0.67	0.53	0.55	0.09	-1.69	7.49
295	08/01	04/01/21	C	M	B	-0.00	0.01	0.00	0.00	0.00	2.54	16.41
296	08/01	04/01/21	C	S	C	0.02	0.08	0.03	0.03	0.01	1.71	6.89
297	08/01	04/01/21	C	C	C	-0.01	0.67	0.53	0.54	0.09	-1.68	7.43

SD : Starting Date (DD/MM). All of the dates are in 2020
ED : Ending Date (DD/MM/YY)
IT: Input Type (S = Sectional , C = Cumulative)
F : Features (M = Mean, S = Standard Deviation, C = Correlation)
CT : Criteria Type (B = Benefit, C = Cost)

After the decision matrix is created, it is necessary to determine whether each criterion is a benefit or a cost type. It is essential to determine the type of the criteria because different normalization equations must be used according to the criteria type (Equations 2, 7, 13, and 18). Following the study of Alali and Tolga (2019), the average return values are considered to be of benefit characteristic, and the standard deviation and correlation values to be of cost characteristic. For benefit (cost) criteria, the higher (lower) the score is, the better for the portfolio.

4.3. Performance Evaluation Set

Carrying out portfolio optimization and performance evaluation on the same data set will result in misleadingly high performance. That is why the weights of the criteria and the determination of the scores of the stocks have been carried out in the data set covering 250 trading days, called the portfolio optimization period. The performance of portfolios was examined over the following 250 trading days. The portfolios created with the weights determined by MCDM techniques are held for one day, two days, three days, ... 250 days following the last day of the portfolio optimization process.

The performance of the MCDM portfolios has been compared with three different benchmark models. These models are;

- Benchmark Model 1. In this model, the rate of return of the S&P500 index is used. The benchmark series is retrieved from finance.yahoo.com.

- Benchmark Model 2. In this model, the portfolio optimization model (Mean-Variance (MV) Model) developed by Markowitz in 1952 is used. The objective function in the model is to maximize the historical return with the following mathematical model.

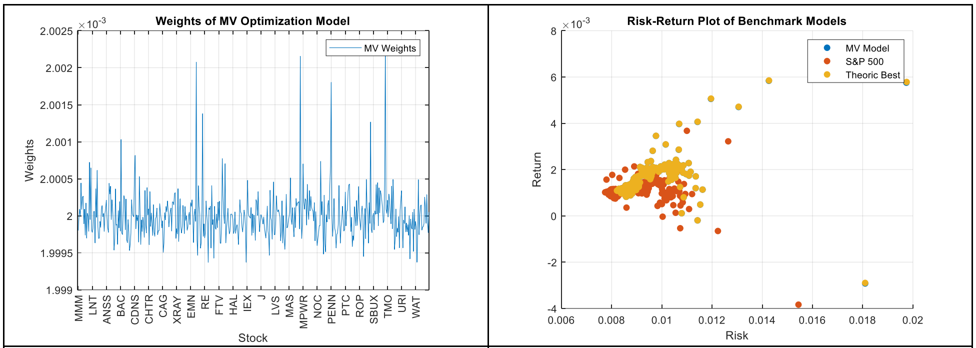
$$\max r_p = \sum_{i=1}^{m} r_i w_i$$

$$s.t. \sum_{i=1}^{m} w_i = 1 \tag{32}$$

$$\forall w_j \leq 1$$

$$\forall w_j \geq 0$$

In this Mean-Variance model (MVM), the portfolio's mean return is maximized, regardless of the variance (risk). The return rates of the past 250 days were used, and the model was run. The weights of the stocks are given in Figure 4 (a).



(a)	(b)

Figure 4. MV weight set (a) performance of benchmark models (b)

- Benchmark Model 3. This benchmark model optimizes the maximum return that can be reached theoretically during a specific period. In MCDM (and Benchmark Model 2), the weights of the securities in the portfolio are determined by using historical data, and the return of the portfolio created with these weights over a future period is examined. However, this model (Benchmark Model 3) differs from the MV model (Benchmark Model 2) in terms of the time of the data set used. In the MV model, the weights that maximize the return in the past period are optimized by using historical rates of return. In the theoretical best model, the weight set that provides the highest return during the evaluation period of the portfolio is optimized.

The start and end dates of the portfolios and the return rates of the benchmark models are listed in Table 5. The risk and return plots are presented in Figure 4 (b).

Table 5. Performance evaluation set and benchmark returns.

	Starting Date	Ending Date	S&P500 Return (%)	MV Return (%)	Theoretical Best Return (%)
Portfolio 1	04/01/21	05/01/21	-0.383610	-0.292110	-0.289680
Portfolio 2	04/01/21	06/01/21	-0.065410	0.575573	0.578624
Portfolio 3	04/01/21	07/01/21	0.322125	0.684094	0.686360
...	...	...	...	...	...
Portfolio 248	04/01/21	28/12/21	0.100825	0.106433	0.106442
Portfolio 249	04/01/21	29/12/21	0.100982	0.107136	0.107145
Portfolio 250	04/01/21	30/12/21	0.099389	0.106008	0.106017
		Minimum	-0.383610	-0.292110	-0.289680
		Maximum	0.367538	0.684094	0.686360
		Mean	0.107338	0.157036	0.157143
		Median	0.107645	0.135839	0.135854
		Std. Dev.	0.048799	0.083602	0.083778
		Skewness	-341.843000	212.854100	216.003700
		Kurtosis	4777.915000	1802.081000	1804.621000

4.4. Criteria Weights

Four MCDM techniques, which were developed to determine the weights of the criteria to be used in MCDM techniques, were run with the decision matrix. As a result of this process, weights were assigned to 297 criteria. The values of the weights are presented in Figure 5.

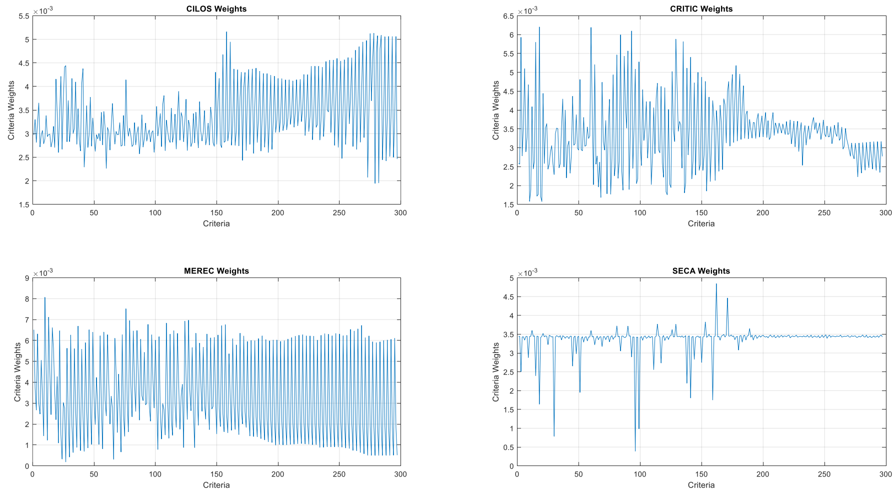


Figure 5. Weights of criteria with four different models

The 5 criteria with the highest and lowest CILOS weights are listed in Table 6. Descriptive statistics of the weights are also presented in the table. According to the CILOS technique, the first five criteria assigned the highest weight are all cumulative criteria. Moreover, all of them are standard deviation values. The three criteria with the lowest weight are correlation coefficients also cumulative.

Table 6. Top 5 and bottom 5 criteria and descriptive statistics of CILOS weights.

Rank	Sectional/Cumulative	Starting Date (DD/MM/YY)	Ending Date (DD/MM/YY)	Group	Weight
1	Cumulative	04/12/20	04/01/21	Std Dev	0.0051596
2	Cumulative	21/02/20	04/01/21	Std Dev	0.0051270
3	Cumulative	28/02/20	04/01/21	Std Dev	0.0051214
4	Cumulative	06/02/20	04/01/21	Std Dev	0.0050858
5	Cumulative	13/02/20	04/01/21	Std Dev	0.0050744
...	...	...	...	...	...
293	Sectional	13/04/20	17/04/20	Correlation	0.0022920
294	Sectional	26/05/20	01/06/20	Correlation	0.0022647
295	Cumulative	06/03/20	04/01/21	Correlation	0.0020765
296	Cumulative	13/02/20	04/01/21	Correlation	0.0019551
297	Cumulative	21/02/20	04/01/21	Correlation	0.0019454
				Minimum	0.0019454
				Maximum	0.0051596
				Mean	0.0033670
				Median	0.0031907
				Std. Dev.	0.0006892
				Skewness	0.8165600
				Kurtosis	2.9089000

The 5 criteria with the highest and lowest CRITIC weights are listed in Table 7. Descriptive statistics of the weights are also presented in the table. The first five criteria with the highest weight are the correlation coefficient, all of which are sectional. The five most minor weighted criteria are of sectional type.

