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Non-Classical Measures of Investment Risk on the Market of Precious Non-Ferrous Metals Using the Methodology of Stable Distributions[†]

A b s t r a c t. The aim of this article is to present some non-classical risk measures which are commonly used in financial investments, including investments in assets from the market of precious non-ferrous metals. The time series of log-returns of gold, silver, platinum and palladium prices are considered. To properly asses the investment risk the measures based on Value-at-Risk methodology have been used (the VaR estimation approach based on values from the tail of the distribution). Additionally, the measure comparing expected profits to expected losses from the opposite tails distribution has been shown – the Rachev ratio. It was assumed that the log-returns of presented assets belong to the family of stable distributions. The results confirm the validity of the use of stable distributions to asses the risk on the precious non-ferrous metals market.

K e y w o r d s: stable distributions, Value-at-Risk, Expected Shortfall, Median Shortfall, Rachev ratio, precious metals.

J E L Classification: G11, C46.

Introduction

Contemporary financial markets represent very complex area both in terms of functional and investigating aspects. There are many ways to increase the value of invested capital, i.e. investments in securities, real estate, works of art, precious metals, etc. Nevertheless, every possibility in investing money is related to uncertainty and risk. Since 2007 the world economy is facing a financial crisis resulting mainly from the situation on the U.S. mortgage market. This situation has rapidly spread to other markets. The potential impact of the crisis

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and the hedging methods against its extension have become the topic of fervent discussions both among politicians, practitioners and scientists.

From the scientific point of view the sudden and unpredictable changes in the research area make inadequate that the current mathematical models used for describing analyzed reality. This raises the necessity of shifting the researcher's attention from a classical to non-classical approach. Similar necessity also applies to the analysis of investment and risk. The investment uncertainty can be considered as a derivative of decisions that have been made by a decision-maker and understood as the risky ones, with consequences in the future. This implies that the probability of occurrence of some particular event may be impossible to identify or be identified with some probability. Thus, the significant difference between risk and uncertainty becomes clear: a risk can be clearly measured whereas an uncertainty is some kind of unmeasurable risk that refers to the investment.

The purpose of this paper is to present some non-classical risk measures, widely used in risk analysis of financial markets, and presented therein with reference to the risk observed in the market of precious non-ferrous metals. The main hypothesis runs that the values of risk measures obtained for stable distributions are closer to the empirical ones if compared to the normal approach. The subject of the study are time series represented by the log-returns of the prices of gold, silver, platinum and palladium. The main reason for choosing this particular market is an increasing interest in investing in precious metals. Deepening global crisis made investors look for alternatives to the traditional financial assets (stocks, bonds, etc.). The most popular precious metal, from the investing point of view, is gold. What investors gain are high availability and the possibility to hedge themselves in situations, where the financial system shows higher level of uncertainty. Purchasing of gold is more secure than typical and popular investments, e.g. in exchange rates market. However, the investment in gold doesn't mean investment only in gold bars. There are some alternative forms, such as golden coins, investment certificates or investing in units of funds connected to the companies actively acting on the gold market. As a result of still worsening situation on the biggest stock markets, majority of investors decide to allocate their money investing in assets that show the upward trend during crisis (e.g. as a hedge against increasing inflation).

1. Methodology

Scientists have developed many advanced methods of measurement and analysis of risk. Unfortunately, the more commonly used risk measures have the drawback of being unclear in terms of their theoretical underpinnings, their use and interpretations. Therefore, the methodological and practical considerations play important role for anyone who is responsible for management of that kind of risk. The risk level is influenced by many factors, some of which can be completely independent. Every theoretical model should accurately reflect the reality, and their construction is a very complex process. The methodological background for estimation and assessing risk, developed in the last century, requires to be modified due to the set of risk factors which are constantly varying over time. It results from the dynamic changes on the market, new possibilities of assets allocation and unpredictability of some unexpected events. Such events are related to the higher risk level and may be impossible to forecast. This is a problem both for individual investors and institutions, which are exposed to huge financial loses or even bankruptcy.

The literature on the subject recognizes a number of risk measures. In this paper the distinction between classical and non-classical measures is presented. As for classical ones all measures which represent a canon in risk analysis and are widely used in practice are considered here. Most of these measures appeared together with the development of certain scientific theories and afterwards were modified according to the external factors which determine their use. Moreover, the set of classical risk measures includes those based on commonly used probability distributions (especially the normal distribution). The set of non-classical risk measures encompasses all measures not included in the set of classical ones. The non-classical measures might be etymologically related to the interdisciplinary nature of science, deriving from the different scientific areas (physics, engineering, bio-medicine, etc.). However, because of their mathematical properties they are used in measuring risk on financial markets.

