Copernican Journal of Finance & Accounting

2019, volume 8, issue 4

e-ISSN 2300-3065
p-ISSN 2300-1240

Akan, M. (2019). Optimal control theoretic approach to investment in doctors. Copernican Journal of Finance & Accounting, 8(4), 91–111. http://dx.doi.org/10.12775/CJFA.2019.019

Mustafa Akan*
Haliç University

OPTIMAL CONTROL THEORETIC APPROACH TO INVESTMENT IN DOCTORS

Keywords: investment in doctors, optimal control theory, patient satisfaction.

J E L Classification: H51, C51, I10.

Abstract: Health care is ever more important with aging population. Assuming the number of doctors per patient is one of the determinants of patient satisfaction, optimal investment in practitioner doctors, specialist doctors and foreign doctors are analyzed given the total number of doctors (domestic) are exogenously determined. The high cost of investment in specialist doctors are weighted against the high salaries of imported foreign doctors.

An optimal control theory is employed to determine the optimal investment plans for the two alternative sources of specialist doctors to maximize the net (of costs) patient satisfaction over a fixed time horizon.

It is found that a nation with insufficient number of specialist doctors at the beginning of the time horizon should increase the investment in local specialist doctors gradually while employing foreign doctors as to equate their salaries to the marginal satisfaction of the patients. An equilibrium point exists, and it is stable.

Date of submission: November 19, 2019; date of acceptance: January 18, 2020.

[*] Contact information: mustafaakan@halic.edu.tr, Halic University Sutluce Mh. Imrahor Sk. No: 82 34445 Beyoglu/Istanbul, Turkey, phone: +905333119405; ORCID ID: https://orcid.org/0000-0002-2900-4932.

INTRODUCTION

Health care is the most important duty of all societies to provide their members with the best quality care within budgetary constraints. In almost all countries, governments are involved directly in providing this service by investing in the health infrastructure and healthcare personnel in addition to passing laws and regulations for the proper functioning of this sector.

Health spending accounted for 8.8% of GDP in 2018 while it accounted for 6.7% in 1990 in OECD countries. Per capita spending increased from 1180 USD in 1990 to 3992 USD in 2018 representing average annual growth rate of 6% (OECD, 2019a). The growth rate of spending for health was around 6% in 2000, decreasing to 3.4% in 2016(OECD, 2019a) which is still much higher than the growth rate of the economies of the OECD countries which was only 1.7% in 2017 (OECD, 2019b). According to another report by (OECD, 2019c), number of people in health and social work as a portion of total employment is more than 10%, which is very significant. It is clear that health sector is already large, and it will get even get larger given that the world population is increasing and getting older.

Major improvements in some key measures of health such as longevity, mortality rates, premature deaths, and infant deaths at birth are being observed due to high and continuous investment in health sector. Life expectancy reached 80.7 years in 2017 (74.7 in 1990) in OECD countries with significant increases in all countries (OECD, 2019d). It is clear that the increase in the life expectancy is positively correlated with the per capita health spending mentioned in the second paragraph. Parallel to this development, infant mortality rate decreased to 5.7 in 2017 from 17 per 1000 in 1990 (World Bank, 2017). Similarly, mortality rates for circulatory diseases have fallen sharply with 50% fewer death rates due to ischemic heart disease since 1990, while cancer mortality rates have fallen by 18% since the same year according to (OECD, 2017).

However, even after so much improvement in some key indicators, more problems remains to be solved. World Health Statistics (2018) shows that less than half world population get essential health services they need and 13 million people die every year before the age of 70 just from cardiovascular disease, chronic respiratory disease, diabetes and cancer. Fifteen thousand died before their fifth birthday every day in 2016, and 303,000 women died due to complications of pregnancy or childbirth in 2015.

It is obvious, even though the current level of investment in healthcare is large and growing, it is not sufficient to address the deficiencies in health such as the ones cited above. This is particularly true if we take into consideration that the population of the World is increasing and aging requiring more care.

There are many factors that go into the production of good quality health care to satisfy the customers such as sufficient number of healthcare facilities, doctors, nurses, laboratories, medical equipment, administrative personnel, in addition to availability of relevant pharmaceuticals at affordable prices. The existence of a well-functioning insurance sector and the quality of its cooperation with the health care system are important in the determination of the quality of the healthcare. We will concentrate only on the number of doctors as one important input into the patient satisfaction function assuming all other factors remain constant.

