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Research Article

Capacity Analysis of Multiple-input-multiple-output System Over Rayleigh and Rician Fading Channel

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ABSTRACT

This paper aims to analyze the channel capacity in terms of spectral efficiency of a multiple-input-multiple-output (MIMO) system when channel state information (CSI) is known using water-filling algorithm and unknown at the transmitter side which it has been shown that the knowledge of the CSI at the transmitter enhancing the performance, the random Rayleigh and Rician channel models are assumed. Ergodic capacity and outage probability are the most channel capacity definitions which are investigated in this study. MATLAB code is devised to simulate the capacity of MIMO system for different numbers of antenna nodes versus different signal-to-noise ratio (SNR) values. In addition, the outage capacity probabilities for vary transmission rate and SNR are discussed.

Keywords: Channel state information, ergodic capacity, multiple-input-multiple-output, outage capacity, rayleigh, rician, water filling

INTRODUCTION

Recently, the evolving technologies demand for multiple-input-multiple-output (MIMO) system is explosively growing; MIMO system offers greater capacity than limited capacity of conventional single-input single-output (SISO). MIMO system can able to improve channel capacity, range, and reliability without requiring any additional bandwidth or transmit power. The large transmission rates associated with MIMO channels are due to the fact that a rich scattering environment provides independent transmission paths from each transmit antenna to each receive antenna, i.e., multipath fading channel is produced. Although fading is caused a performance poverty in wireless communication, MIMO channels use the fading to increase the capacity.[1,2] MIMO channel capacity is related to the Shannon theoretic sense. The Shannon capacity is the maximum mutual information of a single user time-invariant channel corresponds to the maximum data rate that can be transmitted over the channel with arbitrarily small error probability. In the time-varying channel, the channel capacity has multiple definitions, depending on what is known about the instantaneous channel state information (CSI) at both transmitter and receiver, and whether or not capacity is measured based on averaging the rate overall channel states or maintaining a fixed rate for most channel states. When CSI is known perfectly both at transmitter and receiver, the transmitter can adapt its transmission strategy relative to the channel using many power control algorithms for wireless networks.[3,4] Therefore, channel capacity is characterized by the ergodic or outage. Ergodic capacity can be defined as expected value of channel capacity which is suitable for fast varying channels. The outage probability is the maximum rate that can be maintained in all channel states with some probability of outage. When only the receiver has perfect knowledge of the CSI, then the transmitter must maintain a fixed rate transmission strategy based on knowledge of the channel statistics only, which can include the full channel distribution just its mean and variance are known. In this case, ergodic capacity defines the rate that can be achieved through this fixed rate strategy based on receiver averaging overall channel states.[5] Alternatively, the transmitter can send at a rate that cannot be supported by all channel states, in these poor channel states that the receiver declares an outage and the transmitted data are lost.

The remainder of this paper is organized as follows. The next section presents the multipath fading channels (Rayleigh and Rician). Capacity of MIMO system will be studied in section 3. In section 4, ergodic capacity and outage capacity notions are explained. Section 5, the simulation results with discussions will detail. Finally, major conclusion of the paper is discussed.

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FADING CHANNEL MODELS

Rayleigh Channel

The Rayleigh fading channel is composed from the effects of multipath embrace constructive and destructive interference, and phase shifting of the signal. The Rayleigh distribution is commonly used to describe the statistical time-varying nature of the received envelope of the flat fading signal, or the envelop of an individual multipath component, where there is no line of sight (LOS) path means no direct path between transmitter and receiver.

The envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution.[6]

The Rayleigh distribution has a probability density function (pdf) given by:

$$P(r)=\begin{cases}\dfrac{r}{\sigma^2}e^{\frac{-r^2}{2\sigma^2}} & r\geq 0\\ 0 & r<0\end{cases} \quad (1)$$

Where, $\boldsymbol{\sigma}^2$ is the time-average power of the received signal before envelope detection.

Rician Channel

When there is LOS, direct path is normally the strongest component goes into deeper fade compared to the multipath components. This kind of signal is approximated by Rician distribution. Rayleigh fading with strong LOS content is said to have a Rician distribution or to be Rician fading.[7]

The Rician distribution has a probability density function (pdf) given by:

$$P(r)=\begin{cases}\dfrac{r}{\sigma^2}e^{\frac{-(r^2+A^2)}{2\sigma^2}}I_0\left(\dfrac{Ar}{\sigma^2}\right) & A\geq 0, r\geq 0\\ 0 & r<0\end{cases} \quad (2)$$

Where, I_0 is the modified Bessel function of the first kind and zero order. The parameter A denotes the peak amplitude of dominant signal.

