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Somewhat Fuzzy Semi α-Irresolute Functions

V. Seenivasan
P.G. Department of Mathematics, Jawahar Science College
Neyveli – 607 803, Tamilnadu, India
email: `krishnaseenu@rediffmail.com`

G. Balasubramanian
Ramanujan Institute for Advanced Study in Mathematics
University of Madras, Chennai, 600 005, Tamilnadu, India.
and
G. Thangaraj
P.G. Department of Mathematics, Jawahar Science College
Neyveli – 607 803, Tamilnadu, India

ABSTRACT

In this paper the concepts of somewhat fuzzy semi α-irresolute functions and strongly somewhat fuzzy semi-open functions are introduced and studied. Besides giving characterizations of these functions, some interesting properties of these functions are also given.

RESUMEN

En este artículo son introducidos y estudiados lon conceptos de funciones semi fuzzy α-irresoluta y funciones fuertemente fuzzy semi-abiertas. Propiedades de esta clase de funciones son dadas.

Key words and phrases: *Somewhat fuzzy semi α-irresolute, fuzzy semi α-irresolute, fuzzy α-irresolute, somewhat fuzzy semi-open functions.*

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1 Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by L.A.Zadeh [12]. Fuzzy sets have applications in many fields such as information [6] and control [8]. The theory of fuzzy topological spaces was introduced and developed by C.L.Chang [3] and since then various notions in classical topology have been extended to fuzzy topological spaces. The concept of somewhat continuous functions was introduced by Karl R.Gentry and Hughes B.Hoyle III in [4] and this concept was studied in connection with the idea of feeble continuous function and feebly open function introduced and by Zdenek Frolik in [13]. The concept of semi α-irresolute functions was introduced and studied in [11]. The concepts of somewhat fuzzy continuous functions and somewhat fuzzy semi-continuous functions was introduced and studied by G. Thangaraj and G. Balasubramanian in [9] and [10] respectively. In this paper we introduce the concepts of somewhat fuzzy semi α-irresolute functions and strongly somewhat fuzzy semi-open functions and study their properties.

2 Preliminaries

By a fuzzy topological space we shall mean a non-empty set X together with fuzzy topology T [3] and denote it by (X, T). A fuzzy point in X with support $x \in X$ and value p ($0 <$ p ≤ 1) is denoted by x_p. The complement $\mu^{'}$ of a fuzzy set μ is $1-\mu$, defined by $\mu^{'}(x) = (1_X - \mu)(x) = 1 - \mu(x)$, for all $x \in X$ [3]. If λ is a fuzzy set in X and μ is a fuzzy set in Y, then $\lambda \times \mu$ is a fuzzy set in $X \times Y$, defined by $(\lambda \times \mu)(x, y) = \min(\lambda(x), \mu(y))$, for every (x, y) in $X \times Y$ [1]. A fuzzy topological space X is product related to a fuzzy topological space Y [1] if for fuzzy sets γ in X and ξ in Y whenever $\lambda^{'}(= 1 - \lambda) \not\geq \gamma$ and $\mu^{'}\ (= 1 - \mu) \not\geq \xi$ (in which case $(\lambda^{'} \times 1) \vee (1 \times \mu^{'}) \geq (\gamma \times \xi)$) where λ is a fuzzy open set in X and μ is a fuzzy open set in Y, there exists a fuzzy open set λ_1 in X and a fuzzy open set μ_1 in Y such that ${\lambda_1}^{'} \geq \gamma$ or ${\mu_1}^{'} \geq \xi$ and $({\lambda_1}^{'} \times 1) \vee (1 \times \mu_1^{'}) = (\lambda^{'} \times 1) \vee (1 \times \mu^{'})$. Let $f : X \to Y$ be a mapping from X to Y. If λ is a fuzzy set in X, $f(\lambda)$ is defined by

$$f(\lambda)\ (y) = \begin{cases} \underset{x \in f^{-1}(y)}{Sup}\ \lambda(x), & if \quad f^{-1}(y) \neq \phi \\ 0, & otherwise, \end{cases}$$