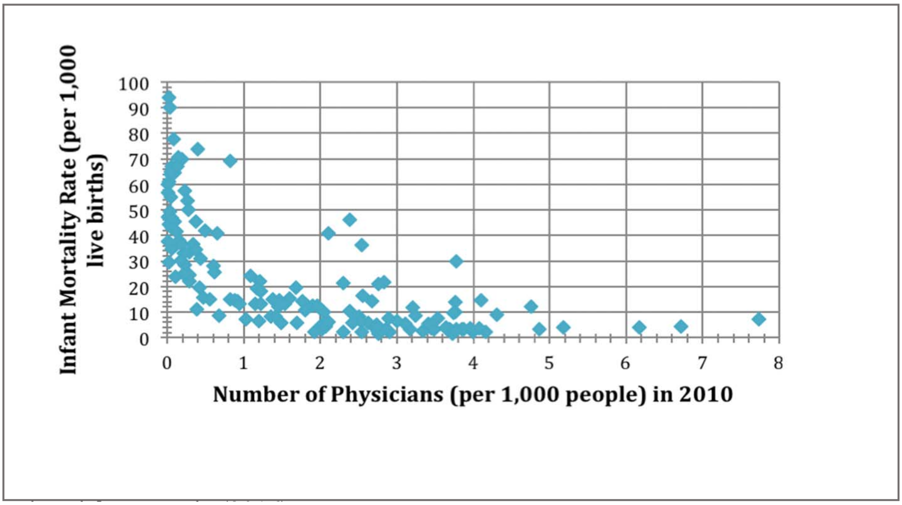
Nylenna, Bjertnaes, Saunes and Lindahl (2015) concluded that it was investment in high quality workforce to deliver good quality health care increasing patient satisfaction. Tehrani, Feldman, Camacho, Rajesh and Balakrishnan (2011) have conducted a study in the USA to measure the outpatient satisfaction and have found that waiting time for the doctor and the time spent with the patient were important factors in determining quality of outpatient care. Vuković, Gvozdenović, Gajić, Gajić, Jakovljević and McCormick (2012) has conducted a survey with over 1300 patients to explore the determinants of patient satisfaction and showed that timelines of the healthcare and the patient centeredness related to doctors' and nurses' commitment to the patients' health were very import determinants of health care quality. Boquiran, Hack, Beaver and Williamson (2015), after a search of 1726 articles, have found that communication, attributes of tangibles, relational conduct, technical skill/knowledge of medical staff, personal qualities, and availability/accessibility were the main factors with respect to patients' satisfaction with the doctors.

Batbata, Dorjdagva, Livsannyam, Savino and Amneta (2017) have shown that health providers' personal care quality was the essential determinant of patient satisfaction. Senić and Marinković (2012) have found that personal relationship, promptness and tangibility were the three most important factors affecting satisfaction, with personal relationship having the highest impact. Merkley and Bickmore (2017) interviewed patients regarding to responsiveness of hospital staff, cleanliness and quietness of the hospital environment, pain management, and communication about medicines, discharge information, and communication with clinicians. Based on their analysis of the answers, they recommended the use patient satisfaction as a balance measure, evaluation of entire care teams instead of individual caregivers, better analysis of data, better use of technology, and improvement in employee engagement.

In all of the studies cited above, dimensions of health care quality and patients' satisfaction such as engaging with patients, quality of health work force, responsiveness, communication with the patients, waiting time for the doctor, timeliness of care, empathy are factors all related to number and quality of health care personnel. It is obvious that the properly educated and experienced doctors are very important human resources to execute the provision of care using relevant medical facilities and administering correct medicine.

A study done by Pando (2016), with data on 147 nations, found that there was a strong relationship between number of physicians and infant mortality rates. The chart representing this result is reproduced in the figure below.

Figure 1. The Effects of Number of Physicians (2010) on Infant Mortality Rate (2013)



Source: reproduced from Pando (2016).

Farahani, Subramanian and Canning (2009) showed that increasing the number of physicians (per 1000 persons) by one percent increases the infant mortality by 15% in 5 years and by 45% in the long term. They also concluded that the effects of human resources on health are higher than previously estimated. Cooper (2009) has also shown that states (USA) with more specialist doctors have better health care. Xesfingi and Vozikis (2016) proved that there is a strong positive relationship between patient satisfaction and number of doctors and nurses.

