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What day of the week is it?[1,2]

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Abstract

C.F. Gauss (1798), Rev. C.J.J. Zeller (1883) and Lewis Carroll (1887) were among the mathematicians of the 19th century who were fascinated by the calculation of how one obtains the weekday from a given date in the Gregorian calendar by some simple method. We present a history of the topic and state the results of the computations.

1. Introduction

The Gregorian calendar is the civil calendar used today. It was promulgated by Pope Gregory XIII on February 24, 1582 with the issuance of the papal bull, *Inter Gravissimas*, upon the advice of scientists, among whom were the prominent astronomers / mathematicians Luigi Lilio and Christoph Clavius. For approximately 1600 years previously, the Julian calendar had been used. The Gregorian came into effect in certain countries [GFK(Band III): 266-279] on Friday, October 5, 1582 (Julian) = October 15, 1582 (Gregorian). Most countries did not adopt it until later, some as late as the 1920s [V3: 274-277]. The Julian calendar is sometimes referred to as Old Style (OS); the Gregorian calendar as New Style (NS). There was the necessity of replacing the Julian calendar because it displaced the time of Easter, eq. # which

[1] This is an updated version of a talk [CH1] given at the Université du Québec à Montréal at a conference of the Canadian Society for the History and Philosophy of Mathematics (Learned Society Conference), 3-5 June 1995.

[2] I would like to thank the following professors for their help in obtaining references: Francine F. Abeles (Kean College, SCNJ), Michael P. Closs (University of Ottawa), Robert E. Dressler (Kansas State University), Ernst J. Kani (Queen's University), Ralph G. Tross (University of Ottawa).

was supposed to be after the vernal (spring) equinox.[3]

The main difference between these two calendars was the placement of leap years: every year divisible by four was a leap year in the Julian calendar; whereas those years divisible by 100, but not divisible by 400, were not leap years in the Gregorian calendar. This slight change (e.g., 1700, 1800, 1900 are not leap years; however 2000 is) almost made up for the past inequities of the Julian calendar. Because the leap years are as stated herein, a complete cycle of the Gregorian calendar could not occur before 400 years and that is precisely what happens. Use any of the methods below to determine that January 1, 1601 and January 1, 2001 fall on the same weekday or note that there are $146097 \equiv 0 \pmod 7$ days in 400 years.

The perennial problem with the calendar is that the earth does not revolve around the sun in exactly $365\frac{1}{4}$ days. In fact, the Gregorian calendar overcompensates by approximately 26 seconds a year in a 400 year period [BFA: 26] and will eventually have to be corrected [BFA: 26]. However, the Julian calendar's error was far worse [BFA: 25]. There was also a day-rotation error, but now that is fixed by astronomers who have occasionally inserted leap seconds since 1972 [GM: 1,3], [V2: 4]. For more information on the history of the Gregorian calendar, see [CH2], [CH3], [CH4], [DN1], [DN2], [MI1], [MI2], [RD: Ch.19], [SA1], [V1].

2. Notation &c.

Let D, d be integers. If $\frac{D}{d} = Q + \frac{R}{d}$ $(0 \le R < d)$, we define $\left[\frac{D}{d}\right] :\equiv Q$ and $\left\{\frac{D}{d}\right\} :\equiv R$. Let k = day of the month; m = month (with numbers associated as specified in each of the methods below); C = century; Y = particular year of the century; N = year = $100C + Y$, where N = the designated year unless otherwise stated; U = weekday (Sunday = 1, Monday = 2, $\cdots$, Friday = 6, Saturday = 0); V = weekday (Saturday = 1, Sunday = 2, $\cdots$, Thursday = 6, Friday = 0); W = weekday (Sunday = 0, Monday = 1, $\cdots$, Friday = 5, Saturday = 6); DL = Dominical (Sunday) Letter (explained later).