Table 7. Top 5 and bottom 5 criteria and descriptive statistics of CRITIC weights.

Rank	Sectional/Cumulative	Starting Date (DD/MM/YY)	Ending Date (DD/MM/YY)	Group	Weight
1	Sectional	13/02/20	20/02/20	Correlation	0.006202
2	Sectional	26/05/20	01/01/20	Correlation	0.006189

Rank	Sectional/Cumulative	Starting Date (DD/MM/YY)	Ending Date (DD/MM/YY)	Group	Weight
3	Sectional	12/08/20	18/08/20	Correlation	0.006096
4	Sectional	22/07/20	28/07/20	Correlation	0.005994
5	Sectional	08/01/20	14/01/20	Correlation	0.005927
...	...	...	...	...	...
293	Sectional	13/02/20	20/02/20	Mean	0.001718
294	Sectional	21/02/20	27/02/20	Mean	0.00171
295	Sectional	16/06/20	22/06/20	Std Dev	0.001684
296	Sectional	30/01/20	05/02/20	Mean	0.001582
297	Sectional	21/02/20	27/02/20	Std Dev	0.001575
				Minimum	0.001575
				Maximum	0.006202
				Mean	0.003367
				Median	0.00329
				Std. Dev.	0.000929
				Skewness	0.67515
				Kurtosis	3.5591

The 5 criteria with the highest and lowest MEREC-weights are listed in Table 8. Descriptive statistics of the weights are also presented in the table. The MEREC technique assigned the top five weights to the averages of the sectional type. All of the criteria that have the least weight are given to the sectional type correlation coefficients.

Table 8. Top 5 and bottom 5 criteria and descriptive statistics of MEREC weights.

Rank	Sectional/Cumulative	Starting Date (DD/MM/YY)	Ending Date (DD/MM/YY)	Group	Weight
1	Sectional	30/01/20	05/02/20	Mean	0.008065
2	Sectional	08/07/20	14/07/20	Mean	0.007517
3	Sectional	06/02/20	12/02/20	Mean	0.007114
4	Sectional	05/11/20	11/11/20	Mean	0.006969
5	Sectional	15/07/20	21/07/20	Mean	0.006955
...	...	...	...	...	...
293	Cumulative	21/02/20	04/01/21	Correlation	0.000500
294	Sectional	13/03/20	19/03/20	Correlation	0.000422
295	Sectional	28/02/20	05/03/20	Correlation	0.000325
296	Sectional	09/01/20	15/06/20	Correlation	0.000316
297	Sectional	06/03/20	12/03/20	Correlation	0.000190
				Minimum	0.000190
				Maximum	0.008065
				Mean	0.003367
				Median	0.002687
				Std. Dev.	0.002129
				Skewness	0.395160
				Kurtosis	1.6703

The 5 criteria with the highest and lowest SECA weights are listed in Table 9. Descriptive statistics of the weights are also presented in the table. The SECA technique assigned the top three weights to the cumulative correlation coefficient. It assigned the bottom four weights to the sectional correlation coefficients.

Table 9. Top 5 and bottom 5 criteria and descriptive statistics of SECA weights.

Rank	Sectional/Cumulative	Starting Date	Ending Date	Group	Weight
1	Cumulative	27-Nov-2020	04-Jan-2021	Correlation	0.004844
2	Cumulative	05-Nov-2020	04-Jan-2021	Correlation	0.004461
3	Cumulative	18-Dec-2020	04-Jan-2021	Correlation	0.003824
4	Sectional	01-Oct-2020	07-Oct-2020	Correlation	0.003767
5	Sectional	05-Nov-2020	11-Nov-2020	Correlation	0.003763
...	...	...	...	...	...

Rank	Sectional/Cumulative	Starting Date	Ending Date	Group	Weight
293	Cumulative	04-Dec-2020	04-Jan-2021	Correlation	0.001751
294	Sectional	13-Feb-2020	20-Feb-2020	Correlation	0.00164
295	Sectional	26-Aug-2020	01-Sep-2020	Correlation	0.000985
296	Sectional	13-Mar-2020	19-Mar-2020	Correlation	0.000785
297	Sectional	19-Aug-2020	25-Aug-2020	Correlation	0.000392
				Minimum	0.000392
				Maximum	0.004844
				Mean	0.003367
				Median	0.003435
				Std. Dev.	0.000382
				Skewness	-4.43572
				Kurtosis	30.4779

Box charts of criteria weights grouped according to the sectional or cumulative type of the criteria are shown in Figure 6. The tailed Wilcoxon rank-sum test was performed to determine whether the median values of the weights assigned to the sectional and cumulative criteria were different (*RQ 1: Do the weights of the different types of inputs (cumulative or sectional) differ from each other?*). The null hypothesis of the tests and the test results are presented in Table 10. According to the data in the table, the CILOS, CRITIC, and SECA techniques assigned higher weight values to the cumulative type criteria. However, the MEREC technique assigned higher weight values to the sectional type criteria. These differences are statistically significant.

Table 10. Hypothesis test results of cumulative and sectional comparison.

Null Hypothesis	Weight Method	Zval	Rank sum	p
The median of sectional weights is ≥ the median of cumulative weights	CILOS	-5.4022	18352	3.2923e-08
The median of sectional weight is ≥ the median of cumulative weights	CRITIC	-1.7750	21036	0.0379
The median of sectional weight is ≤ median of cumulative weights	MEREC	3.2345	24744	6.0921e-04
The median of sectional weight is ≥ the median of cumulative weights	SECA	-4.1400	19286	1.7368e-05

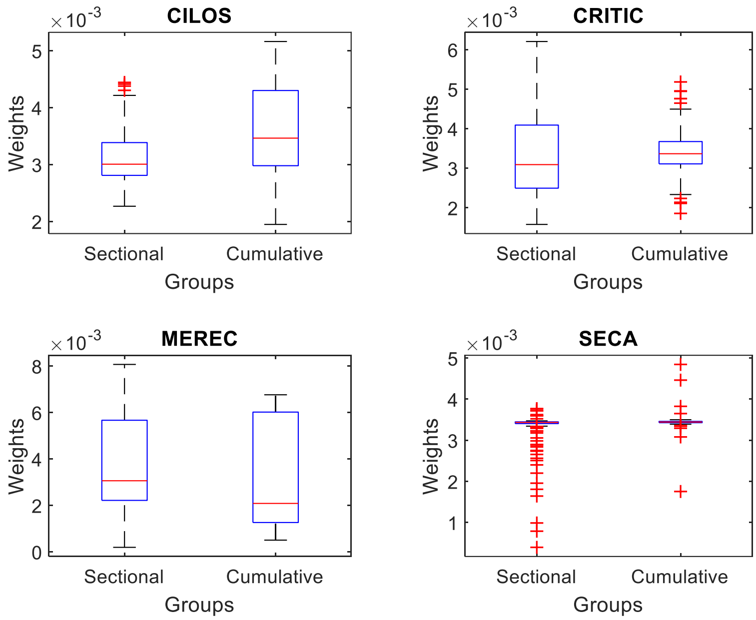


Figure 6. Criteria weight boxplot grouping sectional/cumulative characteristic

Box charts of criteria weights grouped according to the characteristics of the criteria are presented in Figure 7. The mean rank values of the criterion weights for each of the three groups are presented in Table 11. The non-parametric Kruskal-Wallis test was used to determine whether the weights assigned by the techniques differed according to the characteristics of the criteria (*RQ 2: Do the weights of the different groups of inputs (mean, standard deviation, or correlation?*) differ from each other?). A Tukey's test was used to determine whether the weights assigned by the techniques differed according to the characteristics of the criteria. multiple-comparison test was also performed for each method. Test results are presented in Table 12. According to the data in the table, the median value of the weights assigned to the criteria according to their characteristics differs in at least one group.

In the CILOS technique, the highest weights were assigned to the standard deviation, mean, and correlation groups, respectively. In the CRITIC technique, the group with the highest weight values is the correlation group. The median of the weights assigned to this group is higher than the median value of the weights assigned to the other two groups (mean and standard deviation). The MEREC technique assigned the highest weight values to the mean, standard deviation, and correlation groups, respectively. The SECA method assigned the most weight to the standard deviation, mean, and correlation groups, respectively.

