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Newsboy problem with birandom demand

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Abstract: *Estimation of accurate product demand in a single period inventory model (SPIM) is an essential prerequisite for successfully managing the supply chain in large and medium merchandise. Managers/ decision makers (DMs) often find it difficult to forecast the exact inventory level of a product due to complex market situations and its volatility caused by several factors like customers uncertain behavior, natural disasters, and uncertain demand information. In order to make fruitful decisions under such complicated environment, managers seek applicable models that can be implemented in profit maximization problems. Many authors studied SPIM (also known as newsboy problem) considering the demand as a normal random variable with fixed mean and variance. But for more practical situations the mean demand also varies time to time yielding two-folded randomness in demand distribution. Thus, it becomes more difficult for DMs to apprehend the actual demand having two-folded random/birandom distribution. A blend of birandom theory and newsboy model has been employed to propose birandom newsboy model (BNM) in this research to find out the optimal order quantity as well as maximize the expected profit. The practicality of the projected BNM is illustrated by a numerical example followed by a real case study of SPIM. The results will help DMs to know how much they should order in order to maximize the expected profit and avoid potential loss from excess ordering. Finally, the BNM will enhance the ability of the managers to keep parity of product demand and supply satisfying customers' needs effectively under uncertain environment.*

Key words: *Newsboy problem, Uncertain variable, Birandom variable, Expectation.*

1. Introduction

The classical newsboy problem (CNP) aims at determining the optimal order quantity of products, which minimize the expected total cost and / or maximize the

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expected profit in a SPIM. Thus, the characteristics of a SPIM become more complicated due to complex market situations and its volatility caused by several factors like customers uncertain behavior. In response to handle such problems, the CNP is extended in many directions and several researchers have proposed modified versions of CNP. Some of them made proper and thoughtful use of probability theory to solve newsboy problems where product demands follow Poisson and Normal distribution (Hadley and Whitin, 1963), Weibull distribution (Tadikamalla, 1978), Erlang distribution (Mahoney and Sivazlian, 1980), compound Poisson distribution (Dominey and Hill, 2004). Gallego and Moon (1993, 1994) analyzed distribution-free newsboy problems where DMs have no idea about the demand distribution. The only information they have are mean and variance of demand. Their newsboy models can be used as strategic tools in deciding the stock of products that have a limited selling period.

This paragraph is dedicated to articulate the recent developments of SPIM and find the literature gap. Agarwal and Seshadri (2000) worked in CNP where they assumed the demand distribution as a function of selling price and the objective of the risk-averse retailers. To maximize DMs expected utility they presented two models for comparing the risk-neutral retailers (who charge a higher price for less order) with a risk-averse retailer (who charge a low price). The distribution-free newsboy problem under the worst-case and best-case scenario was revealed by Kamburowski (2014). Further, Kamburowski (2015) studied a newsboy problem where the distribution of the random variable is only known when to be non-skewed with given support, mean and variance. For the distribution-free newsboy problem, Gler (2014) extended the model developed by Lee and Hsu (2011). Here, the authors showed the expected profit increases with a proper advertisement policy while an unorganized advertising policy can have its backfire effect or make a very small improvement of the optimal profit value. Ding (2013) proposed a chance constraint multi-product newsboy problem with uncertain demand and uncertain storage capacity. Abdel-Aal et al. (2017) studied a multi-product newsboy problem assuming the service level as a constraint to offer the DMs to select the market to serve. Watt and Vzquez (2017) considered newsboy problem under two new assumptions. First, they assumed that the wholesaler is an expert who sets the wholesale price optimally and a newsboy can return the unsold item with some salvage value. In the second one, the salvage value acts as a standard insurance demand. Sun and Guo (2017) built a newsboy model with fuzzy random demand based on fuzzy random expected value model. Vipin and Amit (2017) proposed a loss aversion SPIM under alternative option and proved the rationality of the decision maker to predict the order quantity by imposing loss aversion in the newsboy model with the change of selling price and purchase cost factors. Additionally, they showed the models based on utility functions perform better in forecasting the rational behavior due to loss aversion. Natarajan et al. (2017) allowed asymmetry and ambiguity in newsvendor models. The effects of decision makers emergency order in SPIM are discussed and analyzed by Pando et al. (2013) and Zhang et al. (2017). Zhang et al. (2017) compared two ways to treat the excess demand and came up with the better one.

