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The Probability of Recession in Poland Based on the Hamilton Switching Model and the Logit Model

A b s t r a c t. In the article dating method for the four phases of economic activity is presented. Comparison of probabilities of recession occurrence in Poland based on the Hamilton switching model and the logit model was conducted in the empirical research. The study shows the convergence of indications based both on the proposed dating method and on the Hamilton model. In the presented version the Hamilton model adequately describes the probability of occurrence of two decline phases. The logit model allows to obtain satisfactory results for the division on four phases of economic activity. However, in the domain of the Polish economy, more research is needed in recognising the symptomatic properties of various macroeconomic indicators. The interest rate spread, used successfully in advanced marked economies, continues to alter its characteristics under Polish economic conditions and is currently not the best possible indicator forecasting a recession.

K e y w o r d s: switching model, logit model, dating of economic activity phases, probability of recession.

J E L Classification: E32, E37.

Introduction

Analysts often emphasise that many financial and economic indicators tend to behave differently during growth and decline. Therefore, it is a well-grounded assumption that the parameters of the models describing the formation of such values change. The switching models allow to test such an assumption. If the turning points, otherwise known as the moments of switching between periods of diversified behaviour of the variables, are known, then the segment model for the quantitative variable or the probability model for the selected variants of the qualitative variable, is estimated. If researchers cannot

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agree upon a single method for establishing such turning points, then the Markov-switching model, as proposed by Hamilton, can be used. In the case of economic activity, the moments of switching depend, among others, on the accepted method of decomposition of time series of selected macroeconomic ratios.

In research on the American market, data provided by the NBER concerning the turning points for growth and decline phases in the U.S. economy is used as the point of reference. Yet in many countries there is no established system to indicate the beginning and end of a recession. That is why it is worthwhile to analyse the convergence of indications resulting from various methods.

Zarnovitz and Ozyildirim (2006) discuss the influence of the accepted method of decomposition on the variability of the course of a U.S. growth cycle. They compare the dating of turning points based on cycles of levels, trend deviations and smoothed growth cycle. The results obtained with the use of the PAT method are very similar to the results obtained on the basis of the Hodrick-Prescott filtering method, local linear trend as well as band-pass filtering method[1]. In Poland, a comparative analysis of business cycles obtained using different methods, made under various assumptions, was described, among others, by Skrzypczyńska (2011) and Burzała (after revives, in press).

This article presents a comparison of the indications for a recession phase based on the two models, with known and unknown switching points. Section 1 presents the dating method of economic activity phases, which allows to determine the moments of switching between the phases of high and low economic activity. Sections 2 and 3 describe the models (Hamilton's switching model and the logit model, respectively) which are used to estimate the probability of a recession. The research results and the comparison of indications based on the accepted method of dating of phases are described in section 4. Section 5 is a summary of the research results.

1. Dating of Economic Activity Phases

In a time of growth-based market economies, it is difficult to clearly determine which of the observed changes have resulted from long-term economic growth and which stem from economic fluctuations. Despite the many research studies and upgraded decomposition methods, the division between 'trend' and 'cycle' has always been accepted as a convention and can be considered somewhat artificial. Therefore, it would seem to make sense to employ an approach based on an analysis of the growth rates of a time series with the seasonal and random fluctuations removed, and with no decomposition into trend and eco-

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[1] PAT (Phase Average Trend) is a 10-step procedure as described by Boschan, Ebanks (1978). It was applied by the NBER to a large number of indicators with generally satisfactory results in terms of timing and conformity to aggregate growth cycles.

nomic fluctuations. Thus, the research is focused on economic activity in general, rather than on the course of a business cycle.

In the empirical studies presented in this paper, the measure of economic activity were the *annual indices of total industrial output sold PR_IR_t* – as recorded on a monthly basis between January 1993 and March 2011 – with seasonal and random fluctuations removed. The selection of output indices mostly resulted from a larger frequency of quotations than from the GNP.

This approach is compatible with the generally accepted growth-based definition of the business cycle as commonly assumed in empirical analyses (Mintz, 1972). The dating rules for economic activity phases, as used in this paper, make use of short- and long-term changes in the output indices (Burzała, 2005).