CAPACITY OF MIMO SYSTEM

CSI Not Available at the Transmitter

MIMO system provides a significant capacity gain over a traditional SISO channel, given that the underlying channel is rich of scatters with independent spatial fading. MIMO systems offered increasing in channel capacity based on the diversity at both the transmitter and the receiver and also on the number of antennas at both sides. When the transmit power is equally distributed among the N transmit antennas in case of CSI is perfectly known at the receiver but unknown at the transmitter, MIMO capacity with N Tx and M Rx antennas can be given in terms of bit per second per Hertz by,

$$C_{MIMO}=\log_2\left[\det\left(I_M+\frac{\rho}{N}HH^*\right)\right] \quad bps/Hz \quad (3)$$

Where, ρ is the signal to noise ratio (SNR), I_M is the identity matrix, (*) means transpose-conjugate, and H is the $M \times N$ channel transmission matrix. This equation can be reduced as follows:[8]

$$C_{MIMO}=\sum\nolimits_{i=1}^{m}\log_2\left(1+\frac{\rho}{N}\lambda_i\right) \quad bps/Hz \quad (4)$$

Where, m = min (M, N) λ_i are the eigenvalues of $H H^*$, known that the square root of λ_i is the diagonal matrix, D. The "diagonalization" process has been achieved by performing the singular value decomposition of matrix H which put it in the form.

$$H=U\,D\,V^* \quad (5)$$

With D being a diagonal matrix, and U and V unitary orthonormal matrices.

CSI Available at the Transmitter (Water-filling Algorithm)

Water-filling algorithm is an optimum solution related to a channel capacity improvement. This algorithm makes CSI known at the transmitter side which can be derived by maximizing the MIMO channel capacity under the rule that more power is allocated to the channel that is in good condition and less or none at all to the bad channels.

Algorithm steps:[9]

1. Take the inverse of the channel gains
2. Water filling has non-uniform step structure due to the inverse of the channel gain
3. At first, take the sum of the total power P_t and the inverse of the channel gain. It gives the complete area in the water filling and inverse power gain

$$P_t+\sum\nolimits_{i=1}^{n}\frac{1}{H_i} \quad (6)$$

4. Decide the initial water level by the formula given below by taking the average power allocated (average water level)

$$\frac{P_t+\sum\nolimits_{i=1}^{n}\frac{1}{H_i}}{\sum channels} \quad (7)$$

5. The power values of each subchannel are calculated by subtracting the inverse channel gain of each channel

$$power\ allocated=\frac{P_t+\sum\nolimits_{i=1}^{n}\frac{1}{H_i}}{\sum channels}-\frac{1}{H_i} \quad (8)$$

6. In case, the power allocated value becomes negative stop the iteration process.

The capacity of a MIMO system is algebraic sum of the capacities of all channels and is given by the formula below.

$$\text{Capacity}=\sum\nolimits_{i=1}^{n}\log_2\left(1+\text{power allocated}*\text{H}\right) \quad (9)$$

ERGODIC CAPACITY AND OUTAGE CAPACITY

Ergodic Capacity

MIMO channels are random in nature. Therefore, H is a random matrix, which means that its channel capacity is also randomly time varying. By taking the ensemble average of the information rate over the distribution of the elements of the channel matrix H, we obtain the ergodic capacity of a MIMO channel. Hence, the MIMO channel capacity can be modeled as follows:[10]

$$C = E\{C(H)\} \quad (10)$$

The ergodic channel capacity without using CSI at the transmitter side, from (6), is given as follows:

$$C = E\left\{\sum_{i=1}^{m}\log_2\left(1+\frac{\rho}{N}\lambda_i\right)\right\} \quad (11)$$

Similarly, the ergodic channel capacity for using CSI at the transmitter side is given as follows:

$$C = E\left\{\sum_{i=1}^{m}\log_2\left(1+\frac{\rho\gamma_i^{opt}}{N}\lambda_i\right)\right\} \quad (12)$$

Where, γ_i^{opt} is the power coefficient that corresponds to the amount of power assigned to the i^{th} channel.