Due to strong assumption about the normality of distribution of stock returns in the set of classical measures, not met for the real data, some alternative measures based on different probability distributions have to be taken into account. The characteristics of empirical time series as high frequency of data, heteroscedasticity of variance, autocorrelation or fat tails of distributions are essential in risk analysis. The stable distributions, developed by B. Mandelbrot in the 60s, have the wide application in this matter. Distributions which belong to the class of stable ones, are described by the shape parameter allowing to model the asymmetry and fatness of the tail of distribution. It makes them useful in many scientific areas (from engineering, through physics to applications on financial markets (Borak, Härdle, Weron, 2005). The main difficulty in application of stable distributions is that their probability distribution function is not clearly defined. To describe stable model the characteristic function is used (and so-called Inverse Fourier Transform with respect to this function). If the random variable X has the cumulative distribution function $F(x)$, then its characteristic function has the form (Rachev, Mittnik, 2000):

$$\varphi(t) = E[\exp(itX)] = \int_{-\infty}^{+\infty} \exp(itx) dF(x), \quad (1)$$

Thus, for stable random variable the characteristic function is presented as:

$$\ln\varphi(t)=\begin{cases} i\mu t-\sigma^{\alpha}|t|^{\alpha}\left[1-i\beta sign(t)tg\left(\frac{\alpha\pi}{2}\right)\right], & for\ \alpha\neq 1 \\ i\mu t-\sigma|t|\left[1+i\beta sign(t)\frac{2}{\pi}\ln|t|\right], & for\ \alpha=1 \end{cases}, \qquad (2)$$

where $sign(t)$ is defined as $sign(t)=\begin{cases} 1\Leftrightarrow t>0 \\ 0\Leftrightarrow t=0 \\ -1\Leftrightarrow t<0 \end{cases}$.

The main characteristic of stable distribution is the shape parameter α allowing to measure the fatness of the tail of distribution. Remaining parameters which describe the stable PDF are skewness parameter $\beta\in\langle -1;1\rangle$, scale parameter $\sigma>0$ and location parameter $\mu\in R$. The practical use of stable distribution is related to complex estimation procedure of all parameters. In literature only three types of stable distributions are explicitly defined: normal distribution (where $\alpha=2$), Cauchy distribution (where $\alpha=1,\ \beta=0$) and Lévy distribution (where $\alpha=1/2,\ \beta=-1$). The shape parameter plays significant role in probability distribution analysis. If it is less than 2, then the variance of the distribution is infinite and the location parameter is equal to the mean of distribution. In the case where the shape parameter is less than 1, both variance and mean are infinite (Samorodnitsky, Taqqu, 1994).

The application of stable distributions is extremely justified if the fat tailed distributions are considered. In that case the probability that random variable takes values at the level significantly outlying from the center of the distribution is higher than in the normal case. Hence, the risk measures which are used in risk analysis have to take into account such values. The most popular and the most widely used risk measure in this case is Value-at-Risk (VaR). Several methods of estimating Value-at-Risk are used, but depend on many aspects, such as statistical assumptions related to the risk factors, dependency between these factors or the portfolio structure. In this paper the quantile based method for estimating VaR is used. In this method it is not necessary to assume the analytical form of function which describes the probability distribution of the risk. The historical data used in this method allows to estimate parameters which describe the real distribution of return. Afterwards, the $\alpha-$quantile of the distribution is estimated which allows to determine the VaR. In this method the most important problem is to select the proper probability distribution function, especially if the financial time series are analyzed. In addition, some tests of the fit of distribution have to be used to verify the convergence between empirical and theoretical distributions.

2. Non-Classical Risk Measures

The tail analysis of empirical distributions requires to define some class of measures which allow to asses risk related to the values significantly distant from the central part of the distribution. In this matter the quantile risk measures are considered. The idea of quantile risk measures is based on the Value-at-Risk (VaR) methodology, which is one of the most popular risk measures used in practice. Its principal advantage is that the VaR conveys straightforward information about potential loss as a result of an investment. However, such information does not include cases where some extreme observation occurs. Hence is not a good measure. Artzner, Delbean, Eber and Heat (Artzner *et al*., 1997) have proposed the set of axioms which have to be met by a good risk measure. These axioms define a coherent risk measure. If ϑ is a coherent risk measure for a set of random variables Y (defined on some probability space), such as $\vartheta : Y \rightarrow R^{+}$, a measure ϑ have to be:

sub-additive, for any $X_1, X_2 \in Y$: $\vartheta(X_1 + X_2) \leq \vartheta(X_1) + \vartheta(X_2)$

positively homogeneous, for any $\lambda \geq 0$ and $X \in Y$: $\vartheta(\lambda X) = \lambda \vartheta(X)$

monotonous, for any $X_1, X_2 \in Y$ if only $X_1 \leq X_2$: $\vartheta(X_1) \leq \vartheta(X_2)$

translation invariant, for any $X \in Y,\ c \in R$: $\vartheta(X + c) = \vartheta(X) + c$