The *annual index PR_IR_t* measures the change in a value PR_t with respect to the corresponding period of the preceding year PR_{t-12} and constitutes the measure of changes 'within a long period of time'. The *monthly index PR_IM_t* measures the change in a value PR_t with respect to the preceding month PR_{t-1} and is a measure of changes 'within a short period of time'. These indices represent the following respective dependencies:

$$PR\_IR_t = \frac{PR_t}{PR_{t-12}} \cdot 100, \quad PR\_IM_t = \frac{PR_t}{PR_{t-1}} \cdot 100. \qquad (1)$$

The first index (PR_IR_t) is a reference series, as has already been mentioned. The proposal of dividing the set of all observations of the reference series into separable subsets (economic activity phases) has been based upon tests which were to determine whether the *short-term growth rate implies long-term changes*. Depending on whether a given index is above or below 100 (which corresponds to a positive or negative growth rate), an observation is classified as characteristic of a given phase of the economy.

This approach uses the rules of *symbolic taxonomy*, in which a phase (state) is described through a conjunction of values of selected indicators (Gatnar, 1998).

These rules allow to distinguish four phases of the economy (Burzała, 2005 a,b):

a) $PR\_IM_t \geq 100$ and, simultaneously, $PR\_IR_t \geq 100$ – *implied growth*, which denotes *high* economic activity (conventionally called the prosperity phase and marked with the code *W_IM*);

b) $PR\_IM_t \geq 100$ and $PR\_IR_t < 100$ – *non-implied growth*, which denotes a rise of the monthly index and most often occurs following a recession (code *W_NIM*);

c) $PR\_IM_t < 100$ and $PR\_IR_t < 100$ – *implied decline*, which denotes an economic recession (code *S_IM*);

d) $PR\_IM_t < 100$ and $PR\_IR_t \geq 100$ – *non-implied decline*, which de-
   notes a dropping monthly index – which is usually a pre-recession phase
   (code *S_NIM*).

The advantage of such a division is that two transitory phases have been identi-
fied, i.e. non-implied decline, which in itself is a warning against adverse
change in business conditions, and non-implied growth, which indicates an im-
proving environment. The growth and decline phases discerned in this paper
may be combined for a less detailed differentiation (prosperity vs. recession).

2. Hamilton's Switching Model

The assumption in this case is that the behaviour of certain macroeconomic
indicators changes as a result of changed economic activity. However, a phase
of economic activity is not directly observable, and thus it is difficult to estab-
lish which phase an economy is in at a given moment. In the terminology of
switching models, a phase of activity is one of the possible states referred to as
regimes. In the original Hamilton model (1989), it was assumed that there were
two possible regimes, *s* (*s = 1, 2*), corresponding to the condition of an econo-
my (prosperity vs. recession). In each regime, indicator values are generated by
two separate and independent processes. In the Hamilton model, these were the
conditional average processes, AR(4), for quarterly GNP changes. Thus, in
a general case:

$$y_t - \mu_{s_t} = \phi_1(y_{t-1} - \mu_{s_{t-1}}) + \phi_2(y_{t-2} - \mu_{s_{t-2}}) + ... \\ + \phi_p(y_{t-p} - \mu_{s_{t-p}}) + \varepsilon_t ,$$

(2)

where: $\varepsilon_t \sim N(0, \sigma^2)$, μ_s denotes the expected value under the *s* regime.

At present, models with switches resulting from the changing variance σ^2,
or the average and a variance are estimated particularly with regard to financial
markets (Doman, Doman, 2009). Such a model, entailing two regimes, was
used in the research studies. Consequently:

$$\varepsilon_t = \sigma_{s_t} \sqrt{h_t} \xi_t ,$$
$$h_t = 1 + \alpha_1 \varepsilon_{t-1}^2 / \sigma_{s_{t-1}}^2 + ... + \alpha_p \varepsilon_{t-p}^2 / \sigma_{s_{t-p}}^2 ,$$
$$\xi_t \sim iid(0,1).$$

(3)

The series of random variables s_i in the subsequent moments in time *t* (*t=1,...,
T*) has the Markov property, i.e. its value at the time moment *t+1*, i.e. s_{t+1}, de-
pends only on the regime at the *t* moment, rather than on all the preceding re-
gimes, which is formally formulated as:

$$P(s_{t+1} = j | s_t = i, s_{t-1} = k,...) = P(s_{t-1} = j | s_t = i) = p_{ij}.$$