Shetty and Shetty (2014) analyzed the relationship between life expectancy and number of doctors per capita in Asian countries and found that the population to physician ratio had a positive correlation with life expectancy. Hockey and Marshall (2009) have concluded that the education of doctors was ever more important in both theory and practice as the health care becomes more complex.

A report by World Health Organization, World Bank and OECD (2018), makes a call to action to all governments in several areas to achieve a quality health care system whereby high-quality health workforce to deliver quality care is of paramount importance.

Babcock, Babcock and Schwartz (2013) have shown that there will be a shortage of doctors and there is a need to increase the number of for-profit medical schools to alleviate this problem. Scheffler and Arnold (2019) forecasted that that there will be dramatic imbalances in the supply of medical personnel in OECD countries. They claimed that there will a shortage of 400,000 doctors and 2,500,000 nurses in 2030. Chojnicki and Moullan (2018) estimated that the supply and demand of doctors will stabilize in the long run but, in the short run, foreign(imported) doctors are needed. Sargen, Hooker and Cooper (2011) states that if the training programs grow as currently projected aggregate per capita supply of advanced clinicians will remain 25% below the demand in 2020. Scheffler, Liu, Kinfu and Poz (2008) have shown that there will be shortages of doctors especially in African countries.

In same study above Pando (2016) report that recommended number of doctors per 1000 persons by World Health Organization is 2.5, while the average for World was 1,83 suggesting that many countries have a ratio less than the target. This result strongly suggests that many nations have challenges in this area. This challenge is even more acute for less developed nations since the investments of educating doctors are very high.

Health spending was about 8.8% of GDP as mentioned above (OECD, 2019a). The breakdown of this percentage shows that it was 12.38% in high-income countries and around 5-6 % in all countries. This implies that the middle and the lower income countries have to increase their investment in health. It is stated in World Health Organization (2019) that 45% of WHO states have less than one physician per 1000 population. In 25 of the 39 OECD countries physicians per population is less than the OECD average of 3.4 (OECD, 2017). Grover, A. (2006) states that the investment in health care has many stakeholders and that politicians must be educated on the issue to be able to assess the need for increasing the number of physicians (specialists and practitioners) work force. The problem, even if there is political will on the part of the government, is optimal allocation of scarce government resources between foreign doctors, specialist doctors, and practitioner doctors.

Eurostat report (2018) defines the class of doctors as generalists (practitioners) and specialists. Specialists are further divided into two categories as medical specialist and surgical specialists. Governments can also employ foreign specialist doctors who specifically educated and experienced in a certain area. In the source just above the share of foreign doctors increased to 22% in 2010 from 20% in 2000.The share of foreign nurses increased to 14% in 2010 from 11% in 2000. The USA was a preferred destination with about 8000 doctors in 2015 and 20,000 nurses in 2014. UK was another favored destination of foreign health care personnel.

Investment in medical education of specialists is very expensive but the local doctors are paid less than the foreign doctors are. A report prepared by OLCDB (2018) presented by the Canadian Ministry of Health analyzed the cost of doctors in some provinces in Canada and found that it was approximately 200,000 USD-500,000 USD. Kerr, E. (2019) showed that the cost of medical education in the top 10 universities in the USA was around 63,000 USD per year. The cost was around 15,000 NZD (around 10,000 USD) for locals around 20,000 USD for foreigners in New Zealand and Australia (Crimson, 2017).

Given that quality of health care and its impact on patient satisfaction is significantly dependent on number of doctors, that there is a shortage of doctors, that the cost of educating doctors is costly, and that the problem is of paramount importance, the determination of optimal strategy for investment in doctors is important.

This paper will address the optimal investment in doctors, the principle elements of health care (foreign doctors, local specialist and practitioner doctors) in the long run, recognizing that the health care system is a complex one involving not only healthcare givers but also investment in health infrastructure, technology and regulations, which are all time dependent. Thus, this is a partial study of dynamic optimization in health care involving only doctors in in a dynamic (the long-term) context. This study is intended for the governments to define a proper health investment strategy in specialist doctors for their countries. Optimal Control Theory is the proper methodology to study dynamic optimization problems.