In the aforementioned works, the product demand is assumed to be either normally distributed with $N(\mu, \sigma^2)$ or exponentially distributed with constant mean (λ) or somewhere distribution free. But the DMs face difficulties to forecast the exact demands of products in many practical problems. The demand distribution changes dynamically from time to time, which yields randomness in the mean demand. For example, the demand (D) is normally distributed with $D \sim N(\mu\,;\,400)$ where $\mu \sim U(3000;\,4000)$ or $D \sim N(\mu\,;\,400)$ with $\mu \sim exp(0.0003)$. However, in newsboy problems, it is largely appreciated to consider demand variable having the standard normal distribution. From the probabilistic viewpoint and the above arguments, it would be more realistic to assume the values of μ and σ^2 should also be treated as random variables. For such cases, it is more convincing and practical to represent the product demand as birandom variable effectively captures two-folded randomness. Traditional probabilistic approaches cannot handle such complicated real world problems. In response, Peng and Liu (2007) developed a birandom theory to tackle such problems. Zhaojun and Liu (2013) showed the common formula on birandom variable and further, several researchers used this theory to solve inventory problems in the birandom environment. In the recent years, birandom theory is widely accepted as the mathematical language of uncertainty. Some more notable extensions and applications of newsboy problem can be found in the recent literature (Abdel-Malek and Otegbeye 2013; Chen and Ho 2013).

From its inception until today, birandom theory has progressed and been applied to different areas. Xu and Zhou (2009) introduced a class of multiple objective decision making problems using birandom variables and by transforming the birandom uncertain problem into its crisp equivalent form through expected value operator and used it in the flow shop scheduling problem. A Portfolio selection problem is analysed by Yan (2009) assumed the security returns as birandom variables. Xu and Ding (2011) developed the general chance constrained multi objective linear programming model with birandom parameters for solving a vendor selection problem. They presented a crisp equivalent model for a special case and gave a traditional method to solve the crisp model. Wang et al. (2012) established a class of job search problem with birandom variables, where the job searcher examined job offers from a finite set jobs having equivalent probability. A multi-mode resource constrained project scheduling problem (Zhang and Xu, 2013) of drilling grounding construction projects considering the uncertain parameters as birandom variables. In the earlier year, Xu et al. (2012) used birandom theory to develop the nonlinear multi objective bi-level models for finding the minimum cost in a network flow problem dealing a large scale construction project. Tavana et al. (2013) measured the efficiencies of decision making units after developing a data envelopment analysis (DEA) model with birandom input and output data. Nevertheless, many more real life applications can be found in the following literature: a multi-objective birandom inventory problem (Tao and Xu, 2013), a birandom multi-objective scheduling problem (Xu et al., 2013) in ship transportation, optimal portfolio selection with birandom returns(Cao and Shan, 2013), a modified genetic algorithm (Maity et al., 2015), chance-constrained programming model for municipal waste management with birandom variables (Zhou et al., 2015), and the CCUS (carbon capture, utilization, and storage) management system in birandom environment (Wang et al., 2017).

To the best of our knowledge, no researcher has investigated the SPIM with birandom demand till date. With these considerations, we discuss an inventory model with single period considering the demand for the birandom variable. We solve a real problem using the presented model in searching the optimum order quantity for maximum profit by using the expected value model. The principal aim of this paper is to deliver basic knowledge and suggest precise results of a complex inventory practical problem for the management.

The remaining part of our paper is presented in the following way. Section 2 presents some basic knowledge of birandom variable and related theorems. Section 3 introduces the birandom simulation for finding the expected value of the birandom variable. In section 4, we provide a mathematical model for newsboy problem with birandom demand. A numerical example is discussed in section 5. To validate the applicability of the proposed model we discuss a real case study in section 6. Finally, the conclusion and future research directions are presented in section 7.

2. Preliminaries

In this section we discuss the basic notations of birandom variables.

2.1. Birandom Variable

Roughly speaking a birandom variable is a random variable of a random variable, i.e., a function defined from a probability space to a collection of random variables is said to be a birandom variable. The formal definition of birandom variable and related theorems are defined in the following way.

Definition 1 (Peng and Liu, 2007). A birandom variable ξ is a mapping from a probability space (Ω, A, Pr) to a collection S of a random variable such that for any Borel subset B of the real line $\Re$ the induced function $Pr\{\xi(\omega) \in B\}$ is a measurable function with respect to ω.

For each given Borel subset B of the real line $\Re$, the function $Pr\{\xi(\omega) \in B\}$ is a random variable defined on the probability space (ω, A, Pr).