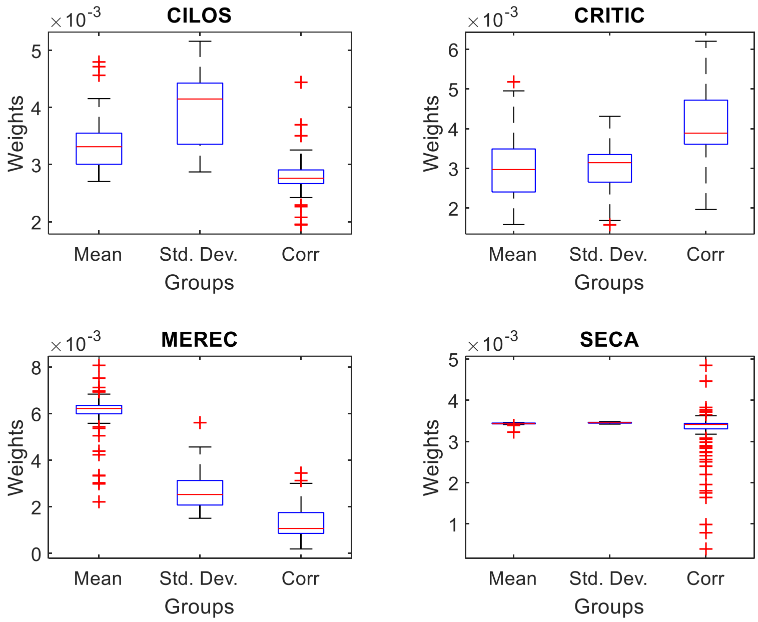


Figure 7. Box plot of weights grouping criteria characteristics

Table 11. Mean ranks of weights across groups.

	Weights			
Mean Ranks	CILOS	CRITIC	MEREC	SECA
Group – 1 (Mean)	161.24	113.14	246.04	135.56
Group – 2 (Standard Deviation)	221.45	113.81	141.07	213.20
Group – 3 (Correlation Coefficient)	64.30	220.05	59.89	98.23

Table 12. Kruskal-Wallis test and multiple comparison results.

CILOS Method

Kruskal-Wallis Results

H_0: mean ranks of the groups are the same (H_0: $\bar{x}_{mean} = \bar{x}_{std\ dev} = \bar{x}_{corr}$)

H_1: At least one of the mean rank of the group differs

	SS	df	MSE	χ^2	p
Columns	1244738	2	622369.2	168.7666	2.25E-37
Error	938409.6	294	3191.87		
Total	2183148	296			

Multiple Comparison Results

			%95 Confidence Interval		
Group -1	Group – 2	$\hat{\mu}_1 - \hat{\mu}_2$	Lower	Upper	p
1	2	-88.8206	-60.2121	-31.6036	2.42E-06
1	3	68.33088	96.93939	125.5479	9.56E-10
2	3	128.543	157.1515	185.76	9.56E-10

CRITIC Method

Kruskal-Wallis Results

H_0: mean ranks of the groups are the same (H_0: $\bar{x}_{mean} = \bar{x}_{std\ dev} = \bar{x}_{corr}$)

H_1: At least one of the mean rank of the group differs

	SS	df	MSE	χ^2	p
Columns	749675.9	2	374837.9	101.6441	8.48E-23
Error	1433472	294	4875.756		
Total	2183148	296			

Multiple Comparison Results

			%95 Confidence Interval		
Group -1	Group - 2	$\hat{\mu}_1 - \hat{\mu}_2$	Lower	Upper	p

1	2	-29.2752	-0.66667	27.94184	0.998
1	3	-135.518	-106.909	-78.3006	9.56E-10
2	3	-134.851	-106.242	-77.6339	9.56E-10

MEREC

Kruskal-Wallis Results

H_0: mean ranks of the groups are the same (H_0: $\bar{x}_{mean} = \bar{x}_{std\ dev} = \bar{x}_{corr}$)

H_1: At least one of the mean rank of the group differs

	SS	df	MSE	χ^2	p
Columns	1724630	2	862314.9	233.8323	1.67E-51
Error	458518.1	294	1559.585		
Total	2183148	296			

Multiple Comparison Results

			%95 Confidence Interval		
Group -1	Group - 2	$\hat{\mu}_1 - \hat{\mu}_2$	Lower	Upper	p
1	2	76.36119	104.9697	133.5782	9.56E-10
1	3	157.543	186.1515	214.76	9.56E-10
2	3	52.57331	81.18182	109.7903	1.04E-09

SECA Method

Kruskal-Wallis Results

H_0: mean ranks of the groups are the same (H_0: $\bar{x}_{mean} = \bar{x}_{std\ dev} = \bar{x}_{corr}$)

H_1: At least one of the mean rank of the group differs

	SS	df	MSE	χ^2	p
Columns	681094.1	2	340547	92.34548	8.86E-21
Error	1502054	294	5109.027		
Total	2183148	296			

Multiple Comparison Results

			%95 Confidence Interval		
Group -1	Group - 2	$\hat{\mu}_1 - \hat{\mu}_2$	Lower	Upper	p
1	2	-106.245	-77.6364	-49.0279	1.55E-09
1	3	8.724822	37.33333	65.94184	0.0063
2	3	86.36119	114.9697	143.5782	9.56E-10

4.5. Hypothesis Tests Results

The one-sided Wilcoxon rank-sum test was used to test whether weights on the criteria increase the rate of return. Three different hypothesis tests were run. First, it is tested whether there is a difference between portfolios to which equal weights are assigned. Then, a comparison was made with the MV model, and lastly, a comparison was made with the theoretical best model.

Portfolios were created with each technique, and each weight and its returns were recorded. In addition, portfolios in which each stock is assigned an equal weight (1/500 = 0.002) are also created. The null hypothesis states that the median value of the returns of the equally weighted portfolios is greater than or equal to the median value of the portfolios created with the weights was tested with the non-parametric Wilcoxon-rank sum test (*RQ 3: Does the returns of the MCDM portfolio greater than or equal to equally weighted portfolio?*). The results are presented in Table 13.

In the ARAS technique, the weights assigned with the CRITIC technique were able to achieve statistically higher returns than in the case where the equal weight was assigned. Weights assigned with MEREC, on the other hand, were able to achieve statistically higher returns in MARCOS, MOORA, OCRA, SAW, and WASPAS techniques compared to the situation where equal weights were assigned.

Table 13. Hypothesis test results: Comparison with the equal weighted model.

		CILOS	CRITIC	MEREC	SECA
ARAS	Rank Sum	61742	66305	58735	61427
	Z Values	-0.54694	2.277826	-2.40845	-0.74194
	p	0.707789	**0.011368***	0.99199	0.770939
COCOSO	Rank Sum	62354	62692	63381	62730
	Z Values	-0.16807	0.041167	0.467699	0.064692
	p	0.566738	0.483581	0.32	0.47421
CODAS	Rank Sum	61966	63003	65008	55083
	Z Values	-0.40827	0.233695	1.474907	-4.66925
	p	0.658462	0.407611	0.070119	0.999998
COPRAS	Rank Sum	62203	62112	64146	62822
	Z Values	-0.26155	-0.31789	0.941279	0.121645
	p	0.603167	0.624715	0.173281	0.45159
EDAS	Rank Sum	62510	62681	62817	62747
	Z Values	-0.0715	0.034358	0.11855	0.075216
	p	0.528501	0.486296	0.452816	0.470022
GRA	Rank Sum	62255	62773	63568	62494
	Z Values	-0.22936	0.091311	0.583463	-0.08141
	p	0.590706	0.463623	0.279791	0.532441
MABAC	Rank Sum	62389	62708	63291	62691
	Z Values	-0.14641	0.051072	0.411983	0.040548
	p	0.5582	0.479634	0.340176	0.483828
MAIRCA	Rank Sum	63177	62335	61076	62764
	Z Values	0.341411	-0.17984	-0.95923	0.08574
	p	0.366397	0.57136	0.831279	0.465837
MARCOS	Rank Sum	61729	63005	65994	55939
	Z Values	-0.55499	0.234933	2.085299	-4.13934
	p	0.710548	0.40713	**0.018521***	0.999983
MOORA	Rank Sum	63356	63100	75896	62440
	Z Values	0.452222	0.293743	8.215217	-0.11484
	p	0.325554	0.384477	**1.06E-16***	0.545712
MOOSRA	Rank Sum	62373	64952	61479	62566
	Z Values	-0.15631	1.44024	-0.70975	-0.03683
	p	0.562107	0.0749	0.761071	0.514691
OCRA	Rank Sum	63104	61501	66666	62491
	Z Values	0.29622	-0.69613	2.501306	-0.08326
	p	0.383531	0.756827	**0.006187***	0.533179
SAW	Rank Sum	61729	63005	65994	55939
	Z Values	-0.55499	0.234933	2.085299	-4.13934
	p	0.710548	0.40713	**0.018521***	0.999983
TODIM	Rank Sum	62978	62179	62887	63145
	Z Values	0.218218	-0.27641	0.161884	0.321601
	p	0.41363	0.608883	0.435699	0.373878
TOPSIS	Rank Sum	62389	63087	63527	62763
	Z Values	-0.14641	0.285696	0.558081	0.085121
	p	0.5582	0.387556	0.288394	0.466083
VIKOR	Rank Sum	63757	63351	60073	63166
	Z Values	0.700465	0.449127	-1.58015	0.334601
	p	0.241819	0.32667	0.942963	0.368963
WASPAS	Rank Sum	61857	62706	68089	57076
	Z Values	-0.47575	0.049834	3.382227	-3.43547
	p	0.682873	0.480127	**0.00036***	0.999704

$$H_0: m_{eq} \geq m_{w-mcdm}$$
$$H_a: m_{eq} < m_{w-mcdm}$$

m_{eq} represents the median of the equally weighted portfolios
m_{w-mcdm} represents the median of the weighted MCDM portfolios
(w is one of the weight methods = CILOS, CRITIC, MEREC, SECA)
($mcdm$ is one of the 17 MCDM techniques)