(4)

The probabilities p_{ij} denote the probability of an economy's switching from regime i into regime j. These are unknown parameters that are estimated, where:

$$
\begin{aligned}
p_{11} &= P(s_{t+1} = 1 | s_t = 1), \\
p_{12} &= P(s_{t+1} = 2 | s_t = 1), \\
p_{21} &= P(s_{t+1} = 1 | s_t = 2), \\
p_{22} &= P(s_{t+1} = 2 | s_t = 2).
\end{aligned}
\tag{5}
$$

Based on conditional probabilities (4 and 5), it is possible to determine the unconditional probability $P(s_t = i)$ of an economy remaining at the t moment under an ith regime. In the case of two regimes, the following is obtained:

$$
P(s_t = 1) = \frac{1 - p_{22}}{2 - p_{11} - p_{22}}, \quad P(s_t = 2) = \frac{1 - p_{11}}{2 - p_{11} - p_{22}}.
\tag{6}
$$

The inverse of probabilities $P(s_t = i)$ is interpreted as the expected time of resumption of regime i:

$$
mt(i) = \frac{1}{P(s_t = i)},
\tag{7}
$$

whereas the expected time of remaining under regime i is provided by means of the dependency:

$$
d_i = \frac{1}{1 - p_{ii}} = \frac{1}{p_{ij}}.
\tag{8}
$$

The parameters of the model (p_{ij}, μ_s, ϕ_i, σ_s^2) are estimated using the maximum likelihood method[2]. If at the t moment the process was under the $s_t = j$ regime, then the conditional probability density function of the explained variable y_t can be represented as $f(y_t | s_t = j, \Psi_{t-1})$, where Ψ_{t-1} denotes the history of the process until the *t-1* moment. Even knowledge of the model's parameters does not allow to establish which regime a given economy is under at the t moment. Any suppositions on the actual regime may only be made by means of a conditional probability:

$$
P(s_t = j | \Psi_t) = \frac{f(y_t | s_t = j, \Psi_{t-1}) \cdot P(s_t = j | \Psi_{t-1})}{\sum_{i=1}^{2} f(y_t | s_t = i, \Psi_{t-1}) \cdot P(s_t = i | \Psi_{t-1})},
\tag{9}
$$

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[2] The switching models were estimated using the TSM programme. The relevant likelihood function is presented as part of the description of the programme (Davidson, 2011).

where:

$$P(s_t = j | \Psi_{t-1}) = \sum_1^2 p_{ij} P(s_{t-1} = i | \Psi_{t-1}). \tag{10}$$

The maximised likelihood function is as follows:

$$L = \sum_{t=1}^T \log \sum_{j=1}^2 f(y_t | s_t = j, \Psi_{t-1}) \cdot P(s_t = j | \Psi_{t-1}). \tag{11}$$

It is not easy to estimate the model's parameters. Numerical problems result from the occurrence of local extrema of the logarithmic likelihood function. This is why normally two, up to three, regimes under which a process may be are distinguished.

3. Logit Model

The dating method of economic activity phases, as proposed in Section 2, allows to date the time periods relating to a recession. Let us assume that this time the explained variable y_t assumes the value of 1 at the *t* moment, if a recession occurs in an economy (*S_IM*); otherwise, the value is 0. The explained variable thus defined is a qualitative binary variable, usually modelled using the logit model.

In the model with a qualitative explained variable, the theoretical probability P_{t1} of occurrence of the first option of the variable at the *t* moment is defined by the cumulative distribution function $(\beta' \mathbf{x}_t)$, i.e.:

$$P_{t1} = F(\beta' \mathbf{x}_t + \xi_t), \tag{12}$$

where $\mathbf{x}_t$ is a vector of explanatory variables, β is the parameters vector and ξ_t is a random disturbance. The type of distribution assumed for the ξ_t random variable generates the type of model under consideration. In the logit model, it is assumed that probability P_{t1} is defined using the cumulative distribution function of a standardised logistic distribution $\xi_t \sim L(0, \pi^2/3)$. Hence:

$$P_{t1} = \int_{-\infty}^{\beta' \mathbf{x}_t} \frac{e^{-\xi_t}}{\left(1 + e^{-\xi_t}\right)^2} d\xi_t = \frac{1}{1 + e^{-\beta' \mathbf{x}_t}}. \tag{13}$$