We are only interested in the assembling of discoveries on weekday-date algorithms. Thus the papers and books here mentioned may include more information than on this subject. In particular, many of the articles discussed also have an Easter component since computationwise the problem can be quite similar. In general, we do not include encyclopedias (e.g., Encyclopedia Americana [EA: 184-191], which has a nice perpetual calendar table (discussed in §6)) and almanacs (e.g., American Ephemeris and Nautical Almanac, which counts October 12, 1492 (Julian) as day 2,266,296 = Friday [PA5]; see §7 for further explanation). Most of these encyclopedias and almanacs no

[3] One acceptable definition of Easter can be found in De Morgan [DM2: 364], [DMI: viii (lst ed.), xii (3rd ed.)]: "In both styles, old and new, Easter Day is the Sunday following that *fourteenth day* of the *calendar* moon which happens *upon* or *next after* the twenty-first of March: so that if the said *fourteenth day* be a Sunday, Easter Day is not that Sunday, but the next."

doubt have a weekday-date formula taken from some mathematical or church document. Bradley [SM] made a survey of the Gregorian-weekday topic in 1955. Included in that paper were [AM], [GW], [J], [MM3], [MT1], [MT2], [S2], [UH] and [Z3].

Converting between the Julian calendar and the Gregorian is fairly easy. Sometimes, in these articles, the Julian dates are given because of the interest in particular ones. However, in all cases, the authors do supply the Gregorian equivalent after the year 1582CE.

3. The Gauss Method

Gauss was probably the first to determine methodically the weekday from the date. Actually, he determined which weekday of any year January 1 happens to be. From this, one can easily obtain the weekday of any other date in the same year. He gave three determinations [GW]:

(a) $$V_{Jan\ 1} \equiv 1+(N-1701)365+\left[\frac{N-1701}{4}\right]-\left[\frac{N-1701}{100}\right]+\left[\frac{N-1601}{400}\right] \ (mod\ 7);$$

which reduces to:

$$\equiv N+2+\left[\frac{N-1}{4}\right]-\left[\frac{N-1}{100}\right]+\left[\frac{N-1}{400}\right] \ (mod\ 7);$$

(b) From N = 1601 until N = 2000,

$$W_{Jan\ 1} \equiv 6+6N+5+\left\{\frac{N-1}{4}\right\}+4\left\{\frac{N-1}{100}\right\} \ (mod\ 7);$$

(c) $$W_{Jan\ 1} \equiv 1+5\left\{\frac{N-1}{4}\right\}+4\left\{\frac{N-1}{100}\right\}+6\left\{\frac{N-1}{400}\right\} \ (mod\ 7).$$

The following usually mimic formula (a) of Gauss. Piper [C1] has similar type formulas for both the Gregorian and Julian calendars. Schocken [PA4] has given such a general formula and it is stated: "With minor alterations this formula can be adopted to the Julian, the Soviet, and even the Moslem Calendar." Schocken [WS: 22-25] refers to his same formula [PA4] and gives other Gauss-like quotient and remainder formulas for both the Julian and Gregorian calendars; also a Carroll-like formula-table is presented [WS: 19-21]. Comstock [S2] gives a formula similar to (a) and from this Running [PA2] presents a formula for February 1 to observe which years have five Sundays in February. Running [MT2] also has a formula for any date of the year, given the exact date numericals; e.g., Feb. 2 = 33, Dec. 25 = 359 (in a non-leap year). Skolnik [MT3] has an unusual formula in the form of Gauss; however, the terms are dissimilar.

A rule given by Krach [ST1] is a variant, but also fits into this Gauss grouping. He takes notable dates obtained from famous history books and encyclopedias to work on. His separation of the Julian and Gregorian calendars begins on September 14, 1752 because England and its American colonies did not change until that day. The starting point is 1 January, 1CE = Saturday.