for each $y \in Y$; and if μ is a fuzzy set in Y, $f^{\ -1}(\mu)$ is defined by $f^{\ -1}(\mu)\ (x) = \mu\ f(x)$, for each $x \in X$ [3]. Let f be a mapping from X to Y. Then the graph g of f is mapping from X to $X \times Y$ sending x in X to $(x,\, f(x))$. For two mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$, we define the product $f_1 \times f_2$ of f_1 and f_2 to be a mapping from $X_1 {\times} X_2$ to $Y_1 \times Y_2$ sending $(x_1,\, x_2)$ in $X_1 \times X_2$ to $(f_1(x_1),\, f_2\,(x_2))$. For any fuzzy set λ in a fuzzy topological space, it is shown in [1] that (i) $1 - \mathrm{cl}\ \lambda = \mathrm{int}(1-\lambda)$, (ii) $\mathrm{cl}(1-\lambda) = 1 - \mathrm{int}\ \lambda$. A fuzzy set λ in fuzzy topological space $(X,\, T)$ is called fuzzy α-dense if there exists no fuzzy α-closed set μ such that $\lambda < \mu < 1$.

Definition 2.1 *[5]: A function $f : (X,\, T) \to (Y,\, S)$ is said to be* ***fuzzy α-irresolute*** *if $f^{-1}(\lambda)$ is fuzzy α-open set in $(X,\, T)$ for every fuzzy α-open set λ of $(Y,\, S)$.*

Definition 2.2 *[7]: A function f from a fuzzy topological space $(X,\, T)$ to a fuzzy topological space $(Y,\, S)$ is said to be* ***fuzzy semi α-irresolute*** *if $f^{-1}(\lambda)$ is fuzzy semi-open set in $(X,\, T)$ for each fuzzy α-open set λ in $(Y\,,\, S)$.*

Definition 2.3 *[10]: Let $(X,\, T)$ and $(Y,\, S)$ be fuzzy topological spaces. A function $f : (X,\, T) \to (Y, S)$ is called* ***somewhat fuzzy semi-open function*** *if and only if for all $\lambda \in T$, $\lambda \neq 0$ there exists a fuzzy semi-open set μ in Y such that $\mu \neq 0$ and $\mu \leq f(\lambda)$.*

Definition 2.4 *Let (X,T) be a fuzzy topological space and λ be any fuzzy set in X.*

1. *λ is called fuzzy α-open set [2] if $\lambda \leq int\ cl\ int\lambda$*
2. *λ is called fuzzy semi-open set [1] if $\lambda \leq cl\ int\lambda$*

The complement of fuzzy α-open (fuzzy semi-open) set is called fuzzy α-closed (fuzzy semi-closed) set.

3 Somewhat fuzzy semi α-irresolute functions

The concept of fuzzy semi α-irresolute functions was introduced and studied in [7]. In this section we shall introduce the concept of somewhat fuzzy semi α-irresolute functions and study their properties.

Definition 3.1 *Let (X,T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X,T) \to (Y, S)$ is said to be* ***somewhat fuzzy semi α-irresolute*** *if for any non-zero fuzzy α-open set λ in Y and $f^{-1}(\lambda\ ) \neq 0$ then there exists a fuzzy semi-open set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda\ )$.*

Clearly every fuzzy semi α-irresolute function is somewhat fuzzy semi α-irresolute, but the converse is not true as the following example shows:-

Example 3.1 *Let μ_1, μ_2 and μ_3 be fuzzy sets on $I = [0, 1]$*

$$\mu_1(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \le x \le 1, \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \le x \le \frac{1}{4} \\ -4x + 2, & \frac{1}{4} \le x \le \frac{1}{2} \\ 0, & \frac{1}{2} \le x \le 1, \end{cases}$$

$$\mu_3(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{4} \\ \frac{1}{3}(4x - 1), & \frac{1}{4} \le x \le 1. \end{cases}$$