It can be proved that the logit model's random component is heteroscedastic (Jajuga, 1990). For this reason, to estimate the model's parameters, the method with the utmost probability is most often used. The maximised logarithmic likelihood function is provided by means of the dependency:

$$L(\beta) = \ln L^*(\beta) = \sum_{t=1}^T \sum_{j=1}^2 y_{tj} \ln P_{tj}. \tag{14}$$

As for the uses of models with the qualitative explained variable, the article by Estrella, Mishkin (1998) is of primary importance for measuring economic activity. These authors tested macroeconomic indicators that preceded the onset of an economic recession in the U.S. by using a binary probit model. As it turned out, the difference in interest rates applied to ten-year treasury notes and three-month treasury bills (the so-called spread) quite reliably indicates, with a one-year advance, an economic recession in the U.S. In the research studies carried out for the Polish economy, apart from the interest rate spread (*SPREAD*), the annual indices of money supply M1 (*M1_IR*), M2 (*M2_IR*), M3 (*M3_IR*) and the WIG stock-exchange index (*WIG_IR*) were tested.

The symptoms of a recession were chosen according to research studies carried out by this author in an unpublished PhD thesis (Burzała, 2005a). It should be emphasised that any given set of leading indicators is not effective for all countries. A different set of indicators is used by NBER or OECD researchers, and specialists from Germany or Japan use yet a different methodology. Under Polish economic conditions, the methods of testing the business cycle are still being improved and the list of quantitative indicators is being expanded. The idea of using the interest rate spread for forecasting economic activity is derived from the rational expectations hypothesis. In a market economy, a negative interest rate spread is one of the symptoms of a recession (short-term interest rates are higher than long-term interest rates)[3]. The growing interest in financial indicators stems from economists' convictions that financial variables have an increasing influence on the real sphere of the economy, and thus the entire economic policy. The meaning of a monetary policy that is handled appropriately results from the function that regulates both the money supply and lending in order to maintain employment, price stability and economic growth. The importance of financial indicators was analysed by Atta-Mensah, Tkacz (1998), Estrella, Hardouvelis (1991), Stock, Watson (1993), Wright (2006), and Nyberg (2009). The purpose of this article is not to determine the best set of indicators, but to check the properties of some of these indicators.

4. Research Results

In order to illustrate the recession indications based on the dating method as proposed in Section 1, the results obtained for the U.S. economy were compared with information from the NBER (Figure 1).

It turned out that the time periods shown for the recession appear to show high convergence, which may prove that the proposed dating method is correct. It is worth emphasising that in dating the cycles, NBER imposes additional restrictions for the duration of the decline and growth phase (a minimum of 6 months) and for maintaining the growths and declines around the turning points

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[3] In the countries of Central and Eastern Europe (including Poland), the interest rate spread does not fulfil the given assumptions during an economic transformation.

(for a period of 5 months). The method proposed in this paper does not take such conditions into consideration in this version, yet it can easily be extend.
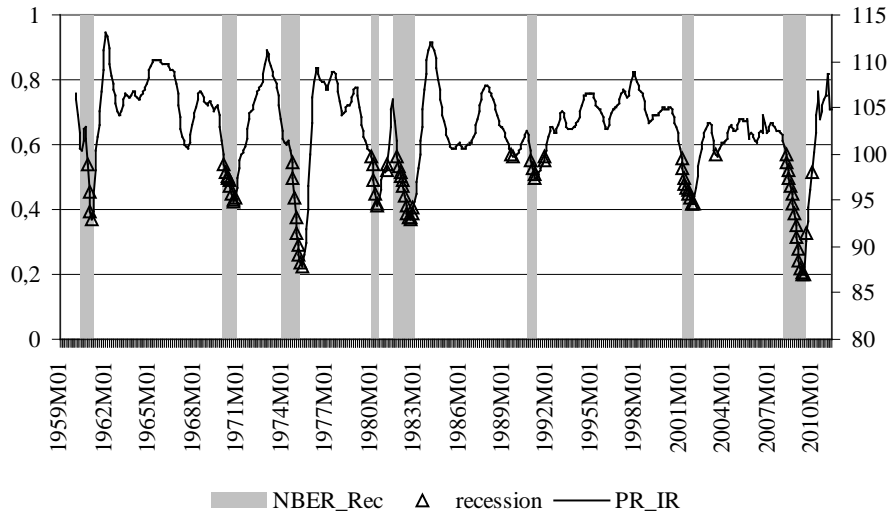
Figure 1. U.S. economy recession phases, January 1959 to March 2011, base on NBER data