Zeller [Z1, Z2, Z3] actually gave two rules in his papers. The not-so-famous one is more like (a) of Gauss. It is noted by Rouse Ball [RB: 242][4] and is quoted by Merrill [PA5]:

$$U = \left\{k + 2m + \left[\frac{3(m+1)}{5}\right] + N + \left[\frac{N}{4}\right] - \left[\frac{N}{100}\right] + \left[\frac{N}{400}\right] + 2\right\} (mod\ 7),$$

where m = the m$^{\text{th}}$ month of the year, starting with March = 3, April = 4,$\cdots$, December = 12. Then January = 13, February = 14 of the preceding year. The other procedure, which we call "The Zeller Method," is discussed below in §4. Zeller retains some notoriety because this latter method is so bizarre that the rule might never have been discovered except by him.

4. The Zeller Method

The Zeller method [Z1, Z2, Z3] is sometimes given in elementary number theory books [R], [UH] to illustrate the modulus concept. It uses the quite unusual concept that the cumulative shift (mod 7) in days from one month to the next (starting in March) is equal to $[2.6m - 0.2] - 2$, a completely arithmetic procedure. The weekday is as follows:

$$W = \left\{k + [2.6m - 0.2] - 2C + Y + \left[\frac{Y}{4}\right] + \left[\frac{C}{4}\right]\right\} (mod\ 7),$$

where m = the month of the year starting with March = 1, April = 2,$\cdots$, December = 10. Then January = 11, February = 12 of the preceding year.

Abeles [GB] notes that the reception into English of Zeller's rule, originally given in Latin [Z1] and German [Z3], was apparently first noticed in an English article by Franklin [MM3] approximately forty years later.

5. The Carroll Method

The method here is to obtain 4 numbers connected with the century, the year of the century, the month, and the day of the month. Add these 4 numbers; divide by 7; consider the remainder, which will be one of the numbers from 0 to 6. These seven numbers will represent Sunday through Saturday in some order. The only deviation seems to be the number attached to the month, m, and whether a leap year is under consideration.

[4] This book (*Mathematical Recreations and Problems*) has gone through a number of editions. In the second edition, the Zeller solution is on page 213; in the fourth edition it is on page 347. The first printing of the first ten editions are as follows: 1892, 1892 (originally reprinted as a first edition), 1896, 1905, 1911, 1914, 1917, 1919, 1920, 1922. These were printed while Rouse Ball was alive (1850 - 1925). From the fourth edition on, the book was called *Mathematical Recreations and Essays*. The 11th, 12th and 13th editions were prepared by and revised by HSM Coxeter, co-author, and were published, respectively, in the years 1939, 1974, 1987. It no longer had the Zeller formula, being revised so as to fit the 'new age' but to keep the book's spirit. As Coxeter states in the 1987 edition: "During the 61 years since Rouse Ball died, mathematical knowledge has increased enormously,$\cdots$"

The same problem (of the months) arose when we were considering Zeller's first method; however, Rev. Zeller had a formula: $[2.6m - 0.2] - 2$. Thus when considering the Carroll method [N1], there are, instead of a formula, 12 numbers (according to the number of months) plus a factor of a leap year.

Carroll seems to have been one of the first to use this method. After his use, there were many others, both amateur and professional mathematicians. A list (which, of course, must be incomplete) follows: [C2], [VH: 124] & [P1: 131-132] (similar, but in tabular form); [PS2] with Julian BCE dates; [N2]; [Sl]; [MTl]; [AM] (examples of Easter dates and ordinary dates, both Julian and Gregorian); Jacoby [J: 381-384] has a formula similar to the one of [AM]; White [PA3] finds that January 1, year 1 (Julian) was on a Saturday; the same Carroll-type method that White [PA3] uses is found in the dictionary under *Calendar* by [FW: 375-376]; Rydzewski [PA1] finds out that December 25, 1642 (OS), the birthdate of Isaac Newton, was a Sunday; [MM4] obtains the Dominical Letter, i.e., essentially finding (for a particular year) which letter from A to G is Sunday and noting that the months in a non-leap year, e.g., are in the order ADD G, BEG C, FAD F. (lf C = Sunday, then D = Monday, etc). For more information on the Dominical Letter, see [BJJ], [P3: 33-40], [A: 4ff.] or [W: §6]. Stark [S: 113-116] in his elementary number theory book uses a Carroll congruence method in the explanation. Conway [E] worked out another Carroll-type method, which is also presented in the book [BCG]. Several Carroll-type methods are given in the Journal of Calendar Reform, which ran from 1931-1956 [JCR1, JCR2, JCR3]. The first of these methods is a repetition from another journal [AMS]. In [P3: x] we find a similar method for the 20th century (a slight modification of which will work for other centuries).

