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On a pencil of K_3 surfaces *

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Resumen.

Una superficie K_3 es una superficie analítica compleja $S(\dim_{\mathbb{C}}S=2)$ que es simplemente conexa y cuyo fibrado canónic K_5 es trivial. Para las superficies K_3 que son algebraicas (es decir, inmersas en P_n), existe un espacio de módulos $\mathcal{M}(K_3)$ que es 19 dimensional.

En esta nota se construye una subvariedad 1-dimensional de $\mathcal{M}(K_3)$ con un grupo de simetrías fijo $(\mathbb{Z}/4\mathbb{Z})^2 \times G_4)$ y se describen explícitamente sus degeneraciones.

Let P_3 be the complex projective space with coordinates $(x_1, z_2, x_3, z_4) \in P_3$. The section $F \in H^o(P_3, O_F_4(4))$ given by $F(x_1, x_2, x_3, z_4) = x_4^4 + x_3^4 + x_4^4$, defines the classical Fermat's quartic. Consider the section $T \in H^o(P_3, O_F_4(4))$ given by $T(x_1, x_2, x_3, x_4) = x_1 x_2 x_3 x_4$, as F and T are linearly independent the 2-dimensional vector subspace $S = (F, T) \subset H^o(P_3, O_{P_3}(4))$ defines a pencil of quartic surfaces that we will denote by:

$$S_{\alpha} = \{F + \alpha T / \alpha \in P_1\}$$

Each generic quartic S_{α} of the pencil is smooth, by Lefschetz's theorem on hyperplane sections, S_{α} is simply-connected, furthermore as $K_{S_{\alpha}} \cong \mathcal{O}_{S_{\alpha}}(4 - (3 + 1)) \cong \mathcal{O}_{S_{\alpha}}$ the canonical sheaf is trivial and S_{α} is a K_{3} surface.

We begin describing the base locus of our pencil.

Proposition 1 The base locus of $(S_a)_{a \in P_1}$ denoted by $B(S_a)$ consists of a stable curve of genus g = 33 with associated graph given by Fig 1.

Proof We consider the four hyperplanes $H_i = \{x_i = 0\}$ with $i \in \{1, ..., 4\}$. The intersection of F = 0 with each hyperplane $H_i, i \in \{1, ..., 4\}$, gives a Fermat's quartic of genus g = 3, that we denote by $F_i, i \in \{1, ..., 4\}$. The intersection $F_i \cap F_j$ is contained in each line $F_i \cap F_j = L_{ij}$ and consists of four differents points.

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Then the base locus is a stable curve, that consists of four curves F_i of genus g = 3, where each F_i cuts each of the other three F_j at four points and the graph associated corresponds to Fig. 1, the genus g = 33 follows from the well known formula [1] for graph curves.



It is well known that the biggest group of projective automorphisms of F = 0(Fermat's quartic surface) is isomorphic to $G \cong (\mathbb{Z}/4\mathbb{Z})^3 \times \mathcal{G}_4$, where \mathcal{G}_4 is the symmetric group. The group G can be represented in $PGL(4, \mathbb{C})$ as $g_{ijk}(x_{1i}, x_{2i}, x_{3j}, x_{4j}) = (\xi^i x_{1i}, \xi^j x_{2i}, \xi^k x_{3i}, x_{4j})$ with ξ a primitive rooth of the unity of order 4 and \mathcal{G}_4 acting by permutation of coordinates.

As the section $T \in H^o(P_3, Q_{P3}(4))$ is also invariant by G, then $G \subseteq Aut(T)$ and the 2-dimensional vector subspace $S = \langle F, T \rangle \subset H^o(P_3, O_{P3}))$ is G-invariant.

We have the following proposition.

Proposition 2 The group G acts on the pencil by multiplication by i and the subgroup $H \cong (\mathbb{Z}/4\mathbb{Z})^2 \times \mathcal{G}_4$ acts trivially.

Proof By definition the subspace $S \subset H^o(P_3, O_{P_3}(4))$ is invariant by the action of G then the group G acts on the pencil.

We consider the epimorphism:

 $\mu: \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \longrightarrow \mathbb{Z}/4\mathbb{Z}$

given by $\mu(\xi^i, \xi^i, \xi^k) = \xi^i \xi^j \xi^k$, ker μ is isomorphic to $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$ and each $h \in H = (\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z})^2$ acts trivially on T, as \mathcal{G}_4 acts also trivially on this section, we have the result.

An elementary calculation allows to obtain the values of $\alpha \in P_1$ where S_α is singular and we have the following proposition.

Proposition 3 For each projective K_3 surface in the pencil $(S_\alpha)_{\alpha \in P1}$ we have that $H \subseteq Aut$ (S_α) . There are only five singular fibers in the pencil, they correspond to the values $\alpha = \{i, -1, -i, 1\}$ and $\alpha = \infty$. For $\alpha \neq \infty$ the singular fibers are Kummer's surface, for $\alpha = \infty$ the singular fiber is of type

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III, a rational surface meeting along rational curves its dual graph is the tetrahedron.

Proof For each $\alpha \in \{i, -1, -i, 1\}$ an easy calculation proves that S_{α} has the maximum of ordinary double points (16 double points) then by [2] S_{α} is a Kummer surface.

For $\alpha = \infty$ it is clear that S_{∞} consists of the union of four rational surfaces (the hyperplanes $H_i = \{x_i = 0\}, i \in \{1, ...4\}$) where two of them intersects along a rational curve.

Bemark. As every Kummer surface is isomorphic to a surface of the form $\mathcal{J}ac(C)/\{\pm 1\}$, where C is a curve of genus 2, and $\mathcal{J}ac(C)$ denotes the Jacobi variety. Then in our case $\mathcal{J}ac(C)$ must admit \mathcal{G}_4 as reduced group of automorphisms.

Finally it would be interesting to study the variation of the Picard number of S_{α} as in [4].

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Proof The most $N \in [1, -1, -1, 1]$ is a case out the last prove that S_0 has the formula of arthmetic provide points (16 double points) that by (2) $S_0 = 0$ Maximize and each sector S_0 and S_0 and S_0 and S_0 and S_0 .

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