As was mentioned above, VaR is one of the most popular quantile measure used by practitioners. It gives an answer to a question of what is the maximum loss incurred by an investor (or institution) from an investment in some specified time horizon. This loss may occur with some probability α called tolerance level. Maintaining previous assumptions and defining Value-at-Risk at the confidence level α, i.e. $VaR_{\alpha}(X)$, it is possible to calculate coherent risk measures determining expected value (in terms of mean and median) of an investment exceeding potential loss beyond VaR: the Expected Shortfall and Median Shortfall. These measures are calculated using formula:

$$ES_{\alpha}(X) = E\left[X - VaR_{\alpha}(X) \middle| X > VaR_{\alpha}(X)\right], \tag{3}$$

$$MS_{\alpha}(X) = Median\left[X - VaR_{\alpha}(X) \middle| X > VaR_{\alpha}(X)\right], \tag{4}$$

These measures inform about the possible expected loss beyond the level represented by VaR. In other words, what follows from (3) and (4) is that for any given α there exists one-step-ahead prediction representing expected value (in terms of mean and median) of loss beyond the VaR.

Additionally, taking into account certain tails' characteristics of empirical distributions it is necessary to compare if expected profits exceed expected losses. An interesting tool has been proposed by Rachev – the Rachev ratio $R - ratio$ (Biglova, Ortobelli, Rachev, Stoyanov, 2004). This measure is calculated using formula:

$$R-ratio=\frac{E\left(|X|\,|\,X\ge -VaR_{\alpha}\left(-X\right)\right)}{E\left(|X|\,|\,X\le VaR_{\beta}\left(X\right)\right)} \tag{5}$$

If the random variable X represents profit or loss from a given investment, the Rachev ratio $R-ratio$ defines the ratio of expected profit above the VaR value (for a given $\alpha^{th}-$quantile) to expected losses below the VaR value (for a given $\beta^{th}-$quantile). If (in a special case) $\beta=1-\alpha$, then the values of $R-ratio$ are interpreted as follows:

- $R-ratio=1$ – expected profits are equals to expected losses,
- $R-ratio>1$ – expected profits exceed expected losses,
- $R-ratio<1$ – expected profits are lower than expected losses.

The risk assessment using $R-ratio$ is based on maximizing its values.

3. Empirical Analysis

The practical application of some non-classical risk measures is presented here in the relation to the market of precious non-ferrous metals, represented by the time series of 2902 daily log-returns of the prices of gold, silver, platinum and palladium in period of January 2000 – June 2011[1]. As presented in Table 1, there is a significant dispersion in prices of precious non-ferrous metals.

Table 1. Descriptive statistics – prices

	Gold	Silver	Platinum	Palladium
Mean	622.89	10.84	1 023.26	387.43
Standard deviation	349.12	7.43	440.08	196.87
Minimum	255.95	4.07	414.00	148.00
Maximum	1 552.50	48.70	2 273.00	1 090.00
Range	1 296.55	44.64	1 859.00	942.00

The highest dispersion level (in terms of range) is observable in prices of platinum and gold. Although, if the coefficient of variation is taken into account, the highest dispersion is related to the prices of silver. The volatility of prices of gold and palladium is presented on Figures 1 and 2 respectively. The prices of gold shows the behavior similar to the stock indices whereas the prices of palladium not.

As is presented in Figure 1, the price of gold has been increasing reasonably till February 2006. Then some significant price changes have been observed. The last four years show that the price of gold hadn't behave in a stable manner.

[1] Data from London Metal Exchange.