Another benchmarking model is the MV model. Portfolios were created with the weights determined by the MV model, and the return values were recorded. A comparison was made with the return rates of the portfolios created with the weights determined by MCDM techniques (*RQ 4: Does the returns of the MCDM portfolio greater than or equal to the Mean-Variance portfolio?*). Hypothesis test results are presented in Table 14. The null hypothesis is that the median value of the returns of the portfolios formed with the MV weights is greater or equal to the median value of the returns of the portfolios formed with the MCDM criterion weights. According to the results in the table, the median value of the returns of the portfolios created with the CILOS weights is higher than the median value of the portfolios created with the MV weights in the MOOSRA and VIKOR techniques. The median value of the returns of the portfolios created with the CRITIC weights is higher than the median value of the returns of the portfolios created with the MV weights in the ARAS, MOOSRA, and VIKOR techniques. The median value of the returns of the portfolios created with the MEREC weights is higher than the median value of the returns of the portfolios created with the MV weights in MOORA and MOOSRA techniques. The median value of the returns of the portfolios created with the SECA weights is higher than the median value of the returns of the portfolios created with the MV weights in MOOSRA and VIKOR techniques. In other comparisons, the differences found are not statistically significant.

Table 14. Hypothesis test results: Comparison with the Mean-Variance model.

		CILOS	CRITIC	MEREC	SECA
ARAS	Rank Sum	64467	68920	61610	64135
	Z Values	1.139996	3.896664	-0.62865	0.934469
	p	0.127144	**4.88E-05**	0.735212	0.175031
COCOSO	Rank Sum	61566	61715	62377	61702
	Z Values	-0.65589	-0.56365	-0.15384	-0.5717
	p	0.744053	0.713505	0.56113	0.716238
CODAS	Rank Sum	60356	61120	62068	54326
	Z Values	-1.40495	-0.93199	-0.34513	-5.13788
	p	0.919982	0.82433	0.635	1.0000
COPRAS	Rank Sum	55629	55389	57072	55979
	Z Values	-4.33124	-4.47982	-3.43794	-4.11457
	p	0.999993	0.999996	0.999707	0.999981
EDAS	Rank Sum	60345	60404	60458	60374
	Z Values	-1.41176	-1.37524	-1.34181	-1.39381
	p	0.92099	0.915471	0.910171	0.918313
GRA	Rank Sum	60748	61056	61791	60986
	Z Values	-1.16228	-0.97161	-0.5166	-1.01495
	p	0.87744	0.834378	0.697284	0.844934
MABAC	Rank Sum	61712	61854	62413	61831
	Z Values	-0.56551	-0.4776	-0.13155	-0.49184
	p	0.714137	0.683534	0.55233	0.688584
MAIRCA	Rank Sum	64873	64335	63105	64495
	Z Values	1.391334	1.058281	0.296839	1.15733
	p	0.082062	0.144964	0.383295	0.123569
MARCOS	Rank Sum	59430	60177	62213	55122
	Z Values	-1.9782	-1.51576	-0.25536	-4.64511
	p	0.976047	0.935211	0.600778	0.999998
MOORA	Rank Sum	54029	53393	71972	52030
	Z Values	-5.32174	-5.71546	5.786031	-6.55924
	p	1.0000	1.0000	**3.60E-09**	1.0000
MOOSRA	Rank Sum	67604	69833	66824	67770
	Z Values	3.081983	4.461865	2.599117	3.184747
	p	**0.001028**	**4.06E-06**	**0.004673**	**0.000724**
OCRA	Rank Sum	54235	53191	57029	53931
	Z Values	-5.19421	-5.84051	-3.46456	-5.3824
	p	1.0000	1.0000	0.999734	1.0000
SAW	Rank Sum	59430	60177	62213	55122
	Z Values	-1.9782	-1.51576	-0.25536	-4.64511

		CILOS	CRITIC	MEREC	SECA
	p	0.976047	0.935211	0.600778	0.999998
TODIM	Rank Sum	62289	61722	62215	62506
	Z Values	-0.20831	-0.55932	-0.25412	-0.07398
	p	0.582508	0.712028	0.6003	0.529486
TOPSIS	Rank Sum	62235	62738	63214	62373
	Z Values	-0.24174	0.069644	0.364316	-0.15631
	p	0.59551	0.472238	0.357811	0.562107
VIKOR	Rank Sum	66492	66126	63301	65936
	Z Values	2.39359	2.167015	0.418174	2.049393
	p	**0.008342**	**0.015117**	0.33791	**0.020212**
WASPAS	Rank Sum	60544	60865	63959	57728
	Z Values	-1.28857	-1.08985	0.825515	-3.03184
	p	0.901226	0.862111	0.20454	0.998785

$H_0: m_{mv} \geq m_{w-mcdm}$
$H_a: m_{mv} < m_{w-mcdm}$
m_{mv} represents the median of the Mean-Variance portfolios
m_{w-mcdm} represents the median of the weighted MCDM portfolios
(w is one of the weight methods = CILOS, CRITIC, MEREC, SECA)
($mcdm$ is one of the 17 MCDM techniques)

Another benchmarking model is the theoretical best return model. Portfolios were created with the weights determined by the theoretical best model, and the return values were recorded. A comparison was made with the return rates of the portfolios created with the weights determined by MCDM techniques (*RQ 5: Does the returns of the MCDM portfolio greater than or equal to the theoretical best return portfolio?*). Hypothesis test results are presented in Table 15. The null hypothesis states that the median value of the returns of the portfolios created with theoretical best return weights, is greater than or equal to the median value of the returns of the portfolios created with the MCDM criterion weights. According to the results in the table, the median value of the returns of the portfolios created with the CILOS weights is higher than the median value of the portfolios created with the MV weights in the MOOSRA and VIKOR techniques. The median value of the returns of the portfolios created with CRITIC weights is higher than the median value of the returns of the portfolios created with the theoretical best weights in ARAS, MOOSRA, and VIKOR techniques. The median value of the returns of the portfolios created with the MEREC weights is higher than the median value of the returns of the portfolios created with the theoretical best weights in MOORA and MOOSRA techniques. The median value of the returns of the portfolios created with the SECA weights is higher than the median value of the returns of the portfolios created with the theoretical best weights in the MOOSRA technique. In other comparisons, the differences found are not statistically significant.

Table 15. Hypothesis test results: Comparison with the theoretical best return model.

		CILOS	CRITIC	MEREC	SECA
ARAS	Rank Sum	64452	68908	61595	64121
	Z Values	1.130711	3.889236	-0.63794	0.925802
	p	0.129088	**5.03E-05**	0.738244	0.177274
COCOSO	Rank Sum	61552	61706	62358	61686
	Z Values	-0.66456	-0.56922	-0.1656	-0.58161
	p	0.746834	0.715398	0.565763	0.719584
CODAS	Rank Sum	60344	61114	62058	54318
	Z Values	-1.41238	-0.93571	-0.35132	-5.14283
	p	0.921081	0.825288	0.637324	1
COPRAS	Rank Sum	55616	55380	57062	55966
	Z Values	-4.33929	-4.48539	-3.44413	-4.12262
	p	0.999993	0.999996	0.999714	0.999981
EDAS	Rank Sum	60332	60388	60438	60364
	Z Values	-1.41981	-1.38514	-1.35419	-1.4

		CILOS	CRITIC	MEREC	SECA
	p	0.922169	0.916996	0.912162	0.919243
GRA	Rank Sum	60733	61044	61772	60973
	Z Values	-1.17157	-0.97904	-0.52837	-1.02299
	p	0.879315	0.83622	0.701378	0.846845
MABAC	Rank Sum	61696	61837	62394	61816
	Z Values	-0.57541	-0.48813	-0.14331	-0.50113
	p	0.717495	0.68727	0.556978	0.691859
MAIRCA	Rank Sum	64863	64318	63084	64479
	Z Values	1.385144	1.047757	0.283838	1.147425
	p	0.083004	0.147375	0.388267	0.125603
MARCOS	Rank Sum	59416	60167	62206	55110
	Z Values	-1.98687	-1.52196	-0.2597	-4.65253
	p	0.976532	0.93599	0.602451	0.999998
MOORA	Rank Sum	54021	53391	71959	52026
	Z Values	-5.32669	-5.7167	5.777983	-6.56171
	p	1.000	1.000	**3.78E-09**	1.0000
MOOSRA	Rank Sum	67581	69820	66812	67750
	Z Values	3.067745	4.453817	2.591689	3.172366
	p	**0.001078**	**4.22E-06**	**0.004775**	**0.000756**
OCRA	Rank Sum	54228	53180	57020	53921
	Z Values	-5.19854	-5.84732	-3.47013	-5.3886
	p	1.0000	1.0000	0.99974	1.0000
SAW	Rank Sum	59416	60167	62206	55110
	Z Values	-1.98687	-1.52196	-0.2597	-4.65253
	p	0.976532	0.93599	0.602451	0.999998
TODIM	Rank Sum	62269	61703	62189	62482
	Z Values	-0.22069	-0.57108	-0.27022	-0.08883
	p	0.587335	0.716028	0.606504	0.535393
TOPSIS	Rank Sum	62217	62722	63191	62355
	Z Values	-0.25289	0.059739	0.350078	-0.16746
	p	0.599822	0.476182	0.36314	0.566494
VIKOR	Rank Sum	66477	66117	63284	65923
	Z Values	2.384304	2.161443	0.40765	2.041346
	p	**0.008556**	**0.015331**	0.341765	**0.020608**
WASPAS	Rank Sum	60534	60845	63948	57714
	Z Values	-1.29476	-1.10223	0.818705	-3.04051
	p	0.902299	0.86482	0.206477	0.998819