Let $S_1 = \{0, \mu_2, \mu_3, (\mu_2 \vee \mu_3), (\mu_2 \wedge \mu_3), 1\}$ and $S_2 = \{ 0, \mu_2, 1\}$. Then S_1 and S_2 are fuzzy topologies on I. Let $f : (I, S_1) \to (I,S_2)$ be defined by $f(x) = x/2$ for each $x \in I$. Let λ be fuzzy set such that $0 < \lambda < \mu_2$. Then λ is not fuzzy α-open set in (I, S_2). Therefore the only non-zero fuzzy α-open sets in (I, S_2) are 1, μ_2 and fuzzy sets ρ such that $\mu_2 < \rho < 1$. Now $f^{-1}(1) = 1$; $f^{-1}(\mu_2) = \mu_1^{'}$ and $f^{-1}(\rho) = 1$. The fuzzy semi-open set μ_2 in (I, S_1) is contained in $f^{-1}(1)$, $f^{-1}(\mu_2)$ and $f^{-1}(\rho)$. This proves f ***is somewhat fuzzy semi α-irresolute function*** *from (I, S_1) to (I, S_2). It can be easily seen that int $\mu_1{}' = \mu_2$ and cl $\mu_2 = \mu_3{}'$ in (I, S_1). Now μ_2 is a fuzzy α-open set in (I, S_2). Since $\mu_1{}' \nleq$ cl int $\mu_1{}'$ in (I, S_1), $\mu_1{}'$ is not fuzzy semi-open set in (I, S_1). Then $f^{-1}(\mu_2) = \mu_1{}'$, which is not a fuzzy semi-open set in (I, S_1). Hence f* ***is not fuzzy semi α-irresolute function****.*

Theorem 3.1 *Let $f : (X,T) \to (Y, S)$ and $g : (Y, S) \to (Z,Q)$ be any two functions. If f is somewhat fuzzy semi α-irresolute and g is fuzzy α-irresolute, then $g \circ f$ is somewhat fuzzy semi α-irresolute.*

Proof: Let λ be non-zero fuzzy α-open set in (Z, Q). Since g is fuzzy α-irresolute, $g^{-\mathbf{1}}(\lambda) \neq 0$ is fuzzy α-open set in (Y, S). Now $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-\mathbf{1}}(\lambda)) \neq 0$. Since $g^{-\mathbf{1}}(\lambda)$ is fuzzy α-open in (Y, S) and f is somewhat fuzzy semi α-irresolute, we conclude that there exists a fuzzy semi-open set $\mu \neq 0$ in (X, T) such that $\mu \le f^{-1}(g^{-\mathbf{1}}(\lambda)) = (g \circ f)^{-1}(\lambda)$. Hence $g \circ f$ is somewhat fuzzy semi α-irresolute.

In above Theorem 3.1 if f is either fuzzy α-irresolute or fuzzy semi α-irresolute and g is somewhat fuzzy semi α-irresolute, then it is not necessarily true that $g \circ f$ is somewhat fuzzy semi α-irresolute as the following example shows:-

Example 3.2 *Define $f : I \to I$ by $f(x) = x/2$. Let μ_1, μ_2 and μ_3 be fuzzy sets in I described in Example 3.1. Let $T_1 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$; $T_2 = \{ 0, \mu_2^{'}, 1\}$ and $T_3 = \{0,$*