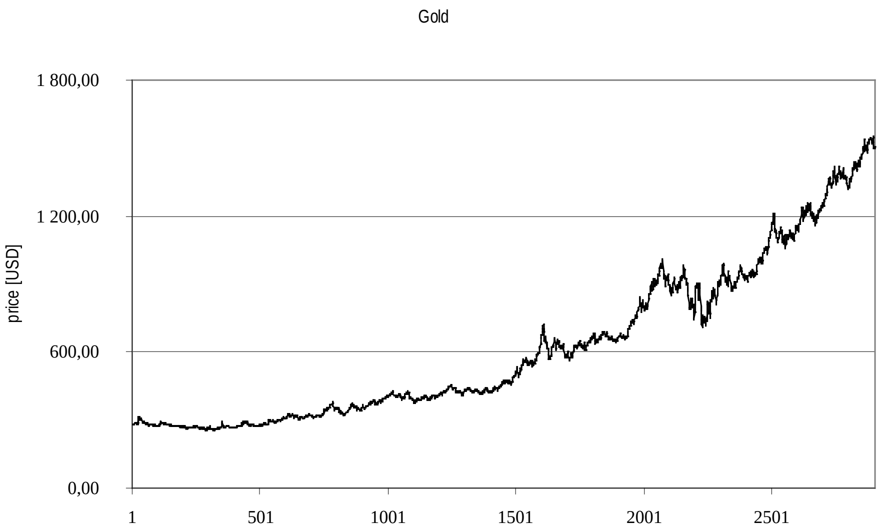

Figure 1. The volatility of price of gold from January 2000 to June 2011

The price of palladium has been changing without specifically defined direction. After first two years of growing the price has fallen dramatically returning to upward trend in first half of 2003. The year 2009 initiated the period of surge of prices that continues till now. In the next step the log-returns of corresponding prices have been calculated. Figure 3 presents the volatility observed in time series of gold log-returns.

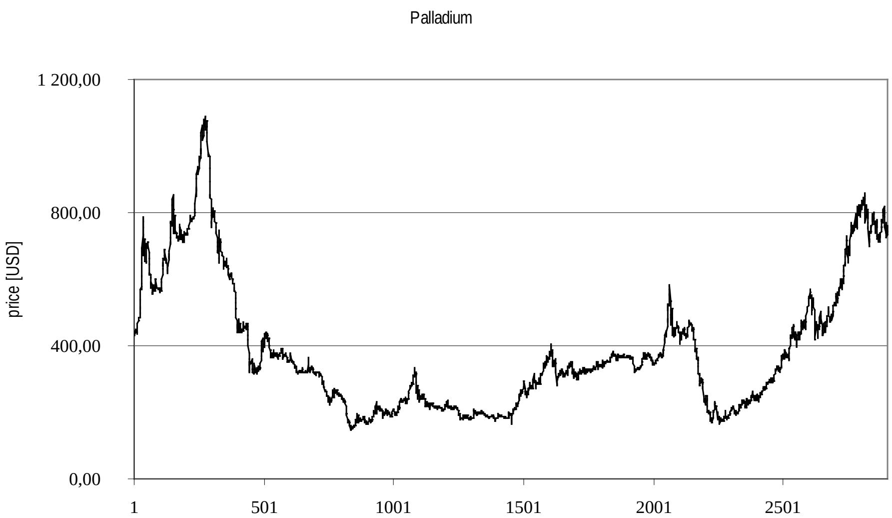

Figure 2. The volatility of price of palladium from January 2000 to June 2011

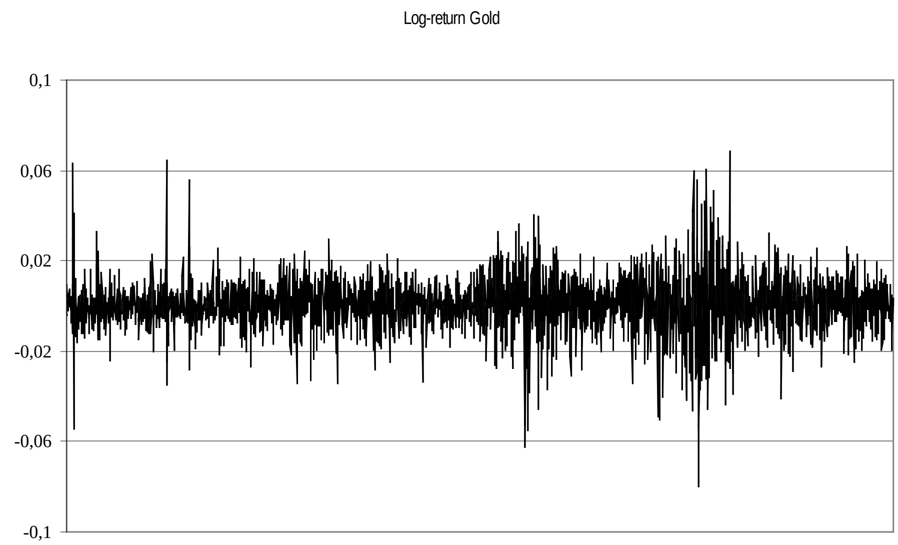

Figure 3. Volatility of log-return of gold

As presented in Figure 3, the period starting in the middle of 2005 reveals increased level of volatility (clustering and significant jumps in returns). The same or even more unstable behavior characterizes returns of palladium. Clustering is observed within entire period, with higher intensity during last year (Figure 4).