Example: December 11, 1935

The rule given by [N2] is one of the simplest and most curious. Let m be the number associated as follows: January = 0, February or March = 1, April = 2, May = 3, $\cdots$, November = 9, December = 10, next January = 11, next February = 12. For a Leap Year, January and February must count as 11 and 12 in the *preceding* year.

Let $4m = 10q + r$; then $R = 10q - r$.

U (the weekday)$\equiv P+Q+R+S$ $(mod\ 7)$, where $P = 5 \times \left\{\frac{C}{4}\right\}$, $\quad Q = Y + \left[\frac{Y}{4}\right]$, R (as above), $S = k$.

For 1935. December 11:

$P = 5 \times 3...... \equiv 1; Q = 35 + 8...... \equiv 1; R = 10q - r = 40 - 0 = 40 \equiv 5;$ $S = 11 \equiv 4$ $(mod\ 7)$.

$U \equiv 1 + 1 + 5 + 4\ (mod\ 7) \equiv 4\ (mod\ 7)$.

Therefore it is a **Wednesday.**

6. The Perpetual Calendar Tables (1601 - 2000)

(i) A perfectly acceptable, but not very mathematical, method to determine the weekday is by tables. If one looks at any of the World Almanacs [WA: 294-295], one can find a Gregorian perpetual calendar from the years 1582-2000. This contains 14 calendars: 7 for regular years with January 1 falling on Sunday through Saturday; 7 for leap years idem. The 14 symbols for these calendars are, respectively: 1,2,3,4,5,6,7,a,b,c,d,e.f,g. [P2: 298-317] does the same for the years 1–2000 (OS); 1582–2000 (NS).

The perpetual Gregorian calendar (mod 400) is as follows:

eq. #
1601-1650
2 3 4 e 7 1 2 c 5 6; 7 a 3 4 5 f 1 2 3 d; 6 7 1 b 4 5 6 g 2 3; 4 e 7 1 2 c 5 6 7 a; 3 4 5 f 1 2 3 d 6 7;
1651-1700
1 b 4 5 6 g 2 3 4 e; 7 1 2 c 5 6 7 a 3 4; 5 f 1 2 3 d 6 7 1 b; 4 5 6 g 2 3 4 e 7 1; 2 c 5 6 7 a 3 4 5 6;
1701-1750
7 1 2 c 5 6 7 a 3 4; 5 f 1 2 3 d 6 7 1 b; 4 5 6 g 2 3 4 e 7 1; 2 c 5 6 7 a 3 4 5 f; 1 2 3 d 6 7 l b 4 5;
1751-1800
6 g 2 3 4 e 7 1 2 c; 5 6 7 a 3 4 5 f 1 2; 3 d 6 7 1 b 4 5 6 g; 2 3 4 e 7 1 2 c 5 6; 7 a 3 4 5 f 1 2 3 4;
1801-1850
5 6 7 a 3 4 5 f 1 2; 3 d 6 7 1 b 4 5 6 g; 2 3 4 e 7 1 2 c 5 6; 7 a 3 4 5 f 1 2 3 d; 6 7 1 b 4 5 6 g 2 3;
1851-1900
4 e 7 1 2 c 5 6 7 a; 3 4 5 f 1 2 3 d 6,7; 1 b 4 5 6 g 2 3 4 e; 7 1 2 c 5 6 7 a 3 4; 5 f 1 2 3 d 6 7 1 2;
1901-1950
3 4 5 f 1 2 3 d 6 7; 1 b 4 5 6 g 2 3 4 e; 7 1 2 c 5 6 7 a 3 4; 5 f 1 2 3 d 6 7 1 b; 4 5 6 g 2 3 4 e 7 l;
1951-2000
2 c 5 6 7 a 3 4 5 f; 1 2 3 d 6 7 1 b 4 5; 6 g 2 3 4 e 7 1 2 c; 5 6 7 a 3 4 5 f 1 2; 3 d 6 7 1 b 4 5 6 g.