Lemma 1 (Peng and Liu 2007). Let an n dimensional birandom vector $\xi = (\xi_1, \xi_2, \dots, \xi_n)$ and $f\colon \Re^n \to \Re$ be a measurable function. Then $f(\xi)$ is a birandom variable.

Let two probability spaces $(\Omega_1, A_1, \mathrm{Pr}_1)$ and $(\Omega_2, A_2, \mathrm{Pr}_2)$, ξ_1 and ξ_2 be two birandom variables respectively taken from that probability spaces. Then $\xi = \xi_1 + \xi_2$ is a birandom variable on $(\Omega_1 \times \Omega_2, A_1 \times A_2, \mathrm{Pr}_1 \times \mathrm{Pr}_2)$ defined by

$$\xi(\omega_1,\omega_2) = \xi_1(\omega_1) + \xi_2(\omega_2), \forall (\omega_1,\omega_2) \in \Omega_1 \times \Omega_2$$

Widely, for the n-tuple operation on birandom variables defined as follows. Let a Borel measurable function defined as $f\colon \Re^n \to \Re$ and ξ_i be birandom variable defined on $(\Omega_i, A_i, \mathrm{Pr_i}), i = 1,2, \dots, n$ respectively. Then $\xi = f(\xi_1, \xi_2, \dots, \xi_n)$ is birandom variable on $(\Omega_1 \times \Omega_2 \times \dots \times \Omega_n, A_1 \times A_2 \times \dots \times A_n, \mathrm{Pr}_1 \times \mathrm{Pr}_2 \times \dots \times \mathrm{Pr_n})$, defined by

$$\xi(\omega_1,\omega_2,\dots,\omega_n) = f\left(\xi_1(\omega_1),\xi_2(\omega_2),\dots,\xi_n(\omega_n)\right)$$
$$\forall (\omega_1,\omega_2,\dots,\omega_n) \in \Omega_1 \times \Omega_2 \times \dots \times \Omega_n$$

2.2. Expected Value of birandom variables

We can transform the complex uncertain problems into their equivalent crisp models, which will be easier to solve. Generally, expected value operator is applied to transform the birandom problem into its deterministic value model for calculating the objective functional value. First, we present the definition of the expected value operator of a birandom variable and then the expected value model of SPIM. The effective tool of an uncertain variable is expectation, which is applied in a different field of applications. Therefore, the idea of expected value of birandom variable is useful.

The expected value operator of birandom variable is defined as follows.

Definition 2 (Peng and Liu, 2007). Let ξ be a birandom variable defined on the probability (Ω, A, Pr). Then the expected value of birandom variable ξ is defined as

$$E\left[\xi\right]=\int_{0}^{\infty}\Pr\left\{\omega\in\Omega|E\left[\xi\left(\omega\right)\right]\geq t\right\}dt-\int_{-\infty}^{0}\Pr\left\{\omega\in\Omega|E\left[\xi\left(\omega\right)\right]\leq t\right\}dt \tag{1}$$

Provided that at least one of the above two integrals is finite.

Lemma 2 (Peng and Liu, 2007). Let ξ be a birandom variable defined on the probability (ω, A, Pr). If the expected value $E[\xi(\omega)]$ of the random variable $\xi(\omega)$ is finite for each ω, then $E[\xi(\omega)]$ is a random variable on (ω, A, Pr).

Lemma 3 (Peng and Liu, 2007). Let us consider two birandom variable ξ and η with finite expected value, then for any two real numbers a and b, we have

$$E\left[a\xi+b\eta\right]=aE\left[\xi\right]+bE\left[\eta\right]$$

We know that a function from a probability space (Ω, A, Pr) to a collection of random variables is called a birandom variable, from the definition of birandom variable. Birandom variables are two type. They are either discrete birandom variable or continuous birandom variable. Expectation theory of birandom variables will be discussed in the current subsection.

Definition 3 (Xu et al., 2009). For a birandom variable ξ, in the probability space (Ω, A, Pr), if $\xi(\omega)$ is a random variable with a continuous distribution function when $\omega \in \Omega$ and its expected value $E[\xi(\omega)]$ is a birandom variable. Then we call ξ continuous birandom variable.