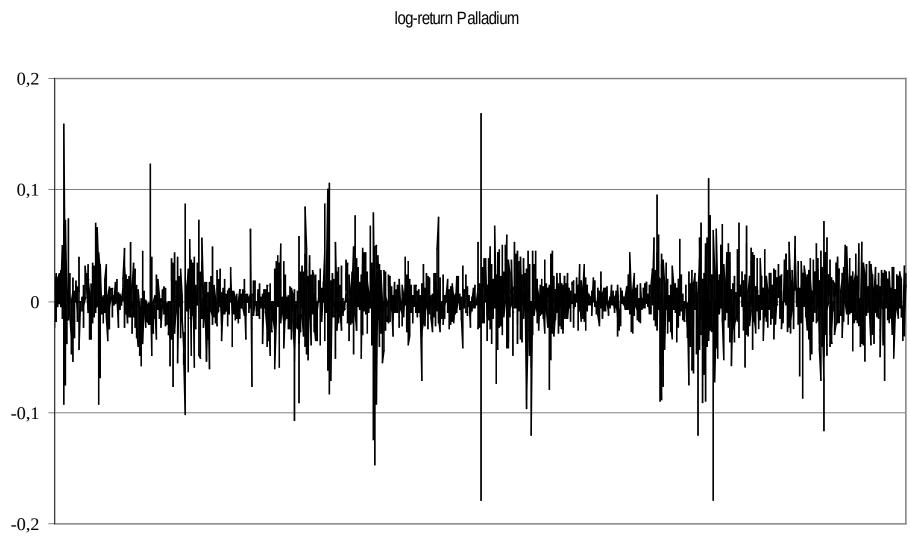

Figure 4. Volatility of log-return of palladium

One of the most important matter related to the risk analysis of financial markets is that, whether there exists a significant relation between assets. It is of great importance especially in portfolio analysis. To see that, Table 2 presents the correlations between log-returns of metals analyzed, whereas Table 3 shows the descriptive statistics.

Table 2. Correlations

	Gold	Silver	Platinum	Palladium
Gold	1.00000	0.56024	0.44953	0.37513
Silver	0.56024	1.00000	0.45243	0.42328
Platinum	0.44953	0.45243	1.00000	0.57408
Palladium	0.37513	0.42328	0.57408	1.00000

The highest values of correlation are between pairs of returns of platinum and palladium and between gold and silver. The lowest values correspond to the relation between gold and palladium. It can be caused by a natural market properties of these precious metals. Gold and silver are considered as investment precious metals while platinum and palladium are more related to industrial environment (especially the latter).

Table 3. Descriptive statistics – returns

	Gold	Silver	Platinum	Palladium
Mean	0.00058	0.00065	0.00048	0.00019
Standard deviation (σ_N)	0.01151	0.02107	0.01603	0.02392
Minimum	-0.07972	-0.18693	-0.17277	-0.17859
Maximum	0.06841	0.18279	0.11129	0.16799
Range	0.14813	0.36971	0.28406	0.34658

As presented in Table 3, the highest value of expected return corresponds to the variables silver and gold. The comparable level reaches the return of platinum, whereas the return of palladium oscillates around zero. The dispersion analysis showed that the most risky (in terms of standard deviation) are investments in palladium and silver (in terms of range as well) while the safest one correspond to gold (Figure 5).

Further analysis related directly to risk measurement requires to match the theoretical probability distribution to the real market data. Classical risk analysis is based on the normal distribution, so this case has to be verified. To test the normality of log-returns of the selected precious metals the following goodness-of-fit methods have been used: Jarque-Bera test and Anderson-Darling test. The results of comparing empirical and normal distribution show that the assumption of normality has to be rejected (at the significant level of 0.01). Therefore the empirical distributions do not belong to the family of normal ones. The rejection of normality requires the use of risk measures which are not based on this strong assumption and allow to use other statistical distributions.

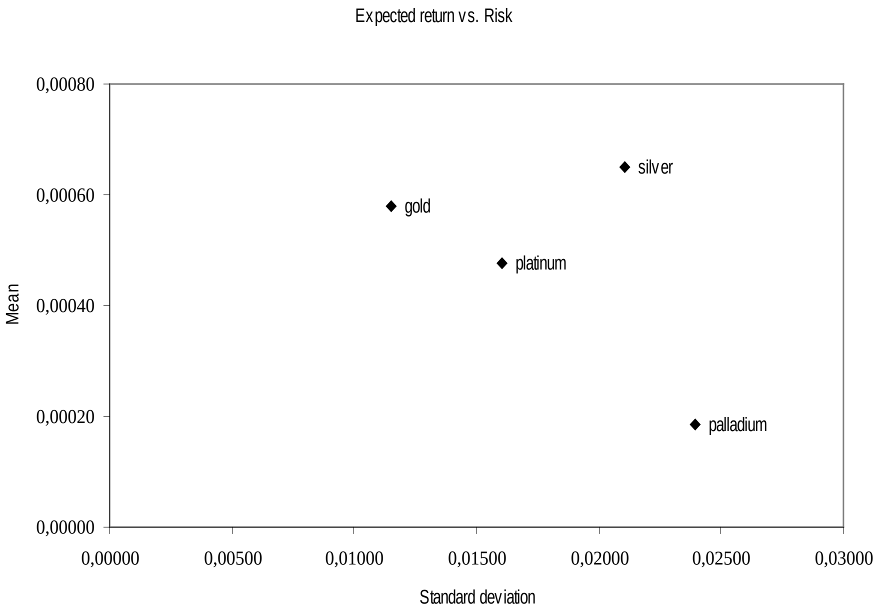

Figure 5. Expected return vs. Risk

To solve this problem, the family of stable distribution is used. To estimate the parameters of stable distribution, the Maximum Likelihood Methodology is applied and the results are presented in Table 4.