Fundamental research studies on Polish economic activity were based on statistics from the International Monetary Fund (CEIC). Figure 2 shows the economic activity phases for Poland. The bar chart highlights the recession phases.
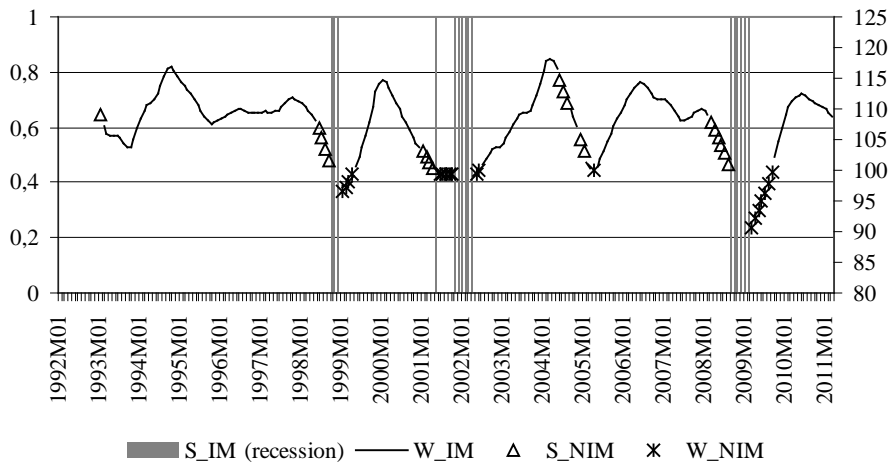

Figure 2. Economic activity phases for Poland, January 1993 to March 2011, based on CEIC data

The regularities that occur are worth consideration: a recession phase and a prosperity phase are always preceded by a transitory phase, which signalises a deterioration or improvement of the economic conditions. The economic activity phases determined as such were compared with the probability of occurrence of two regimes (prosperity and recession, by definition) based upon the Hamilton model.

The results Hamilton presented referred to the differences between the GNP logarithms. The proposed pattern, once directly applied to the output indices in the Polish economy, has not yielded satisfactory results. Therefore, it was proposed that the explained variable under the Hamilton model be the logarithm of the annual output index PR_IR_t (1), as recorded on a monthly basis. The parameters of the model as estimated are shown in Table 1. In the construction of a dynamic model describing changes in output, a conclusion was applied which is used in the dating of economic activity phases. The tests that were carried out have shown that the model which takes into account the first and twelfth lag (a counterpart of the short- and long-term changes) gives the best results.

Table 1. Parameters of Hamilton's switching model

Parameters (standard error)	Probability of transition	
ϕ_1 = 0.952 (0.007)	p_{11} = 0.920	p_{12} = 0.027
ϕ_{12} = - 0.547 (0.032)	p_{21} =0.079	p_{22} = 0.973
	Expected regime i duration	
μ_1 = 317.204 (6.548)	d_1 = 12.577	d_{22} = 36.873
μ_2 = 316.875 (6.545)	Unconditional probability	
σ_1 = 0.736 (0.052)	$P(s_t = 1)$ = 0.254	$P(s_t =2)$ = 0.746
σ_2 = 0.175 (0.011)	Expected time of regime i resumption	
R^2 = 0.9829	$mt(1)$ = 3.932	$mt(2)$ = 1.341