Definition 4 (Xu et al., 2009). Suppose ξ is a birandom variable, then $\xi(\omega)$ is a random variable. If $f(x,\xi)$ be the density function of $\xi(\omega)$, and

$$E\left[\xi\left(\omega\right)\right]=\int_{x\in\Omega}xf\left(x\right)dx \tag{2}$$

Then the density function of birandom variable ξ is $f(x,\xi)$.

Definition 5 (Xu et al., 2009). For the continuous birandom variable ξ, if its density function is $f(x)$, we can define the expected value of ξ as follows

$$E\left[\xi\right]=\int_{0}^{\infty}Pr\left\{\int_{x\in\Omega}xf\left(x\right)\geq r\right\}dr-\int_{-\infty}^{0}Pr\left\{\int_{x\in\Omega}xf\left(x\right)\leq r\right\}dr \tag{3}$$

Definition 6 (Xu et al., 2009). If the density function of a birandom variable ξ is $f(x,\xi)$ and $g(x)$ is a continuous function. Then expectation for the birandom variable $g(\xi)$ is defined as

$$E\left[g\left(\xi\right)\right]=\int_{0}^{\infty}Pr\left\{\int_{x\in\Omega}g\left(x\right)f\left(x\right)\geq r\right\}dr-\int_{-\infty}^{0}Pr\left\{\int_{x\in\Omega}g\left(x\right)f\left(x\right)\leq r\right\}dr \tag{4}$$

Theorem 1 (Xu et al., 2009). Let $f(x,\xi)$ is the density function of the birandom variable ξ. Then the expected value of ξ exists if only if the expected value of random variable $\xi(\omega)$ exists.

Theorem 2 (Xu et al., 2009). The expectation of a birandom variable. $\xi\sim N(\mu,\sigma^2)$, where, $\mu\sim \mathrm{U(a,b)}$ is $\frac{a+b}{2}$.

Theorem 3. The expectation of a birandom variable $\xi \sim N(\mu, \sigma^2)$ where $\mu \sim \exp(\lambda)$ is $\frac{1}{\lambda}$.

Proof: By definition (2), we know

$$E[\xi] = \int_0^\infty \Pr\left\{\omega \in \Omega | E\left[\xi(\omega)\right] \geq t\right\} dt - \int_{-\infty}^0 \Pr\left\{\omega \in \Omega | E\left[\xi(\omega)\right] \leq t\right\} dt$$

Since $\mu \sim \exp(\lambda)$, and obviously $E[\xi(\omega)] = \mu$, by definition 4 and 5, the above function can be transformed as follows,

$$E[\xi] = \int_0^\infty \Pr\{\mu \geq t\} dt - \int_{-\infty}^0 \Pr\{\mu \leq t\} dt$$

Since, $\mu \sim \exp(\lambda)$, and we know that the density function and the distribution function of exponential distribution are as follows,

$$f(x) = \lambda e^{-\lambda x}, x \in [0, \infty) \; and$$

$$F(x) = 1 - e^{-\lambda x}, \; x \in [0, \infty).$$

According to the definition of the distribution function we can obtain the following two functions from the distribution function

$$\Pr(\mu \leq x) = 1 - e^{-\lambda x}, \;\; 0 \leq x < \infty \; and$$

$$\Pr(\mu \geq x) = e^{-\lambda x}, 0 \leq x < \infty$$

Obviously, $\int_{-\infty}^0 \Pr\{\mu \leq t\} \, dt = 0$

Therefore

$$E[\xi] = \int_0^\infty \Pr\{\mu \geq t\} \, dt = \int_0^\infty e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{\lambda}\right]_0^\infty = \frac{1}{\lambda}.$$

However, it is very hard to accomplish the mathematical expression of expected value for all types of birandom variables. But using birandom simulation, we could calculate the expected value of birandom variables, with the help of Strong Number Law.

3. Birandom Simulation

Let (Ω, A, Pr), be a probability space and a $f: \Re^n \rightarrow \Re$ be a measurable function. Consider that ξ is an $n -$ dimension birandom vector on the given probability space. Now we have to find the expectation $E[f(\xi)]$ of birandom variable. Using stochastic simulation, we can find the expected value for every $\omega \in \Omega$. Here, we have used an algorithm for birandom simulation to find the expectation of $E\left[f\left(\xi(\omega)\right)\right]$, which is defined as follows.