*$\mu_3{}'$, 1}. Then T_1, T_2 and T_3 are fuzzy topologies on I. Consider the mapping $f : (I, T_3) \to (I, T_1)$. It can be easily seen that int $\mu_1^{'} = \mu_3^{'}$; cl $\mu_3^{'} = 1$; cl $\mu_1^{'} = 1$ in (I, T_3). Since int $\mu_1^{'} = \mu_3^{'}$ and cl $\mu_3^{'}= 1$, $\mu_1^{'}$ is fuzzy semi-open set and also fuzzy α-open set in (I, T_3). Let λ, ρ and δ be fuzzy sets such that $0 < \lambda < \mu_1$, $\mu_1 < \rho < \mu_2$ and $\mu_2 < \delta < (\mu_1 \vee \mu_2)$. Then λ , ρ and δ are not fuzzy α-open sets in (I, T_1). Therefore the only fuzzy α-open sets in (I, T_1) are 0, 1 , μ_1, μ_2, $\mu_1 \vee \mu_2$ and fuzzy sets μ such that $(\mu_1 \vee \mu_2) < \mu < 1$. Now $f^{-1}(0) = 0$; $f^{-1}(1) = 1$; $f^{-1}(\mu_1) = 0$; $f^{-1}(\mu_2) = \mu_1^{'} = f^{-1}(\mu_1 \vee \mu_2)$ and $f^{-1}(\mu) = 1$ are fuzzy α-open sets in (I, T_3) and also fuzzy semi-open sets in (I,T_3). Therefore f **is fuzzy α-irresolute** from (I,T_3) to (I,T_1) and also f **is fuzzy semi α-irresolute** from (I, T_3) to (I, T_1). Let $g : (I, T_1) \to (I, T_2)$ be defined by $g(x) = x$, for each $x \in I$. Let λ be fuzzy set such that $0 < \lambda < \mu_2^{'}$. Then λ is not fuzzy α-open set in (I,T_2). Therefore the only non-zero fuzzy α-open set are 1, $\mu_2^{'}$ and fuzzy sets ρ such that $\mu_2^{'} < \rho < 1$. Now $g^{-\mathbf{1}}(1) = 1$; $g^{-\mathbf{1}}(\mu_2^{'}) = \mu_2^{'}$ and $g^{-\mathbf{1}}(\rho) = 1$. The fuzzy semi-open set μ_1 in (I,T_1) is contained in $g^{-\mathbf{1}}(1)$, $g^{-\mathbf{1}}(\mu_2^{'})$ and $g^{-\mathbf{1}}(\rho)$. Therefore g **is somewhat fuzzy semi α-irresolute function.***

*Now consider the functions $(g \circ f) : (I,T_3) \to (I, T_2)$. Then $(g \circ f)^{-1}(\mu_2^{'}) = f^{-1}(g^{-\mathbf{1}}(\mu_2^{'})) = f^{-1}(\mu_2^{'}) = \mu_1$ and $(g \circ f)^{-1}(1) = f^{-1}(g^{-\mathbf{1}}(1)) = f^{-1}(1) = 1$. But $(g \circ f)^{-1}(\mu_2^{'}) = \mu_1$ and there is no non-zero fuzzy semi-open set in (I,T_3) such that it is contained in $(g \circ f)^{-1}(\mu_2^{'}) = \mu_1$. Therefore $(g \circ f)$ **is not somewhat fuzzy semi α-irresolute function.***

Definition 3.2 *[10]: A fuzzy set λ in fuzzy topological space (X, T) is called **fuzzy semi-dense** if there exists no fuzzy semi-closed set μ such that $\lambda < \mu < 1$.*

Theorem 3.2 : *Let (X,T) and (Y,S) be any two fuzzy topological spaces and $f : (X,T) \to (Y,S)$ be a function. Then the following assertions are equivalent.*

(1) f is somewhat fuzzy semi α-irresolute.

(2) If λ is a fuzzy α-closed set in Y such that $f^{-1}(\lambda) \neq 1$, then there exists a fuzzy semi-closed set $\mu \neq 1$ in X such that $\mu \geq f^{-1}(\lambda)$.

(3) If λ is a fuzzy semi-dense set in X, then $f(\lambda)$ is fuzzy α-dense set in Y.

Proof: (1) $\Rightarrow$(2): Suppose f is somewhat fuzzy semi α-irresolute and λ is a fuzzy α-closed set in Y such that $f^{-1}(\lambda) \neq 1$. Therefore clearly $1 - \lambda$ is fuzzy α-open set in Y, and $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda) \neq 0$ (since $f^{-1}(\lambda) \neq 1$). By (1), there exists a fuzzy semi-open set η in X such that $\eta \leq f^{-1}(1 - \lambda)$. That is, $\eta \leq 1 - f^{-1}(\lambda)$ which implies that $f^{-1}(\lambda) \leq 1 - \eta$. Clearly $1 - \eta$ is fuzzy semi-closed set and taking $\mu = 1 - \eta$, we have therefore $f^{-1}(\lambda) \leq \mu$. Thus we find that (1) $\Rightarrow$(2) is proved.