Changes of output indices PR_IR_t, phases of recession S_IM and the probability of the occurrence of regime 1 (Rg1_SmProbs) are shown in Fig. 3. Our analysis implies that regime 1 corresponds with decreased economic activity. The high probability of regime 1 covers not only the recession periods S_IM but also non-implied decline periods S_NIM. The non-implied decline phase usually precedes an economic recession. The period of June 2004 to May 2005 was one of significant turbulence in the Polish economy and was connected with the EU accession boom. The period's prosperity was disturbed by a decline as well as by non-implied growth. None of the observations, however, has been considered a time of economic recession. Under the Hamilton model a high probability of regime 1 occurring can be observed for this period. Assuming that regime 1 was to appear, with the probability of its occurrence exceeding 0.5, then the accuracy of indications was evaluated. Higher accuracy rates were obtained with the assumption that regime 1 corresponds to a combination of two decline phases (indications for both decline phases proved to be 90% accurate, with a total accuracy of 97%; as compared to the accuracy for the recession phase equal to 94%, with a total accuracy of 81%). It is worth noting that both regimes

differ insignificantly in terms of the expected value. The period of lower economic activity is characterised by definitely higher variance. Both activity states are relatively durable (with the probability of remaining in the lower activity phase equal to 0.920, during a prosperity phase equal to 0.973), whereas the probabilities of transition between the states are small. The table also shows the unconditional probabilities which indicate a higher probability of prosperity occurring (0.746). Prosperity is also characterised by a longer expected duration period (*c*.37 months) and a shorter time of return from the state of lower economic activity (*c*.1.3 months).
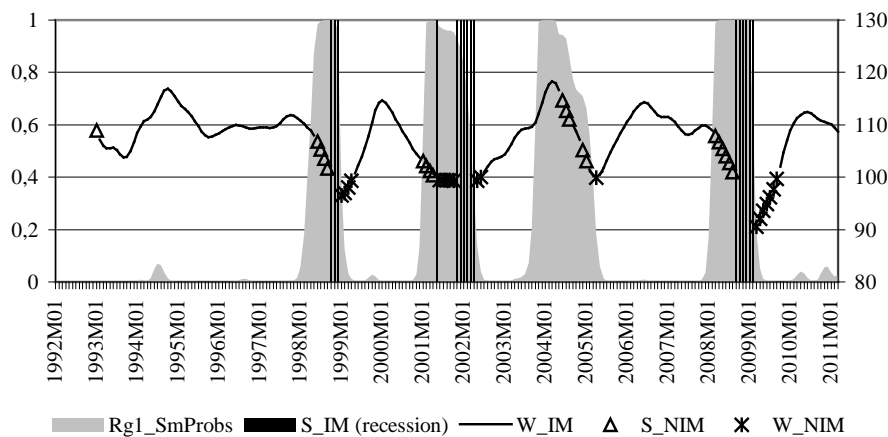
Figure 3.   Hamilton model-based probability of regime 1, set against the background of the economic activity phases in Poland

The second research approach using the logit model assumed that the moments of switching to the recession phase are known. The qualitative explained variable y_t assumes a value of 1 if the observation concerned an implied decline *S_IM*, otherwise it takes on the value of 0. The model was used to estimate the probability of a recession occurring on the basis of values assumed by selected leading indicators. They were represented in the model by annual indices of change – the exception being the interest rate spread as the difference in the interest rate of the ten-year treasury notes and the three-month treasury bills. Should the sample be balanced (the number of 1s being identical with the number of 0s for the explained variable), we assume that a recession is to occur if the probability of it occurring exceeds 0.5. For an unbalanced sample (which appeared in the research study), the limit value is usually assumed to be the share of 1s in the sample. Measures of fit and the accuracy of estimations from the logit models are shown in Table 2. The forecast of symptomatic variables (leading indicators) was determined based on the maximum value of McFadden's R^2 (determination coefficient) and Akaike's information criterion.