Table 4. The parameters of stable distributions

	α	β	σ	μ
Gold	1.63592	-0.04440	0.00620	0.00063
Silver	1.60458	-0.03883	0.01074	0.00103
Platinum	1.58989	-0.09054	0.00790	0.00061
Palladium	1.54917	-0.01818	0.01192	0.00049

Note: the parameters are significant at the level of 0.01.

As was mentioned in section “Methodology”, the parameter α plays the most important role. It is responsible for the thickness of tail of distribution. As results from calculations, each of log-returns distributions of analyzed assets is fat-tailed. The index of stability takes the highest value for the variable gold, and the lowest for the variable palladium. Moreover, each of the distributions has a left-sided asymmetry which means that the left-side tail is heavier than the right-side one. If α takes values below 2, the location parameter is equal to the mean of distribution. Therefore the more accurate information about the expected return of empirical data is provided rather by the value of location parameter than by the value of mean (which is more suitable if the assumption

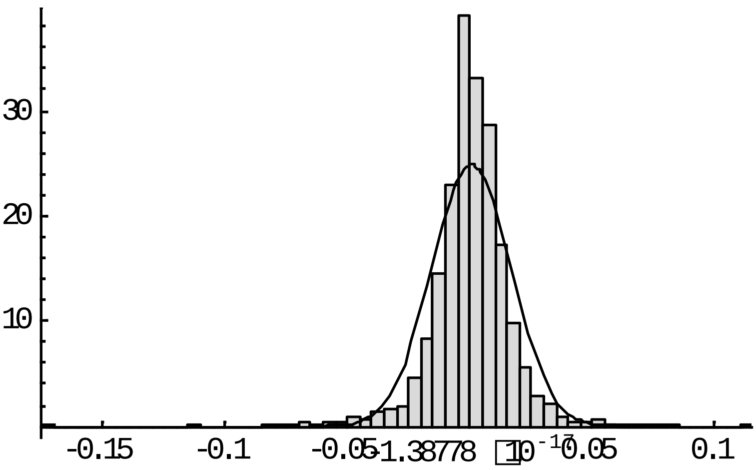

Figure 6. Fitting of normal distribution – platinum

of normality is not rejected). The scale parameter σ can be interpreted as a dispersion parameter (just like the standard deviation in normal case) as the relation between location parameter of stable distribution and standard deviation of normal distribution is as follows: $\sigma = \sigma_N \sqrt{2}$. The fitting of normal and stable distributions to the empirical data of variable platinum is presented using histograms and QQ-plots (Figures 6–9).

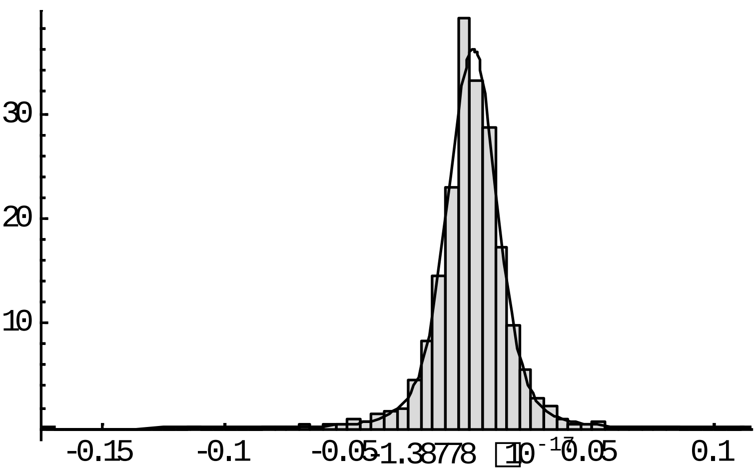

Figure 7. Fitting of stable distribution – platinum

The Figures 6–7 show that stable distribution is better fitted to the empirical data of platinum than in the normal case. QQ-plots confirm that in terms of quantile analysis. The more dotted line covers the solid one the better fit to the theoretical distribution. This property of QQ-plot is rejected for normal case.