(2) $\Rightarrow$ (3): Let λ be a fuzzy semi-dense set in X and suppose $f(\lambda)$ is not fuzzy α-dense in Y. Then there exists a fuzzy α-closed set η(say) in Y such that

$$f(\lambda) < \eta < 1. \qquad (A)$$

Since $\eta < 1$, $f^{-1}(\eta) \neq 1$ and so by (2) there exists a fuzzy semi-closed set δ ($\delta \neq 1$) such that $\delta \geq f^{-1}(\eta) > f^{-1}(f(\lambda)) \geq \lambda$ (from (A)). That is, there exists a fuzzy semi-closed set δ such that $\delta > \lambda$ which is contradiction to the assumption on λ .Therefore (2) $\Rightarrow$ (3) is proved.

(3) $\Rightarrow$(1): Suppose λ is fuzzy α-open set in Y and $f^{-1}(\lambda) \neq 0$ and therefore $\lambda \neq 0$. Suppose there exists no fuzzy semi-open set μ in X such that $\mu \leq f^{-1}(\lambda)$. Then $1 - f^{-1}(\lambda)$ is fuzzy set in X such that there is no fuzzy semi-closed set δ in X with $1 - f^{-1}(\lambda) < \delta < 1$(otherwise $1 - f^{-1}(\lambda) < \delta \Rightarrow 1 - \delta \leq f^{-1}(\lambda)$ and $1 - \delta$ is fuzzy semi-open set, which is a contradiction). This means $1 - f^{-1}(\lambda)$ is fuzzy semi-dense in X. Then by (3), $f(1 - f^{-1}(\lambda))$ is fuzzy α-dense in Y. But $f\ (1 - f^{-1}(\lambda)) = f\ (\ f^{-1}(1 - \lambda)) < 1 - \lambda < 1$(since $\lambda \neq 0$). This is contradiction to the fact that $f\ (1 - f^{-1}(\lambda))$ is fuzzy α-dense. Therefore, there exists a fuzzy semi-open set μ in X such that $\mu \leq f^{-1}(\lambda)$. Hence f is somewhat fuzzy semi α-irresolute function.

Theorem 3.3 *Let (X_1, T_1), (X_2, T_2), (Y_1, S_1) and (Y_2, S_2) be fuzzy topological spaces such that X_1 is product related to X_2 and Y_1 is product related to Y_2. Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ be somewhat fuzzy semi α-irresolute functions. Then $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is somewhat fuzzy semi α-irresolute function.*

Proof: Let $\lambda = \bigvee_{i,j}(\lambda_i \times \mu_j)$ be fuzzy α-open set in $Y_1 \times Y_2$ (where λ_i and μ_j are fuzzy α -open sets in Y_1 and Y_2, respectively). We can assume that λ_i's and μ_j's are not all zeros. If any one is zero, that factor can be omitted. Now $(f_1 \times f_2)^{-1}(\lambda) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\lambda_i \times \mu_j)) = \bigvee_{i,j}(f_1 \times f_2)^{-1}\ (\lambda_i \times \mu_j) = \bigvee_{i,j}(f_1^{-1}(\lambda_i) \times f_2^{-1}(\mu_j))$. Since $f_1 : X_1 \to Y_1$ is somewhat fuzzy semi α-irresolute and λ_i is fuzzy α-open set in Y_1 and $f_1^{-1}(\lambda_i) \neq 0$, there exists a fuzzy semi-open set δ_i in X_1 such that $\delta_i \leq f_1^{-1}(\lambda_i)$. Also since $f_2 : X_2 \to Y_2$ is somewhat fuzzy semi α-irresolute and μ_j is fuzzy α-open set in Y_2 and $f_2^{-1}(\mu_j) \neq 0$, there exists a fuzzy semi-open set η_j in X_2 such that $\eta_j \leq f_2^{-1}(\mu_j)$. Therefore $\delta_i \times \eta_j \leq f_1^{-1}(\lambda_i) \times f_2^{-1}(\mu_j) = (f_1 \times f_2)^{-1}\ (\lambda_i \times \mu_j)$. Then by Theorem 4.3 and Theorem 4.6 in [1] $\bigvee_{i,j}(\delta_i \times \eta_j)$ is a fuzzy semi-open set and $\bigvee_{i,j}(\delta_i \times \eta_j) \leq \bigvee_{i,j}\ (f_1 \times f_2)^{-1}\ (\lambda_i \times \mu_j) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\lambda_i \times \mu_j)) = (f_1 \times f_2)^{-1}(\lambda)$. This proves $f_1 \times f_2$ is somewhat fuzzy semi α-irresolute.