Algorithm:

Step 1. Start
Step 2. Set l=0 and N= Number of iteration
Step 3. Sample ω from Ω according to the probability measure Pr

Step 4. $E[f(\xi(\omega))]$ *is find by the stochastic simulation.*
Step 5. Then $l \leftarrow l + E[f(\xi(\omega))]$
Step 6. Repeat the steps from second to fifth steps N *ttimes.*
Step 7. $E[f(\xi(\omega))] = \frac{l}{n}$.
Step 8. Stop

4. Mathematical Formulation

We are assuming a single period inventory problem with single product. Here all the costs (buying cost and selling cost) are deterministic. Salvage value is taken which is also deterministic. But the demand is birandom variable. The mathematical notation of a birandom newsboy problem is defined as follows:

$\bar{\bar{\xi}}$	:	The demand of market, a birandom variable
x	:	The quantity which to be order, a decision variable
$p = c(1 + m)$	:	Selling price per unit
$s = c(1 - d)$	:	Salvage value per unit
c	:	Purchasing cost per unit
$g\left(x, \bar{\bar{\xi}}\right)$	:	The profit for the order quantity x and demand $\bar{\bar{\xi}}$
μ	:	Expected value of birandom demand $\bar{\bar{\xi}}$
σ^2	:	Variance of the birandom demand $\bar{\bar{\xi}}$
m	:	Mark-up, i.e., return per dollar on unit sold
d	:	Discount rate, i.e., loss per dollar on unit unsold
$x^+ = \max\{x, 0\}$	:	The positive part of x

Then the profit can be expressed as

$$g\left(x, \bar{\bar{\xi}}\right) = p\left[\min\left\{x, \bar{\bar{\xi}}\right\}\right] + s\left(x - \bar{\bar{\xi}}\right)^+ - cx \tag{5}$$

Now $\min\left(x, \bar{\bar{\xi}}\right) = \bar{\bar{\xi}} - \left(\bar{\bar{\xi}} - \mathrm{x}\right)^+$

Where

$$\left(x - \bar{\bar{\xi}}\right)^+ = \left(x - \bar{\bar{\xi}}\right) + \left(\bar{\bar{\xi}} - x\right)^+ \tag{6}$$

$$g\left(x, \bar{\bar{\xi}}\right) = p\bar{\bar{\xi}} + s\left(x - \bar{\bar{\xi}}\right) - cx - (p - s)\left(\bar{\bar{\xi}} - x\right)^+ \tag{7}$$

Since the demand for the product is birandom, the profit function $g\left(x, \bar{\bar{\xi}}\right)$ is also consists of birandom variable. Hence, the expectation criteria is used for handling the birandom variable. Therefore, to find the optimal quantity, the decision maker will maximize the total expected value.

We can write the expected profit as

$$\Pi(x) = (p - s)\mu + (s - c)x - (p - s)E\left(\bar{\bar{\xi}} - x\right)^+ \tag{8}$$

Or using the definition of m and d, as

$$\Pi(x) = c[(m+d)\mu - xd - (m+d)E\left(\bar{\bar{\xi}} - x\right)^{+} \tag{9}$$

The information of $\bar{\bar{\xi}}$ is known. To maximize the profit function, we need the following lemma.

Lemma 4. For given a birandom variable $\bar{\bar{\xi}}$, we have the following inequality,

$$E\left(\bar{\bar{\xi}} - x\right)^{+} \leq \frac{\left[\sigma^2 + (x-\mu)^2\right]^{\frac{1}{2}} - (x-\mu)}{2} \tag{10}$$

Proof: Notice that $\left(\bar{\bar{\xi}} - x\right)^{+} = \frac{\left|\bar{\bar{\xi}} - x\right| + \left(\bar{\bar{\xi}} - x\right)}{2}$

The result follows by taking expectations and by using the Cauchy-Schwarz inequality

$$E\left[\bar{\bar{\xi}} - x\right] \leq \left[E\left|\bar{\bar{\xi}} - x\right|^2\right]^{\frac{1}{2}} = [\sigma^2 + (x-\mu)^2]^{\frac{1}{2}}$$

By using the lemma (4) the equation (9) will be rewritten as

$$\Pi(x) = c\left[(m+d)\mu - xd - (m+d)\frac{\left[\sigma^2 + (x-\mu)^2\right]^{\frac{1}{2}} - (x-\mu)}{2}\right] \tag{11}$$

It is easy to validate that equation no (11) is strictly convex in x. Upon setting the derivative to zero and solving for x we obtain the ordering rule

$$x^* = \mu + \frac{\sigma}{2}\left(\left[\frac{m}{d}\right]^{\frac{1}{2}} - \left[\frac{d}{m}\right]^{\frac{1}{2}}\right) \tag{12}$$

5. Numerical Example

Assume that the unit purchase price of a perishable product is $c = \$40$, the unit selling price is $p = \$60$, and there is no salvage value ($s = 0$). Thus, $m = \frac{p}{c} - 1 = \frac{60}{40} - 1 = \frac{1}{2}$. Discount rate $d = 1 - \frac{s}{c} = 1$. Further assume that the product demand is a birandom variable with normal distribution $N(\mu_1, 400)$, and $\mu_1 \sim exp(0.0003)$. From theorem (3), we have $\xi \sim N(\mu_1, 400)$, where, $\lambda = 0.0003$.