$H_0: m_{tbr} \geq m_{w-mcdm}$

$H_a: m_{tbr} < m_{w-mcdm}$

m_{tbr} represents the median of the theoretical best-return portfolios
m_{w-mcdm} represents the median of the weighted MCDM portfolios
(w is one of the weight methods = CILOS, CRITIC, MEREC, SECA)
($mcdm$ is one of the 17 MCDM techniques)

4.6. Correlation among Techniques

Correlation coefficients among the returns of techniques are presented as a heatmap in Figure 8 (*RQ 6: What are the correlation coefficients of the returns of the portfolios created with MCDM techniques?*) There are 5 rows and columns for each technique indicating returns with equal weights, CILOS, CRITIC, MEREC, and SECA weights, respectively. CODAS and MOORA techniques have small correlation coefficients with other techniques. The top and bottom 10 technique combinations and their average correlation calculated with the help of the equation are presented in Table 16. The MOOSRA technique was able to produce higher correlation coefficients.

Table 16. Average correlation coefficients of returns.

MCDM Technique	Weight Technique	Average Correlation
MOOSRA	MEREC	0.92976
MOOSRA	CILOS	0.92606
MOOSRA	SECA	0.92509
MOOSRA	EQ	0.92480
ARAS	MEREC	0.92411
MOOSRA	CRITIC	0.92152
COPRAS	CRITIC	0.92087
EDAS	CRITIC	0.91791
EDAS	MEREC	0.91764
COPRAS	CILOS	0.91757
...	...	...
SAW	CRITIC	0.83605
MOORA	MEREC	0.82695
CODAS	CILOS	0.81229
CODAS	CRITIC	0.81187
CODAS	EQ	0.80900
CODAS	MEREC	0.79679
MOORA	SECA	0.72448
MOORA	EQ	0.72258
MOORA	CRITIC	0.71290
MOORA	CILOS	0.70743

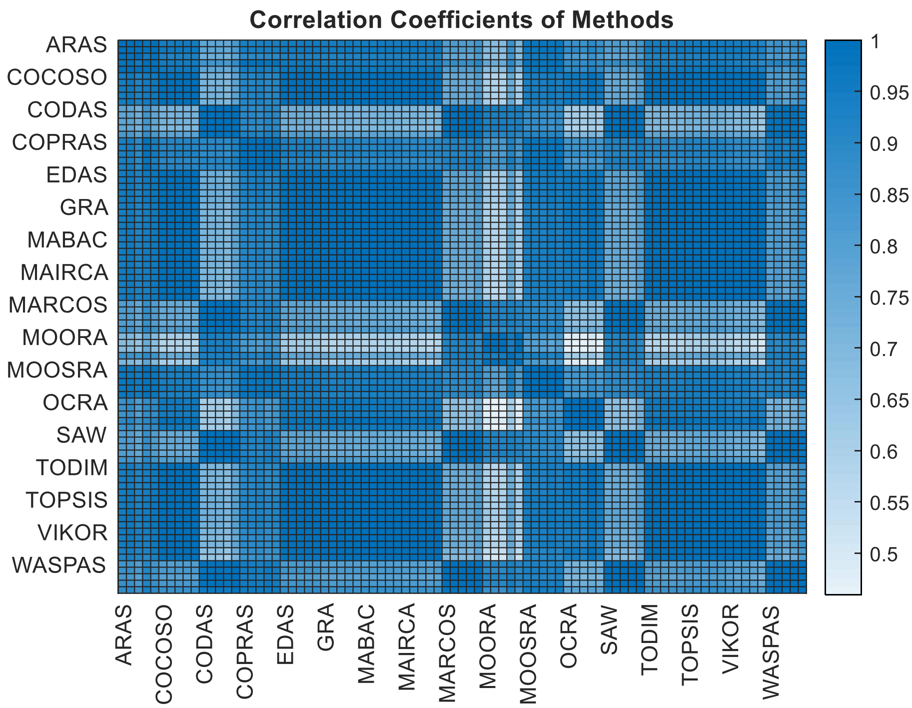


Figure 8. Correlation coefficients among techniques

4.7. Performance Measurement

The return of the techniques was compared with the theoretical best return. Performance measurement was carried out with MAPE, MSPE, and RMSPE values. The performance metrics of the 15 models with the lowest MAPE values (best performing) and the SP500 and MV models are listed in Table 17.

According to Table 17, the MCDM technique that produced the closest results to the theoretical best was MOORA, where CILOS determined the criterion weights. However, no MCDM model has been able to outperform the MV model.

Table 17. Performance measurement results.

Method	Weight	MAPE*	MSPE	RMSPE
		$\frac{100}{n}\sum_{k=1}^{z}\left\lvert\frac{e_k}{t_k}\right\rvert$	$\frac{100}{n}\sum_{k=1}^{z}\left\lvert\frac{e_k}{t_k}\right\rvert^2$	$\sqrt{\frac{100}{n}\sum_{k=1}^{z}\left\lvert\frac{e_k}{t_k}\right\rvert^2}$
MOORA	CILOS	0.534909	0.031644	0.177888
MOORA	CRITIC	0.534909	0.031644	0.177888
MABAC	MEREC	2.706524	0.607484	0.779413
COCOSO	MEREC	3.153058	0.802829	0.896007
MABAC	EQ	5.331937	3.068918	1.751833
MABAC	SECA	5.359261	3.132539	1.769898
GRA	MEREC	5.539326	2.807246	1.675484
MABAC	CRITIC	5.729293	3.659542	1.912993
TODIM	SECA	5.768634	3.005054	1.733509
COCOSO	SECA	6.121478	4.494593	2.120046
COCOSO	EQ	6.12351	4.452624	2.110124
MABAC	CILOS	6.230026	4.944004	2.223512
TOPSIS	CRITIC	6.52587	3.717531	1.92809
COCOSO	CRITIC	7.042785	5.996119	2.448697
COCOSO	CILOS	7.110823	6.920949	2.63077
	MV Model	0.498091	0.026368	0.162381
	S&P 500	101.0263	2539.648	50.39492

$e_k = p_k - t_k$;
p_k : Return of MCDM model,
t_k : Return of the theoretical best model,
z : number of the portfolio.

4.8. Financial Performance Results

The financial performance of the portfolios created with the models was also examined. At the start of the test set, $1000 is invested in the stocks at the rate of determining weights. The amount of the portfolio at the end of the test set was calculated. When $1000 was invested equally in S&P 500 stocks on January 5, 2021, the portfolio value on December 30, 2021 was $1,301.6. The model that showed the best financial performance was the model in which the MEREC technique determined the weights of the criteria and the CODAS technique determined the weights of the stocks. At the end of the period, the portfolio value was $1,501.6.

The model with the lowest financial performance is where equal weights are assigned to the criteria and the COPRAS technique determines the scores of the stocks. The portfolio value created by the model at the end of the period was $1,289.7. The changes in the value of the portfolio are presented in Figure 9.

Five weight sets (including equal weights) and 17 MCDM techniques were used in the study. The average return of portfolios created with the 17x5 = 85 model was $1,339, higher than the SP500's return at the end of the period.