Table 2. Logit models and their verification

Model	Variable	Lag	Parameter estimate	P-value	McFadden R-squared	Akaike information criterion	Frequency of 1s (empirical)	Accuracy, in total	Accuracy of 1s
M1	SPREAD	12	-0.345	1.40E-03	**0.122**	78.36	0.105	76%	**54%**
	Const		-2.771	1.88E-11					
M1A	SPREAD	12	-0.340	1.89E-02	0.412	55.82	0.099	82%	85%
	PR_IR	1	-0.333	3.00E-04					
	Const		30.710	6.00E-04					
M2	M1_IR	7	-0.105	4.40E-03	0.083	97.98	0.105	65%	44%
	Const		9.647	1.78E-02					
M2A	M1_IR	7	-0.06	1.72E-01	0.322	75.49	0.105	78%	88%
	PR_IR	1	-0.269	4.98E-05					
	Const		32.07	1.00E-04					
M3	M2_IR	4	0.077	3.31E-02	0.046	102.44	0.103	63%	56%
	Const		-11	9.00E-03					
M3A	M2_IR	4	0.023	6.23E-01	0.309	77.24	0.103	80%	100%
	PR_IR	1	-0.267	4.12E-05					
	Const		22.481	1.76E-02					
M4	M3_IR	4	0.069	6.08E-02	**0.035**	103.58	0.103	63%	**63%**
	Const		-10.08	1.85E-02					
M4A	M3_IR	4	0.026	5.63E-01	0.280	80.23	0.103	78%	100%
	PR_IR	1	-0.254	3.82E-05					
	Const		20.914	1.91E-02					
M5	WIG_IR	2	-0.058	5.95E-06	**0.312**	79.91	0.083	80%	**94%**
	Const		2.981	4.20E-03					
M5A	WIG_IR	2	-0.082	4.20E-03	0.409	70.00	0.087	84%	94%
	PR_IR	1	0.017	8.71E-01					
	Const		3.443	6.96E-01					
M6	SPREAD	12	-0.354	1.65E-02	**0.462**	**53.59**	**0.099**	**88%**	**100%**
	M3_IR	4	0.006	9.59E-01					
	WIG_IR	2	-0.085	3.80E-03					
	Const		3.811	7.85E-01					

Taking the individual indicators into account, the WIG stock-exchange index appears to have the highest symptomatic properties. Unfortunately, from a practical standpoint, a two-month forecast is too short. Due to the length of the forecast period, the interest rate spread better fits the purpose. A higher accuracy of indications for the recession phase, given a four-month forecast, was obtained for annual changes in the index of money supply M3. In all cases the accuracy of indications was clearly improved if a lagging industrial output index was

added to the model. An attempt was also made in this research to combine the symptomatic properties of three leading indicators (i.e. index of money supply M3, the stock-exchange index and the interest rate spread). This latter model is characterised by the highest degree of measure of fit, the lowest Akaike's criterion value as well as the highest accuracy of indications within the sampling period. The probability of occurrence of a recession based on the M6 model is shown in Figure 4.
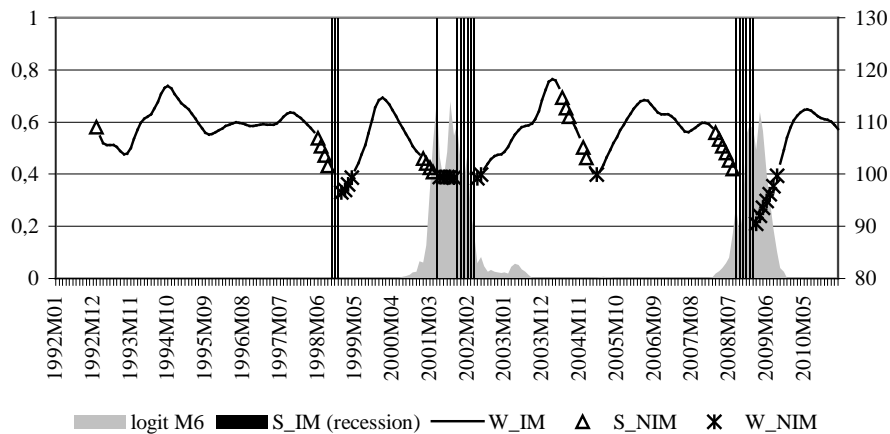
Figure 4. Probability of a recession based on the logit model M6 and the recession phases in Poland

Summarising the results of the research – the duration of a recession was analysed based on the proposed dating method, the Hamilton model and one of the logit models – the M6 (Table 3).