(ii) Woolhouse [W: 149-157] obtains the Dominical Letter (DL) for any particular Gregorian year [A if the year begins on Sunday, B on Saturday,···, G on Monday] in a perpetual table [W: 154] or by arithmetical calculations and uses another table to show the day of the week [W: 155]. To find the DL by calculations, a formula like that of Gauss is employed [W: 152-153]. De Morgan [DM1: 7-9] gives the DL for the years 1582 - 2000 in a table. In this preface [DM1] De Morgan credits Francœur [F2] with the "effective plan of uniting the thirty-five almanacs, and indicating the proper one for each year by an index." Francœur [F2] has the DL for the Gregorian calendar on pages 34-45 for the years 1582 to 2200. Bond [BJJ] was, like the previous two [DM1, F2], another famous calendar book of the 19th century, written by the Assistant Keeper of the Public Records, with DLs ranging from 1CE (OS) and 1582 (NS). VanWijk [VW], although mainly concerned with the Jewish calendar, has on page 27 the DL (OS) for the purpose of obtaining the weekday for any year CE.

(iii) Fitch [F1] expounds on the perpetual calendar for both the Julian and Gregorian years. Since England (and hence the American Colonies) adopted the NS only in 1752, he gives the coronation of George II on June 11, 1727 (OS) as occurring in the calendar "1" on Sunday, and the coronation of George III on October 25, 1760 (NS) as occurring in the calendar "c" on Saturday. June 11, 1727 (OS) is the same as June 22, 1727 (NS), occurring in calendar "4"–again a Sunday.

(iv) Frisby [PS1] gives a movable perpetual table calendar. Roman [MM1] has a short discussion of adjustable perpetual calendars, of which many have been patented. Morris [MM2] presents a particular perpetual calendar which he patented on November 26, 1918. Franklin [MM3] notes, in regard to the Morris [MM2] article, Zeller's first formula [Z3], which he considers an "arithmetical" perpetual calendar. It technically belongs in §4.

(v) Kraitchik [K] has two (of many) nomograms (graphs with movable straight edge) for perpetual calendars. In [MM5], the author states three methods to obtain a perpetual calendar. The first one is typical of Zeller's second formula in his famous paper [Z3] to obtain the Dominical Letter and technically should be in §4; from this one constructs a nomogram to calculate a calendar from the year 1CE to the year 2299CE (Julian or Gregorian); lastly, a slide rule perpetual calendar is shown going from the year 1CE to 2099CE.

7. Julian Period

Julian days (JD) were devised by Joseph Scaliger in 1582 [WA: 296] and named after his father Julius. Julian day #1 began at noon on January 1, 4713BCE. Noon of January 1, 2000 = the beginning of day 2,451,545. Astronomers find it convenient to use this Julian period. Weekdays are found by the modulus of a division by 7. A similar concept is the "fixed date" or *rata die* (DR) described by [SA2], beginning on the Gregorian date 1 January 1582. Nice explanations of both DL and JD can be found in [EB: 664-682] under *Calendar.*

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