Hence, by theorem (3) we can say that the mean (μ) of the birandom variable (ξ) is $\frac{1}{\lambda} = \frac{1}{0.0003} = 3333.33$, and $\sigma^2 = 400$. Now it remains to calculate the optimal order quantity and expected profit. For this purpose, we apply equation (12) and obtained the optimal order quantity, $x^* = 3326$. Hence, the expected profit is, $\Pi = \$66100$.

6. A Case Study

To endorse the model developed in this study, we sent our projected framework to five leading fish merchants in West Bengal, India. They sell only the freshest and best quality fishes, and maintains quality control at every stage of packaging and delivery in many different parts or areas of West Bengal. Among them two firms positively responded to explore this research proposal and we conducted necessary preliminary tasks on these companies. We selected a reputed fish merchant (*"Lakshmi fish enterprise", name changed*), situated in the "southern" West Bengal, which has several operational units nationwide. Our objective is to incorporate the perceptions of all participants (customer/retailer/company mangers) in the fish industry and to achieve its comprehensive outcomes since this research is purely grounded on birandom product demand information obtained from experts in the business. In this paper, we consider the perspectives of a wholesale fish merchant who buys fishes from the company, having a large market share in Kolkata zone.

In West Bengal, fish merchants generally sell a special fish in monsoon. The name of this fish is Hilsa. The business of this fish is a good example of SPIM. The business of this fish totally depends on its demand and supply in the monsoon season. Merchants have to decide how many fishes should be purchased from his or her supplier depending on the customer's demand. Buying more amount of Hilsa may not bring him more profit. Rather it can cause him a great loss since it cannot be preserved for long periods and the expired fish has no market value. If he/she buys too few amount of Hilsa he/she will lose the opportunity of making a higher profit. Thus, the actual inventory level cannot be determined precisely in such complex situation. It may be assumed for simplicity that the fish demand follows normal distribution. But under such circumferences, the manager looks after of some previous data, and finds the mean demand of "Hilsa" is also a random variable. This leads us to consider the "Hilsa" demand as a birandom variable. Each such fish sells for \$60 and costs for the shop owner \$40. Investigating the previous year data, the decision maker decides the demand follows the two types two- folded random variable (birandom).

Scenario 1: Normal distribution $N\sim(\mu_1, 400)$, with $\mu_1 \sim U(3000, 4000)$.

Scenario 2: Normal distribution $N\sim(\mu_1, 400)$, with $\mu_1 \sim N(3500, 500)$.

Therefore, according to our proposed model we have $c = 40, p = 60$, and there is no salvage value i.e., $s = 0$. In scenario 1, by theorem (2) the mean of the birandom variable is $\mu = 3500$, and $\sigma^2 = 400$. Therefore the optimal order quantity $x^* = 3492$. Expected profit $\Pi = \$69434$. And in scenario 2, using birandom simulation we get the mean of the birandom simulation $\mu = 3508$, and $\sigma^2 = 400$. Therefore, the optimal order quantity $x = 3500$, expected profit = \$69594. Finally, we shared the outcomes of this research work with the managers of our case enterprise, they are satisfied with the outcomes and willing to accept this result for their monsoon business of "Hilsa".

7. Conclusion

In this paper we have proposed a newsboy problem where the demand is considered as birandom variable. The market volatility and uncertainty in customers' behavior make the demand of the product a birandom variable. We use the expected value model for handling this birandom variable and to convert the BNM into its equivalent deterministic model. We discuss a case study of fish merchant to validate the usefulness and applicability of the proposed model. In this proposed model demand is the only parameter considered as birandom variable. However, in reality cost may be assume as a birandom variable. This is the one aspects to be considered for further investigation in the current model. Also, for future research work, one can develop chance constraint technique to convert the birandom model into deterministic one.

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