Table 3. Recession time periods

Beginning of recession				End of recession			
S_IM, S_NIM	S_IM	Hamilton Model	Logit Model M6	S_IM, S_NIM	S_IM	Hamilton Model	Logit Model M6
1998-06		1998-04		1998-12		1998-12	
2001-01	2001-05	2001-01	2001-02	2002-04	2002-04	2002-03	2002-04
2004-06		2003-10		2005-01		2005-02	
2008-03	2008-09	2008-02	2008-08	2009-02	2009-02	2009-02	2009-09

The classic approach was additionally adopted in the comparisons, which states that every phase should last at least half a year (Bry, Boschan, 1971). This allowed us to exclude short periods of implied decline in the comparisons and means that the rise or drop of a measure must remain on both sides of the turning point for at least six months. For the Hamilton model this was the rise/drop

of the probability of recession above/below the limit value of 0.5. For the logit model, as a result of not balancing the statistic sample, the limit value was the empirical frequency of 1s occurring for the qualitative explained variable. In the proposed dating method it was the change in behaviour of the annual and monthly output indices.

All of the analysed dating methods show significant convergence in terms of dating the end of recession (a 1-month difference). The only exception is the logit model, which shows the end of the last recession to be later. However, the dating of the beginning of recession by the Hamilton model shows convergence with the results from the logit model in 2001 and the two phases of decline *S_NIM, S_IM* in 1998 and 2001. In the case of the last economic recession, the Hamilton model and the combined phases *S_NIM* and *S_IM* indicate the beginning of recession in February/March 2008, while the logit model and the phase of implied decline *S_IM* indicate it as late as in August/September. Yule's coefficient of association, based on Chi-square statistics, which is a measure of interdependence of qualitative variables measured in a nominal scale, was used to quantitatively estimate the similarity of indications (it has a value from -1 to 1). The indications of the Hamilton model demonstrate the highest level of association with the indications of two phases of decline *S_NIM* and *S_IM* – 0.55, however, the logit model indications demonstrate the highest level of association with the indications of implied decline *S_IM* – 0.42.

5. Conclusions

The research conducted shows the convergence of indications based both on the proposed dating method and on the Hamilton model. In the presented version the Hamilton model adequately describes the probability of occurrence of two decline phases. The dating of phases as proposed in Section 2 is simple in empirical applications and allows for a division into four phases of economic activity, which is beneficial as it distinguishes two transitory phases.

The logit model allows to gain satisfactory results for a more detailed division of the recession phase *S_IM*, based on the value of selected leading indices. However, in the domain of the Polish economy, more research is needed in recognising the symptomatic properties of various macroeconomic indicators. The interest rate spread, used successfully in advanced marked economies, continues to alter its characteristics under Polish economic conditions and is currently not the best possible indicator forecasting a recession.

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Prawdopodobieństwo kryzysu w Polsce z modelu przełącznikowego Hamiltona i modelu logitowego

Z a r y s t r e ś c i. W wielu krajach brakuje wypracowanego systemu oznaczania początku i końca kryzysu. W proponowanej metodzie periodyzacji każda z czterech faz aktywności gospodarczej opisywana jest przez koniunkcję wartości rocznych i miesięcznych indeksów produkcji przemysłowej. Analitycy rynku zwracają szczególną uwagę na zróżnicowanie zachowania się większości wskaźników makroekonomicznych w czasie spadków i długookresowego wzrostu. W związku z tym uzasadnione jest założenie o zmieniających się parametrach modeli opisujących kształtowanie się tych wielkości. Realizację takiego założenia umożliwiają zarówno modele przełącznikowe jak i modele logitowe. W badaniach empirycznych przeprowadzono porównanie prawdopodobieństwa wystąpienia kryzysu z obu modeli Analiza wyników pokazuje duże podobieństwo wskazań z zaproponowanej metody periodyzacji i modelu Hamiltona. Model Hamiltona w prezentowanej wersji dobrze opisuje prawdopodobieństwo wystąpienia dwóch faz spadkowych. Model logitowy pozwala na uzyskanie zadawalających rezultatów dla podziału bardziej szczegółowego. Na gruncie gospodarki polskiej należy jednak w dalszym ciągu prowadzić badania nad rozpoznaniem własności symptomatycznych różnych wskaźników makroekonomicznych.

S ł o w a k l u c z o w e: model przełącznikowy, model logitowy, periodyzacja faz aktywności gospodarczej, prawdopodobieństwo kryzysu.