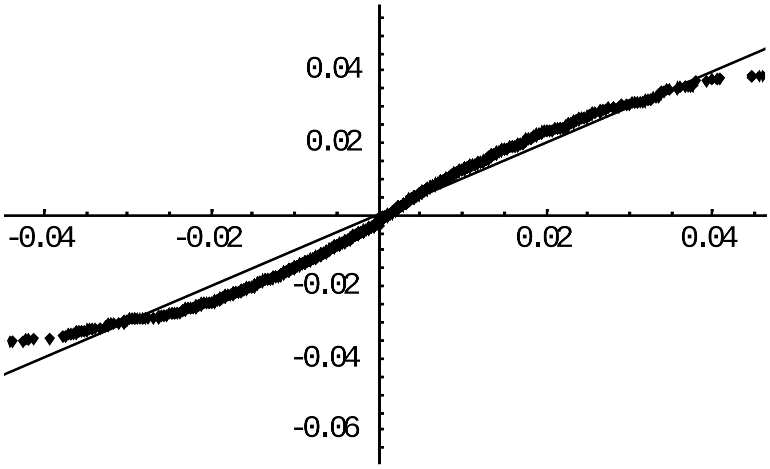

Figure 8. QQ-plot of normal distribution – platinum

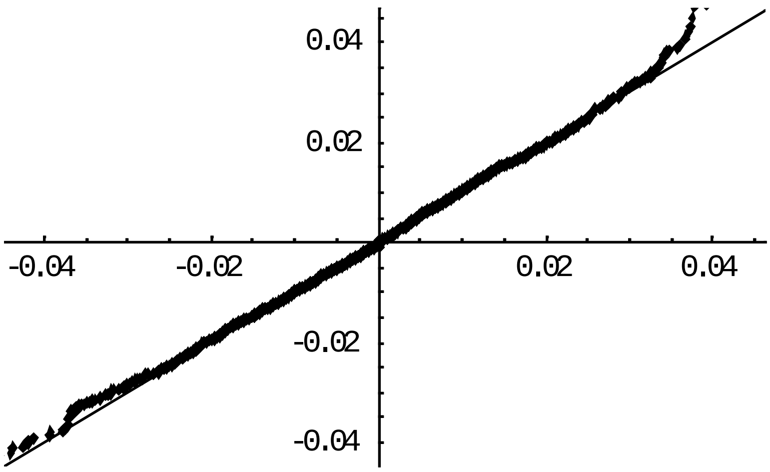

Figure 9. QQ-plot of stable distribution – platinum

Further risk analysis is based on the non-classical measures arising from the methodology of Value-at-Risk. The concept of VaR is based on the estimation of quantile of arbitrary distribution. In the first step the Value-at-Risk for each variable is calculated. For calculation one-period forecast is used. The results presented in Table 5.

Table 5. Value-at-Risk estimates

Risk measure	Type of distribution	Quantile	Gold	Silver	Platinum	Palladium
Value-at-Risk	empirical distribution	0.01	-0.03298	-0.05946	-0.04992	-0.07466
		0.05	-0.01727	-0.03093	-0.02266	-0.03747
	normal distribution	0.01	-0.02414	-0.04929	-0.03684	-0.05663
		0.05	-0.01815	-0.03344	-0.0262	-0.04074
	stable distribution	0.01	-0.03638	-0.06713	-0.05238	-0.08346
		0.05	-0.01651	-0.02931	-0.02218	-0.03453

Table 5 presents Value-at-Risk estimates for quantiles 0.01 and 0.05 for three types of distributions. The results obtained for the stable model are similar to those obtained for the empirical distribution. However, comparing results of normal estimates, the differences are significant. For example, the VaR for gold at 0.01 confidence level for normal distribution is -0.02414 while for empirical and stable distribution is -0.03289 and -0.03638 respectively. Therefore, taking into account the normal case, potential loss seems to be lower than it actually is (comparing to the empirical values). Moreover, the results are corroboration of the fitting quality of stable models to the empirical data.

Table 6. Expected Shortfall estimates

Risk measure	Type of distribution	Quantile	Gold	Silver	Platinum	Palladium
Expected Shortfall	empirical distribution	0.01	-0.04349	-0.09007	-0.07171	-0.09979
		0.05	-0.02707	-0.05209	-0.04001	-0.05930
	normal distribution	0.01	-0.02715	-0.05739	-0.04209	-0.06523
		0.05	-0.02207	-0.04352	-0.03286	-0.05051
	stable distribution	0.01	-0.04429	-0.09507	-0.08766	-0.08540
		0.05	-0.03563	-0.06547	-0.06267	-0.06288

Similar results obtained for the Expected Shortfall (Table 6) and Median Shortfall (Table 7). Expected loss exceeding VaR (in terms of mean and median) is similar for normal and stable distributions while it differs for the normal one.