Outage Capacity

Outage capacity is used for slowly varying channels where the instantaneous SNR is assumed to be constant for a large number of transmitted symbols. Unlike the ergodic capacity definition, schemes designed to achieve outage capacity allow for channel errors. Hence, in deep fades, these schemes allow the data to be lost and a higher data rate can be thereby maintained than schemes achieving Shannon capacity, where the data need to be received without an error overall fading states.[11] The parameter P_{out} indicates the probability that the system can be in outage, i.e., the probability that the system cannot successfully decode the transmitted symbols. Corresponding to this outage probability, there is a minimum received SNR, SNR_{min} given by $P_{out} = P(SNR<SNR_{min})$, for received SNRs below SNR_{min}, the received symbols cannot be successfully decoded with probability 1, and the system declares an outage. In other words, the system is said to be in outage if the decoding error probability cannot be made arbitrarily small with the transmission rate of R bps/Hz. The definition of the outage probability as follows:

$$P_{out}(R) = P(C(H)<R) \quad (13)$$

RESULTS AND DISCUSSION

MIMO channel capacity and outage probability over Rayleigh and Rician fading channel are simulated using MATLAB m-file for different antenna configurations, different SNR, and with and without CSI at the receiver. The transmission channel matrix H is chosen randomly from the probability distribution of the considered fading channels. Assuming Rayleigh distribution of unity σ and Rician distribution of unity σ and unity peak amplitude (A) except in Figures 1 and 2, where σ has values 1 and 2.

Ergodic Capacity

The ergodic capacity average of 10,000 random generated matrix H in case of CSI unknown and known at the transmitter side, i.e. without and with using water filling of the system (in terms of b/s/Hz) is calculated for Rayleigh and Rician channels over a wide range of SNR.

The capacity of SISO system is compared in Figures 1 and 2 with 2 × 2 and 4 × 4 MIMO over Rayleigh channel and Rician channel, respectively, for a two different values of σ. It is noticeable that the capacity is increased as SNR increases and the capacity is increased for increases in number of antennas in both transmitter and receiver sides. In addition, the capacity increases as σ value increases.

Figure 3 shows that the water-filling algorithm has improved the transmission capacity for both channels of 4 × 4 MIMO, especially for low SNR. This figure depicts that the Rician of unity σ and unity peak amplitude (A) has a higher capacity compared with Rayleigh of unity σ. Moreover, the improvement is achieved over Rayleigh rather than Rician channel.

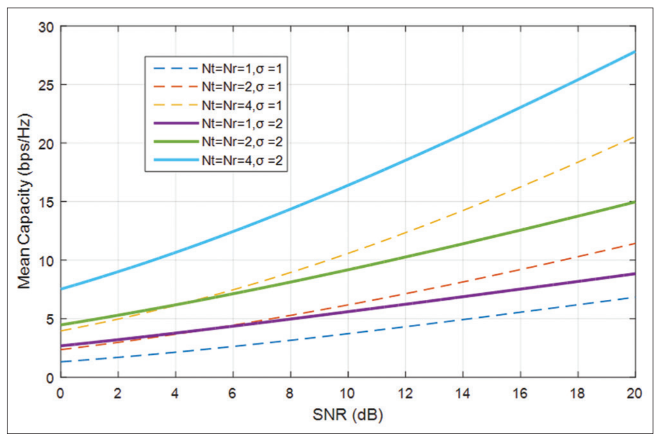

Figure 1: Single-input single-output and multiple-input-multiple-output capacity over Rayleigh channel

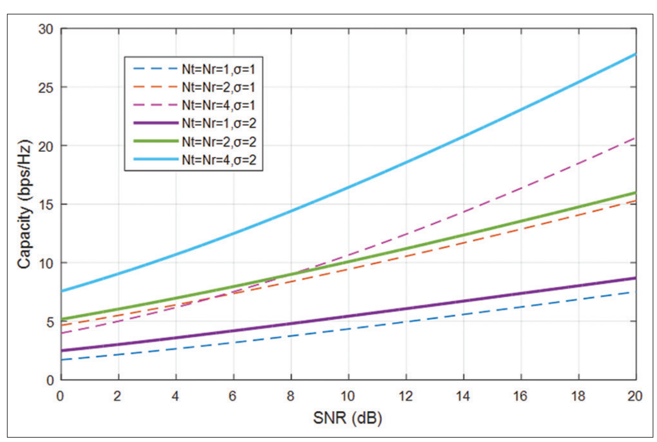

Figure 2: Single-input single-output and multiple-input-multiple-output capacity over Rician channel