The following lemma which is established in [1] is required to prove Theorem 3.4.

Lemma 3.1 *[1]: Let $g : X \to X \times Y$ be the graph of a function $f : X \to Y$. If λ is a fuzzy set in X and μ is a fuzzy set in Y, then $g^{-\mathbf{1}}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu)$.*

Theorem 3.4 *Let $f : (X,T) \to (Y,S)$ be a function from fuzzy topological space (X,T) to another fuzzy topological space (Y,S).If the graph $g : X \to X \times Y$ of f is somewhat fuzzy semi α-irresolute, then f is somewhat fuzzy semi α-irresolute.*

Proof: Let λ be a non zero fuzzy α-open set in Y. Then, by Lemma 3.1, we have $f^{-1}(\lambda)$ $= 1 \wedge f^{-1}(\lambda) = g^{-\mathbf{1}}(1 \times \lambda)$. Since g is somewhat fuzzy semi α-irresolute and $1 \times \lambda \neq 0$ is a fuzzy α-open set in $X \times Y$, there exists a fuzzy semi-open set $\mu(\neq 0)$ (say) in X such that μ $\leq g^{-\mathbf{1}}(1 \times \lambda) = f^{-1}(\lambda)$. This proves that f is somewhat fuzzy semi α-irresolute function.

4 Strongly somewhat fuzzy semi-open functions

The concept of somewhat fuzzy semi-open function was introduced and studied in[10]. In this section we shall introduce a strongly notion as follows:-

Definition 4.1 : *Let $(X,\ T)$ and $(Y,\ S)$ be any two fuzzy topological spaces. A function f : $(X,\ T) \to (Y,\ S)$ is called **strongly somewhat fuzzy semi-open** if and only if for each non-zero fuzzy α-open set λ in (X,T), there exists a fuzzy semi-open set μ in (Y,S) such that $\mu \neq 0$ and $\mu < f(\lambda\ )$.*

Clearly every strongly somewhat fuzzy semi-open function is somewhat fuzzy semi-open function. However the converse is not true as the following example shows:-

Example 4.1 *Let μ_1, μ_2 and μ_3 be fuzzy sets in I described in Example 3.1. Clearly T_1 $= \{0,\ \mu_1,\ 1\}$ and $T_2 = \{0,\ \mu_2,\ 1\}$ are fuzzy topologies on I. Let $f : (I,\ T_1) \to (I,\ T_2)$ be defined by $f(x) = min\{2x,\ 1\}$ for each $x \in I$. It can be easily seen that int $\mu_3 = \mu_1$; cl $\mu_1 = 1$ in $(\ I,\ T_1)$. Simple computations gives $f(0) = 0$; $f(1) = 1$; $f(\mu_1) = 0$. Thus f **is somewhat fuzzy semi-open function**. Since $\mu_3 \leq$ int cl int μ_3 in (I,T_1), μ_3 is fuzzy α-open set in $(I,\ T_1)$. But $f(\mu_3)(y) = \begin{cases} 0, & 0 \leq y \leq \frac{1}{2}, \\ \frac{1}{3}, & \frac{1}{2} \leq y \leq 1. \end{cases}$ Let λ be any non-zero fuzzy set such that $\lambda \leq f(\mu_3)$ in (I,T_2). Then cl int $\lambda = 0$, this shows that λ is not fuzzy semi-open set. Thus there is no non-zero fuzzy semi-open set such that it is contained in $f(\mu_3)$. Hence f **is not strongly somewhat fuzzy semi-open** functions.*