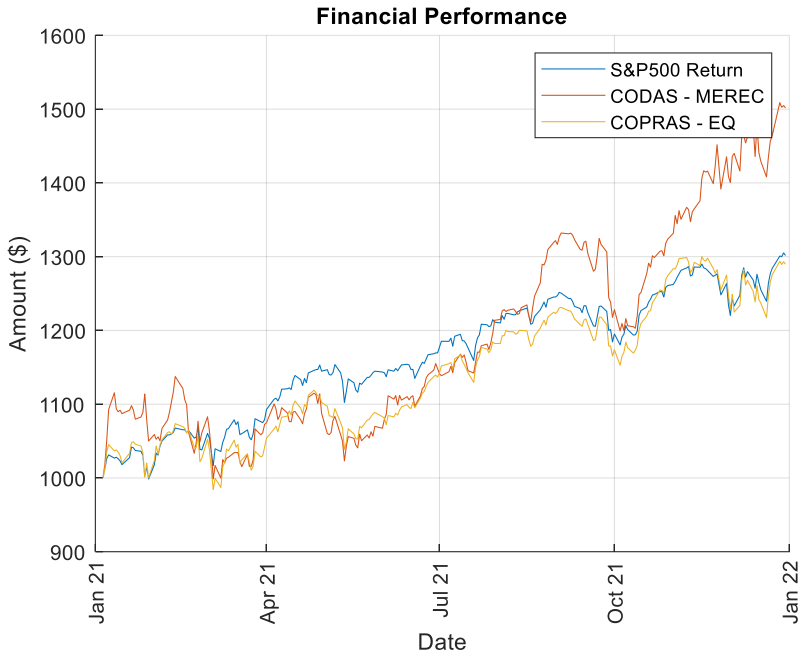


Figure 9. Financial performance of selected models

4.9. Stocks with Highest Scores

In 85 models, the weights of the stocks are combined, and each stock's average weight value is calculated. These values are listed in order from largest to smallest, and the stocks with the highest average weight are listed in Table 18 (*RQ 7: Which stocks has the highest average weight?*). Returns on portfolio optimization and performance evaluation periods are reported in the last two columns, respectively. Stocks with the highest average weight have positive returns, except for SE and TYL in the portfolio optimization period and SHW in the performance evaluation period. It is a preferred situation for the investor if the average returns of the stocks with the highest weight are positive.

Table 18. Top 10 Stocks with the Highest Average Weights.

Average Weight	Ticker	Security	GICS Sector	POSP	POEP	PESP	PEEP	rPO	rPE
0.141	JNPR	Juniper Networks	Information Technology	100.78	132.09	131.18	20.68	30.16	57.65
0.049	STE	Steris	Health Care	83.56	74.26	72.31	78	-13.46	7.87
0.045	TGT	Target	Consumer Discretionary	938.12	705.67	729.77	1070.34	677.91	46.67
0.010	EW	Edwards Lifesciences	Health Care	29.99	175.47	172.24	186.41	474.32	8.23
0.009	TYL	Tyler Technologies	Information Technology	20.7	17.17	17.35	21.04	-16.18	21.27
0.007	PLD	Prologis	Real Estate	39.57	43.12	42.21	56.65	6.67	34.21
0.004	DVN	Devon Energy	Energy	117.69	139.51	135.29	176.09	14.95	30.16
0.004	BF.B	Brown–Forman	Consumer Staples	71.07	136.43	135.76	188.25	91.02	38.66
0.003	HBAN	Huntington Bancshares	Financials	266.12	499.87	491.77	661.32	84.79	34.48

Average Weight	Ticker	Security	GICS Sector	POSP	POEP	PESP	PEEP	rPO	rPE
0.003	SHW	Sherwin-Williams	Materials	101.79	319.12	311.35	282.35	205.87	-9.31

POSP : Portfolio Optimization Starting Price ($)
POEP : Portfolio Optimization Ending Price ($)
PESP : Performance Evaluation Starting Price ($)
PEEP : Performance Evaluation Ending Price ($)
rPO : Return on Portfolio Optimization Period (%)
rPE : Return on Performance Evaluation Period (%)

The correlation coefficients between the return and price values of the ten stocks with the highest average weight, both in the portfolio optimization process and the portfolio performance evaluation process, are shown as a heatmap in Figure 10. In the performance evaluation dataset, the price correlations turned out to be slightly higher than in the portfolio optimization dataset. Similarly, return correlation values in the performance evaluation dataset are slightly higher than in the portfolio optimization dataset.

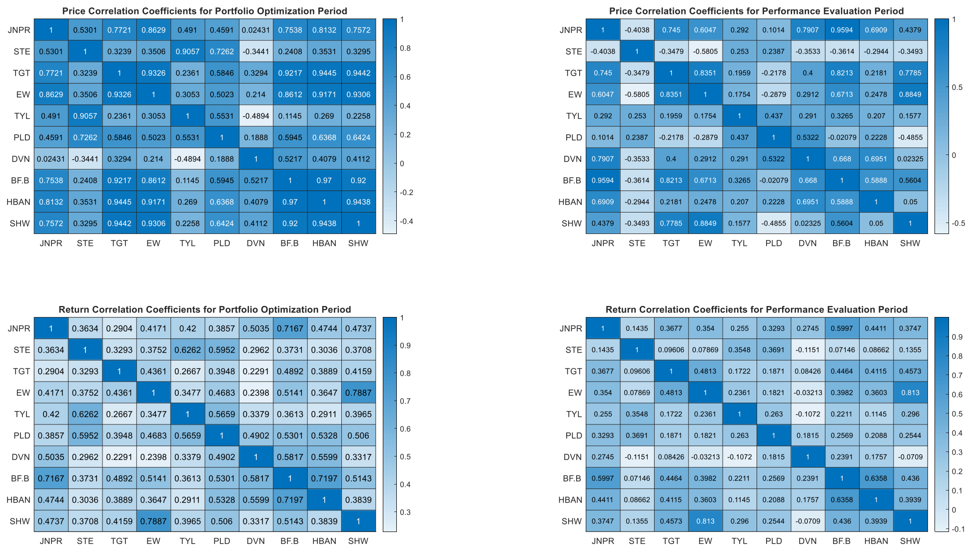


Figure 10. Price and return correlations of top 10 stocks.

5 sets of weights (equal weights, CILOS, CRITIC, MEREC, and SECA) were calculated to be used with 17 different MCDM techniques. When run with these weight sets, five different weights are calculated for the stocks in each MCDM. The average of these weights has been calculated. For example, in the MOORA technique, the average weight value of the stocks in the consumer discretionary industry was calculated as 0.2959. The distribution of these averages according to the sectors is presented in the figure. The weights assigned to industries in the CODAS, MARCOS, MOORA, and SAW techniques differ markedly. The difference between industries is not as high as in the four techniques mentioned in other techniques.

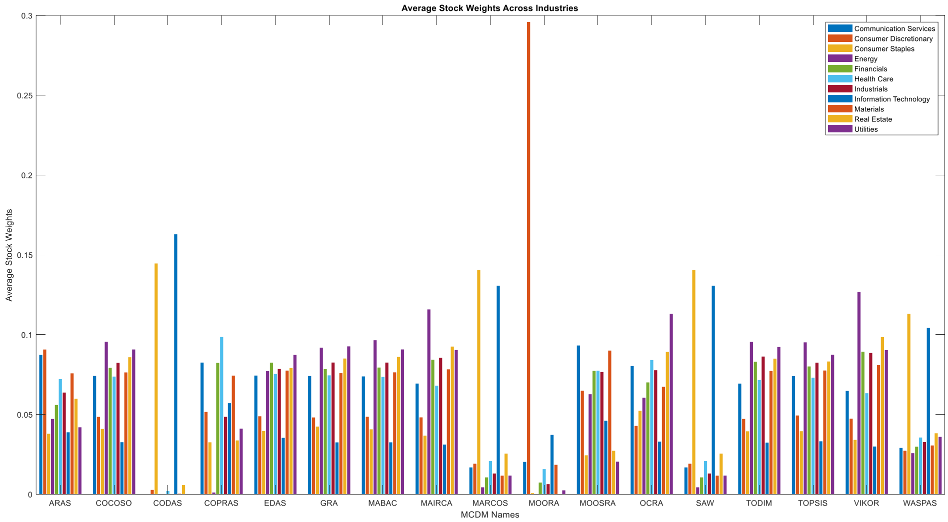


Figure 11. Average stock weights across industries.

The portfolio in best harmony with the theoretical best return values was the portfolio in which the weights of the criteria were determined with CILOS, and the weights of the stocks were determined with MOORA (Table 17). The weights of the sectors in the portfolio created according to this technique are shown in Figure 12 with a pie chart (*RQ 8: What is the weight distribution of industries in the best portfolio (or any portfolio)?*). The industry with the highest weight in this portfolio was the Information Technology industry with 14%. The point to be considered here is that the sector distribution in Figure 12 and the sector weights in Figure 11 will be different. While the distribution of only one portfolio based on the sectors is given in Figure 12, the weights of five different portfolios are included in the Figure 11.

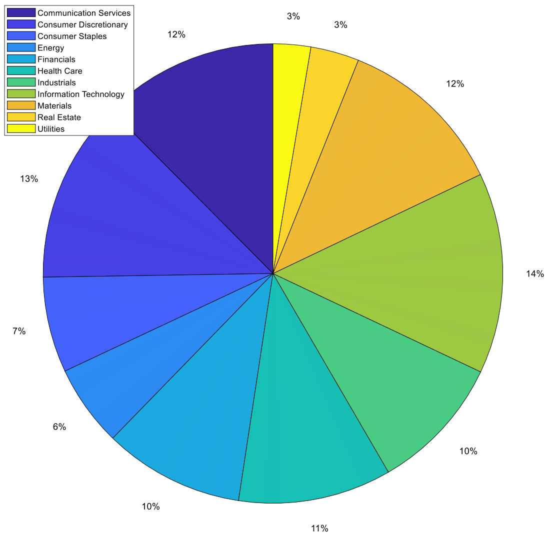


Figure 12. The industrial weight distribution of the best portfolio (CILOS-MOORA)

4.10. Experiments with Different Periods

The experiment was repeated with datasets covering different periods.