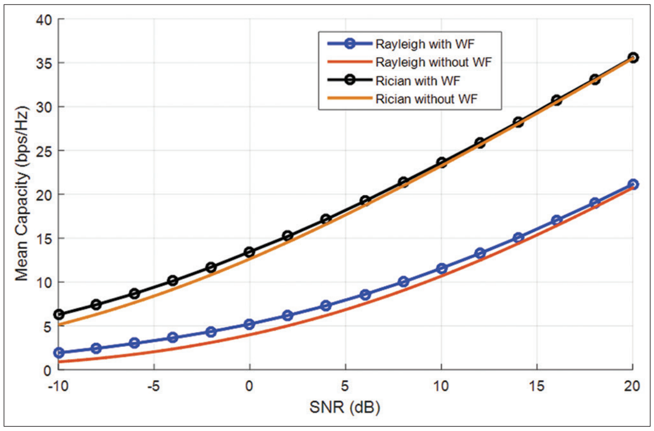

Figure 3: The capacity with and without using water-filling algorithm over Rayleigh and Rician channel

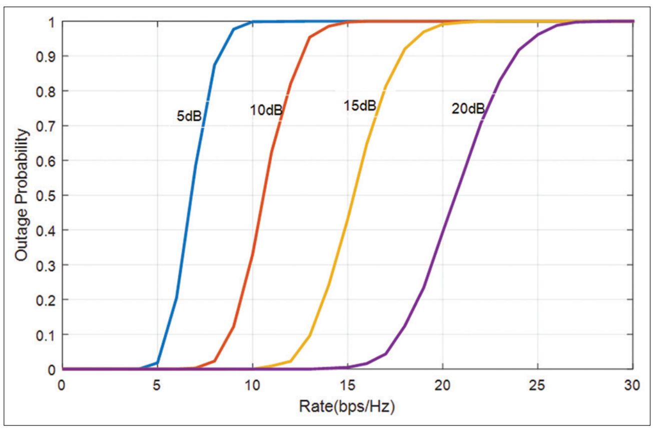

Figure 4: The outage probability versus transmission rate with different signal-to-noise ratio threshold

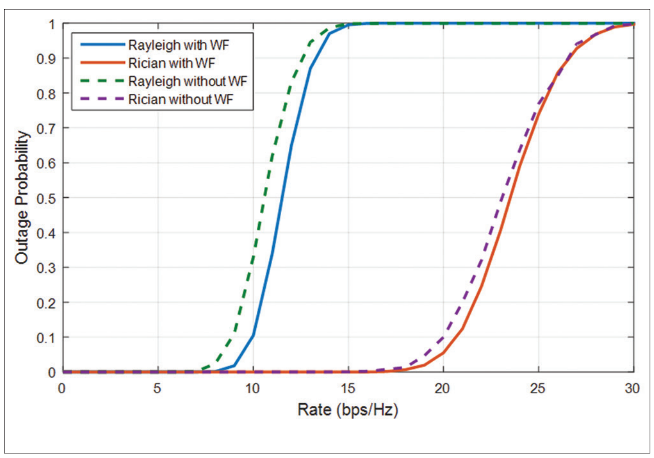

Figure 5: Outage probability versus transmission rate with signal-to-noise ratio threshold 10 dB in Rayleigh and Rician channel

Outage Capacity

Outage capacity is useful to estimate when CSI is known at the receiver but not at the transmitter. Therefore, the capacity is a random variable, and no matter how small the rate threshold would be, there is always a non-zero probability that the channel is incapable of supporting arbitrarily low error rates. In this section, the outage capacity probability versus transmission rate is simulated for 4×4 MIMO system.

The outage capacity probabilities are shown in Figure 4 for a MIMO channel for various value of SNR (5, 10, 15, and 20). The figure shows the variance of the instantaneous capacity increased as the SNR increased for the same system.

The outage capacity probabilities are shown in Figure 5 for Rayleigh and Rician channel with and without applying water-filling algorithm, and it has been clear that Rician is better compared with Rayleigh channel due to the LOS is existing between the antennas in the formal channel which makes a higher receiving signal level. The figure depicts the outage capacity probabilities mitigated much more when applying water filling on the Rayleigh channel.

CONCLUSION

In this paper, analysis and comparison of the MIMO was presented when CSI is known/not known at the transmitter over fading channel models by calculating the ergodic capacity and outage capacity probabilities. The results show that the capacity was enhanced when CSI known at the transmitter by applying water-filling algorithm on both assumed channel models. This study concludes that water-filling algorithm gives more ergodic and outage capacity improvement over the Rayleigh channel than the Rician channel considering their simulation parameters although the performance of Rician channel is obviously better, in general, than Rayleigh channel.

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