Theorem 4.1 *Suppose (X, T) and (Y, S) be fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an onto function. If f is strongly somewhat fuzzy semi-open function and λ is a fuzzy semi-dense set in Y, then $f^{-1}(\lambda)$ is fuzzy α-dense in X.*

Proof: Suppose λ is a fuzzy semi-dense set in Y. We want to show that $f^{-1}(\lambda)$ is a fuzzy α-dense set in X. Suppose not. Then there exists a fuzzy α-closed set μ in X such that $f^{-1}(\lambda) < \mu < 1$. Then $1 - f^{-1}(\lambda) > 1 - \mu > 0$. $f\,(1 - f^{-1}(\lambda)) > f\,(1-\mu)$ which implies that $f(\,f^{-1}(1-\lambda) > f(1-\mu)$. That is, $f(1-\mu) < f\,(f^{-1}(1-\lambda)) = 1 - \lambda$. Now μ is fuzzy α-closed set $\Rightarrow 1-\mu$ is fuzzy α-open set in X. Since f is strongly somewhat fuzzy semi-open, $1-\mu$ is fuzzy α-open in $X \Rightarrow$there exists a fuzzy semi-open set $\delta \neq 0$ in Y such that $\delta < f(1-\mu)$. Therefore $\delta < f\,(1-\mu) < 1-\lambda \Rightarrow \delta < 1-\lambda \Rightarrow \lambda < 1-\delta$. Now $1-\delta$ is fuzzy semi-closed set and $\lambda < 1-\delta \Rightarrow \lambda$ is not a fuzzy semi-dense set in Y, which is a contradiction to our hypothesis. Therefore $f^{-1}(\lambda)$ must be a fuzzy α-dense in (X, T).

Theorem 4.2 *Suppose (X, T) and (Y, S) be fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be a* 1-1 *and onto function. Then the following conditions are equivalent.*

(1) *f is strongly somewhat fuzzy semi-open.*

(2) *If λ is a fuzzy α-closed set in X such that $f\,(\lambda) \neq 1$, then there exists a fuzzy semi-closed set μ in Y such that $\mu \neq 1$ and $f(\lambda) < \mu$.*

Proof: (1)$\Rightarrow$(2). Let λ be a fuzzy α-closed set in X such that $f(\lambda) \neq 1$. Then $1-\lambda$ is fuzzy α-open set and since f is 1-1 and onto $f(1-\lambda) = 1 - f(\lambda) \neq 0$ [3]. As f is strongly somewhat fuzzy semi-open, there exists a fuzzy semi-open set η in Y such that $\eta \neq 0$ and $\eta < f\,(1-\lambda) = 1 - f(\lambda)$. That is $f(\lambda) < 1-\eta = \mu$ (say) and μ is fuzzy semi-closed set. This proves (1)$\Rightarrow$(2).
(2)$\Rightarrow$(1). Let λ be a fuzzy α-open set in X such that $\lambda \neq 0$. Then $1-\lambda$ is fuzzy α-closed set and $1-\lambda \neq 1$. Now $f(1-\lambda) = 1 - f(\lambda) \neq 1$ (for, if $1 - f(\lambda) = 1$, then $f(\lambda) = 0 \Rightarrow \lambda = 0$). Hence by (2) there exists a fuzzy semi-closed set μ in Y such that $\mu > f(1-\lambda)$. Then $\mu > 1 - f(\lambda\,)$. That is $f(\lambda) > 1-\mu = \delta$ (say). Clearly δ is fuzzy semi-open set in Y such that $\delta < f(\lambda)$ and $\delta \neq 0$ (since $\mu \neq 1$). This completes the proof of (2)$\Rightarrow$(1).

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