4.10.1. Experiment 1 (January 11, 2019 – January 05, 2021)

In this experiment, 250 days between January 11, 2019, and January 8, 2020, were used for portfolio optimization, and 250 days between January 9, 2020, and January 5, 2021, were used to test the portfolios' performance. A performance comparison with the theoretical best set is presented in Table 19. As in the previous data set, the MV model has succeeded in producing the rate of return in best harmony with the theoretical best model. On January 9, 2020, it reaches $1,177.5, and on January 5, 2021, it reaches $1,177.5. The model with the best financial performance in this period was the one in which SECA determined the weights, and the MOORA technique determined the scores of the stocks. The $1,000 to be invested in this model reached $7,636.1 after 250 days. The model with the worst financial performance in this period was the model in which SECA determined the weights of the criteria, and the CODAS technique determined the scores of the stocks. After 250 days, the $1,000 portfolio created with these weights is worth $1,030.7. The average value of 85 models at the end of the period was calculated as $1,240.1.

Table 19. Performance Measurement Results (January 11, 2019 – January 05, 2021).

Method	Weight	MAPE	MSPE	RMSPE
MOORA	EQ	0.509158	0.030874	0.175711
MOORA	CILOS	0.509158	0.030874	0.175711
COCOSO	CILOS	2.548074	2.018169	1.420623
COCOSO	MEREC	2.565769	0.833146	0.912769
MABAC	CILOS	2.782055	1.877622	1.370263
COCOSO	SECA	2.822000	1.934425	1.390836
COCOSO	EQ	2.866778	1.982592	1.408046
MABAC	MEREC	3.105209	1.069271	1.034056
MABAC	SECA	3.155448	1.901175	1.378831
MABAC	EQ	3.193452	1.939244	1.392568
COCOSO	CRITIC	3.507706	2.833204	1.683212
MABAC	CRITIC	3.629398	2.497097	1.580220
GRA	MEREC	4.537579	3.222231	1.795057
GRA	SECA	4.570835	6.691955	2.586881
GRA	EQ	4.595009	6.738409	2.595844
	MV Performance	0.448359	0.025327	0.159144
	S & P Performance	66.07595	426.0556	20.64111

The heatmap of the correlation coefficients between the returns of the models is presented in Figure 13. Average correlation coefficients calculated with the help of the equation are presented in Table 20. Returns with the MOORA technique produced a lower correlation with other returns.

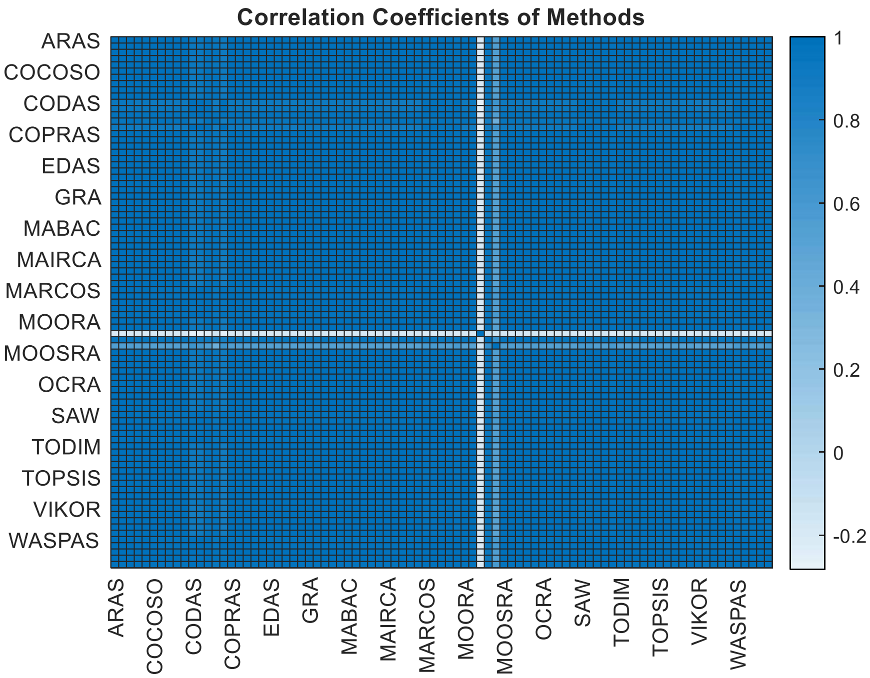


Figure 13. Correlation coefficients of returns

Table 20. Average correlation coefficients of returns.

MCDM Technique	Weight Technique	Average Correlation
COPRAS	CRITIC	0.97249
ARAS	CILOS	0.97245
ARAS	SECA	0.97241
ARAS	CRITIC	0.97241
ARAS	EQ	0.97239
COPRAS	CILOS	0.97224
COPRAS	EQ	0.97215
COPRAS	SECA	0.97213
EDAS	CRITIC	0.97188
OCRA	CRITIC	0.97187
...	...	...
MARCOS	SECA	0.95632
SAW	SECA	0.95632
CODAS	CRITIC	0.95342
CODAS	MEREC	0.93758
MOORA	MEREC	0.92681
CODAS	CILOS	0.91684
CODAS	EQ	0.90375
CODAS	SECA	0.89787
MOORA	SECA	0.52938
MOORA	CRITIC	-0.19156

The weights of the industries according to 17 criteria are presented in Figure 14. It can be stated that the shares of industries in the portfolio are relatively stable compared to the previous period. The highest weights are assigned to the materials and consumer discretionary sectors in the MOORA technique.

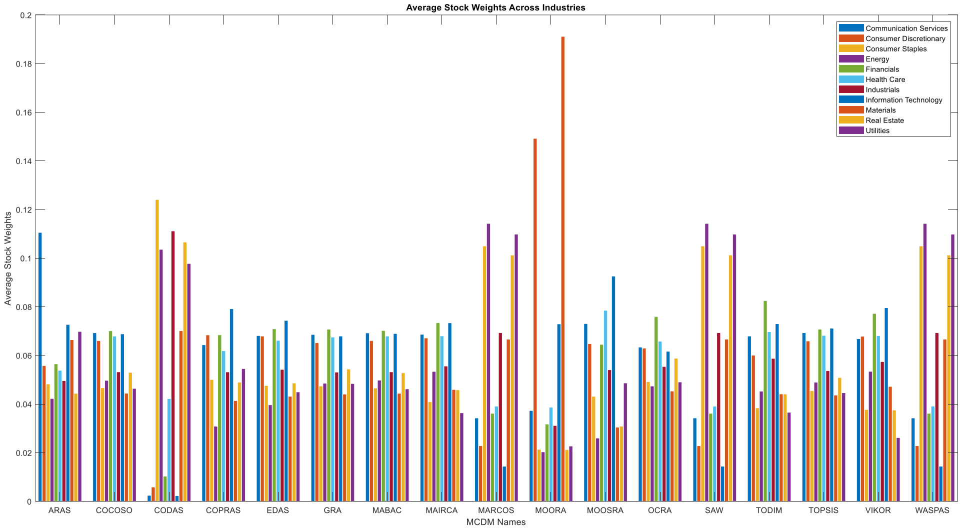


Figure 14. Average stock weights across industries

4.10.2. Experiment 2 (July 13, 2018 – July 08, 2020)

This experiment used 250 trading days between July 13, 2018, and July 11, 2019, to determine the portfolio weights. The following 250 trading days were used to test the portfolio performance between July 12, 2019 and July 08, 2020. Performance comparison with the theoretical best set is presented in Table 21. Tuesday, July 12, 2019: On July 12, 2019, a $1,000 investment in equal weight S&P 500 securities grows $1054.6 after 250 trading days on July 8, 2020. 85 models created, the best financial performance was found in the model where the criteria were equally weighted and the MOORA technique determined the weights of the stocks. The value of this portfolio was $1,842.6 at the end of the period. CRITIC was used to figure out how much weight each criterion should have, and the OCRA method was used to figure out how well each stock did. The value of this portfolio was $1,0422.2 at the end of the period. The average value of portfolios made with 85 different models at the end of the time period was $1,108.5.

Table 21. Performance measurement results (January 11, 2019 – January 05, 2021).