Table 7. Median Shortfall estimates

Risk measure	Type of distribution	Quantile	r-gold	r-silver	r-platinum	r-palladium
Median Shortfall	empirical distribution	0.01	-0.04017	-0.07918	-0.06704	-0.09038
		0.05	-0.02359	-0.04459	-0.03377	-0.05055
	normal distribution	0.01	-0.02591	-0.05627	-0.03951	-0.06276
		0.05	-0.02147	-0.04145	-0.03137	-0.05051
	stable distribution	0.01	-0.04341	-0.07851	-0.07346	-0.09073
		0.05	-0.02594	-0.03991	-0.03263	-0.04142

Using values of VaR for a given pairs of quantiles, the results obtained for $R-ratio$ are presented in Table 8 and Table 9.

Table 8. R-ratio estimates (quantiles 0.99 vs. 0.01)

Risk measure	Type of distribution	Quantile	r-gold	r-silver	r-platinum	r-palladium
R-ratio	empirical distribu-tion	0.99 vs.0.01	1.22435	1.12766	0.99684	1.00492
	normal distribution	0.99 vs. 0.01	1.13321	1.19392	0.98224	1.02183
	stable distribution	0.99 vs. 0.01	1.21334	1.14118	0.99733	1.01432

Taking into account quantiles 0.99 and 0.01 the values of ratio obtained for stable distribution are closer to the values obtained from empirical distribution. This shows that the approach based on stable distribution is more adequate to the real behavior of stock returns.

Table 9. R-ratio estimates (quantiles 0.95 vs. 0.05)

Risk measure	Type of distribution	Quantile	r-gold	r-silver	r-platinum	r-palladium
R-ratio	empirical distribu-tion	0.95 vs.0.05	1.01766	1.04638	1.00231	1.01233
	normal distribution	0.95 vs. 0.05	1.12235	1.11321	0.99112	1.09121
	stable distribution	0.95 vs. 0.05	1.02152	1.08199	0.99984	1.04523

The data presented in Table 9 confirmed the results obtained for quantiles 0.99 and 0.01 – the advantage of using stable approach over the normal one.

Conclusions

The application of non-classical risk measures plays very important role in financial market analysis. Both scientists and practitioners confirm that the risk analysis has to be extended beyond the normal case, and this approach covers not only financial market but also other ones, e.g. non-ferrous metals market. This paper has presented the analysis of daily log-returns of gold, silver, platinum and palladium time series. As confirmed, the assumption of normality of distribution for each variables has been rejected and the stable models have been applied. The results show that stable distributions are better fitted to the empirical data than the normal ones, complying with leptokurtosis, heavy tails (the range of parameter α reflecting thickness of the tail is 1.55–1.64) and asymmetry.

In the case of risk measurement the methodology of Value-at-Risk is the basis of calculating non-classical risk measures. Both VaR and measures such as Expected Shortfall and Median Shortfall prove discrepancy between normal and empirical distributions. That is the reason why the other type of distributions should be taken into account. Similar results for calculating coherent risk measures were obtained by Trzpiot, Krężołek (2009) for analyzing daily log-returns on Polish financial market using stable models and by Krężołek (2010) on precious non-ferrous metals market, but using geo-stable distributions (more precisely – Asymmetric Laplace distribution). The results, although straightforward in terms which class of theoretical distribution should be chosen, have to be interpreted very carefully. Moreover, considering $R-ratio$ the results obtained for stable distributions are closer to the values for empirical one comparing to the normal ones.

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Nieklasyczne mierniki ryzyka inwestycyjnego na rynku szlachetnych metali nieżelaznych z wykorzystaniem metodologii rozkładów stabilnych

Z a r y s t r e ś c i. Celem artykułu jest prezentacja wybranych nieklasycznych miar ryzyka, które mają szerokie praktyczne zastosowanie w przypadku inwestycji finansowych, w tym inwestycji w walory notowane na rynkach surowców (nieżelazne metale szlachetne). Przedmiotem badania są szeregi czasowe reprezentowane przez stopy zwrotu cen złota, srebra, platyny oraz palladu. Do oceny ryzyka inwestycyjnego wykorzystano miary wyznaczone w oparciu o metodologię Value-at-Risk stosując estymację opartą na wartościach z ogona rozkładu. Dodatkowo do oceny ryzyka wykorzystano wskaźnik Rachev'a. Przyjęto założenie, że stopy zwrotu wykorzystanych zmiennych należą do rodziny rozkładów stabilnych. Wyniki badania potwierdzają stosowność wykorzystania rozkładów stabilnych do oceny ryzyka na rynku metali nieżelaznych.

S ł o w a k l u c z o w e: rozkłady stabilne, Value-at-Risk, Expected Shortfall, Median Shortfall, wskaźnik Rachev'a, metale szlachetne