Method	Weight	MAPE	MSPE	RMSPE
MOORA	CILOS	0.534909	0.031644	0.177888
MOORA	CRITIC	0.534909	0.031644	0.177888
MABAC	MEREC	2.706524	0.607484	0.779413
COCOSO	MEREC	3.153058	0.802829	0.896007
MABAC	SECA	5.328738	3.066025	1.751007
MABAC	EQ	5.331937	3.068918	1.751833
GRA	MEREC	5.539326	2.807246	1.675484
MABAC	CRITIC	5.729293	3.659542	1.912993
COCOSO	SECA	6.113243	4.442141	2.107639
COCOSO	EQ	6.123510	4.452624	2.110124
MABAC	CILOS	6.230026	4.944004	2.223512
TOPSIS	CRITIC	6.525870	3.717531	1.928090
COCOSO	CRITIC	7.042785	5.996119	2.448697
COCOSO	CILOS	7.110823	6.920949	2.630770
VIKOR	CRITIC	8.012394	4.387633	2.094668
	MV Performance	0.498091	0.026368	0.162381
	S & P Performance	101.0263	2539.648	50.39492

The heatmap of the correlation coefficients between the returns of the models is presented in Figure 15. Average correlation coefficients calculated with the help of the equation are presented in Table 22. Returns with the MOORA technique produced a lower correlation with other returns.

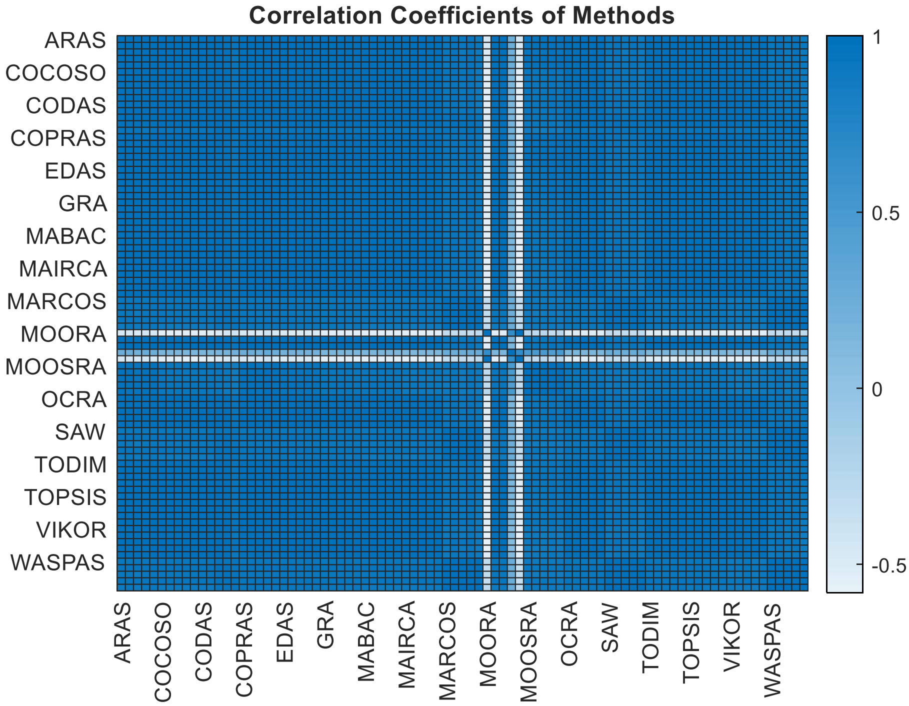


Figure 15. Correlation coefficients of returns

Table 22. Average correlation coefficients of returns.

MCDM Technique	Weight Technique	Average Correlation
EDAS	CILOS	0.93501
EDAS	EQ	0.93492
EDAS	SECA	0.93491
EDAS	CRITIC	0.93480
EDAS	MEREC	0.93446
TODIM	CRITIC	0.93389
COPRAS	CILOS	0.93366
COPRAS	CRITIC	0.93356
COPRAS	EQ	0.93343
COPRAS	SECA	0.93341
...	...	...
SAW	CRITIC	0.88582
MOOSRA	CRITIC	0.88533
MOOSRA	SECA	0.88108
MOOSRA	EQ	0.88058
WASPAS	SECA	0.88050
SAW	SECA	0.88050
MARCOS	SECA	0.88050
MOORA	MEREC	0.21272
MOORA	SECA	-0.43926
MOORA	EQ	-0.43930

The weights of the sectors according to 17 criteria are presented in Figure 16. It is clear from the figure that the different weights are assigned to the industries. In some techniques, there can't be a big difference in the weights given to different industries. In the MOORA technique, it can be seen that the information technology industry gets the most weight.

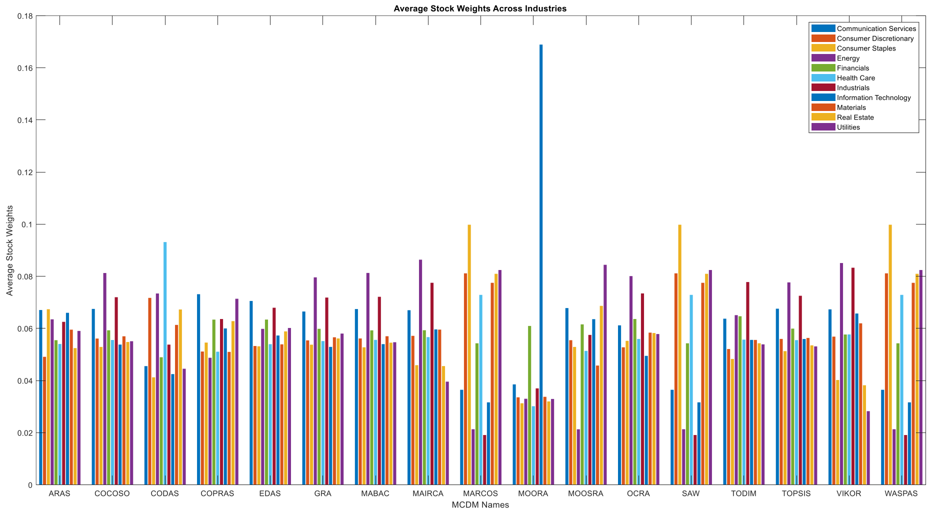


Figure 16. Average stock weights across industries

5. Result and Conclusion

In this study, asset allocation is performed with MCDM techniques, and the effect of weights of criteria on performance has been examined. MCDM allows considering portfolio manager preferences and all relevant criteria for portfolio selection. As the weights of the criteria are determined objectively, the allocation of the asset is made without any normative restrictions. Return values were calculated by using the historical closing prices of S&P500 securities. 297 criteria, both sectional and cumulative types, are calculated based on the return values. The monetary amount and the statistical performance indicators are utilized as performance measures. Four different MCDM techniques determined the weights of the criteria, and it was proved with the help of hypothesis tests that these weights provide a performance increase. This paper also utilizes an index called theoretical-best return to measure the performance of any portfolio. The theoretical-best return is to optimize the weights of the assets in the portfolio so that the return will be maximized in the portfolio holding period. However, these weights may not be assigned by the investor at the beginning of the portfolio holding period since the weights are optimized by using the future return values.

The cumulative criteria were determined to have higher weights in three of the four different MCDM techniques considering the input variables. Thus, it may be helpful to focus on cumulative criteria in further studies. Neither group of mean, standard deviation, or correlation coefficients differed evidently from the other. Although different techniques have assigned high weights to different groups, there is no consensus among the four techniques. Results indicate that for the first experiment, the MCDM technique that produced the closest results to the theoretical best return was MOORA, where CILOS determined the criterion weights. 14% of this portfolio consists of information technology firms. The performance of the portfolios was compared with the theoretical best model and it was determined that no MCDM model produced better results than the MV model. The most compatible results with the theoretical best model belong to the MV model. The best financial performance belongs to the CODAS technique where the criteria weights are determined with the MEREC technique. It accomplished to produce a 50 % return in 2021. In other words, a single technique was not able to produce the best results on both statistical and financial performance metrics. The minimum correlation coefficient among the techniques for the first experiment is calculated as 0.7074. This finding indicates that the techniques have returns that are positively correlated with each other.

The experiments were repeated using the data set of different periods. As a result, it can be stated that portfolios in which the criteria weights are determined with MCDM techniques and the assets are allocated with the help of MCDM techniques can provide higher returns than the market returns.

It is an essential advantage that expert knowledge is not used in the study. When using techniques such as AHP, BWM, FUCOM, DEMATEL, and MACBETH, experts need to make a pairwise comparison between the criteria. However, in this study, techniques that do not require expert knowledge have been selected while determining the weights of the criteria. Future studies can predict with the help of artificial intelligence techniques which weight and method combination will perform best. In this study, it is accepted that only long positions can be taken in the market, and short positions are not allowed. Transaction costs and dividends are omitted from the calculations. The proposed system can be evaluated in different countries' market data in future studies. Some constraints may be considered. For example, the minimum weight of a stock in a portfolio can be considered a constraint. Commodity prices or exchange rates can also be added to the initial asset pool. In this study, the number of assets in the portfolio is fixed at 500 stocks. In further studies, it can be examined how the system's performance will change when fewer stocks are used. In this study, 297 criteria were used in the decision matrix. Studies can be carried out to reduce or transform the number of the criteria so that calculations can be repeated with a more compact decision matrix.

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