Optimal Control Theory has been used in medical area but mostly in relation to disease control. A study by Vernon and Hughen (2006) was an introduction of the use of optimal control theory in Pharma economics. They stated that optimal control theory technique had not yet been fully exploited by this field although it has been used extensively in mathematical biology and disease management. It has not been exploited in economic modelling in this sector such as optimal allocation of scarce health resources over time. Sharomi and Malik (2015) studied the use of optimal control specifically in epidemiology. Lin, Muthuraman and Lawley (2010) applied optimal control theory to non-pharmaceutical interventions. Hansen (2011) used optimal control to study infectious disease modelling. Momoh and Fügenschuh (2018) used optimal control theory in the modelling of intervention strategies to control Zika virus. Rowthorn and Toxvaerd (2012) employed optimal control for infectious disease prevention.

Faezipour and Ferreira (2013) addressed the patient satisfaction from holistic point of view considering the dynamic nature of complex systems affecting health care. They emphasize that the demands of the society, economic and environmental needs should be balanced with the available resources to ensure sustainable quality of life. However, no mathematical output function for patient satisfaction is proposed in any study to the best of author's knowledge.

We develop the optimal control theoretic model and the necessary conditions for its solution the next section. Results follow in the subsequent section. Conclusions and the Recommendations for Further Research section is the last section intended for academicians interested in the subject.

THE RESEARCH METHODOLOGY AND THE COURSE OF THE RESEARCH PROCESS

Following assumptions are made on the model formation:

- It is assumed that the government determines the number of new graduates such that the total number of doctors (specialist and practitioners) grow at the same rate (net of retirement) as the population. This assumption simplifies the solution reducing the solution to finding the optimal number of specialist and the foreign doctors.

- Satisfaction of the population from the quality of health care is a function of per capita number of specialist doctors, local (T) or foreigners (F), and the number of practitioner (P) doctors. Foreign and local specialist are considered to have the same effect on patient satisfaction.

Mathematically:

$$A(T(t)+F(t))^{\alpha}P(t)^{\gamma}$$

This is a variant of Cobb-Douglas production function with three inputs where γ≤α, α<1,and γ<1 which implies that the impact of specialist doctors on satisfaction is larger than that of the practitioners with decreasing return to scale. It also implies that the local and foreign specialists provide same degree of satisfaction. Cobb-Douglas function is used for its simplicity even though the existence of other production functions such as Constant Elasticity of Substitution and Leontief is recognized.

- Population is constant. This implies that the doctors per capita will increase by increasing of number of doctors.
- Salaries of all doctors are constant. Cost (Salaries and other expenses) of foreign doctors are assumed larger than that of the local specialists.
- Foreign doctors can be hired and fired without significant costs.
- It is recognized that the satisfaction of patients from health care depends on multiple factors and their complex relationships as mention in Faezipour and Ferreira (2013). This model is a partial model, which sets out to define the long-term strategy of investing in specialist doctors (foreign and local) who are the most important input into health care given everything else, remains the same.
- Education of practitioner doctors for specialization is very costly.

Therefore, the model is:

$$Max\int_{0}^{T_1}(T(t)+F(t))^{\alpha}P(t)^{\gamma}-cF(t)-dT(t)-eP(t)-f(\beta(D-T(t))dt$$

$$st.$$

$$T'=\beta(D-T(t))$$

$T(0)=T_0$, a small number or zero.

$$0\le\beta\le1$$

For the purposes of a better exposition, without losing generality, we will assume:

$f(\beta (D-T)) = \beta^2 (D-T)^2$

By optimally choosing investment in new local specialist doctors, practitioners, and in foreign doctors. However, since the number of practitioners is the difference between the total number of doctors (D>0) and the specialist (T), we can define P as:

$P=D-T$.

$(T+F)^{\alpha}P^{\gamma}$ is the patient satisfaction function, where α and γ are numbers between zero and one and their sum is less than one. This makes the satisfaction function a concave function.

T: Number of total local specialist doctors

F: Number of foreign doctors. We will assume, for all practical purposes that there are always foreigner doctors working in the country (F>0).

P: Number of practitioner doctors

β: Percentage of total practitioner doctors (D-T) chosen to be educated as specialists.

f (β(D-T)): Cost of educating new specialist doctors, a convex function.

c: Salary of foreign doctors, a constant,

d: Salary of local specialist doctors, a constant,

e. Salary of practitioner doctors

T_0: Number of local doctors at time zero.

The relevant Lagrangian (omitting the t in relevant variables):

$$L = (T+F)^{\alpha}(P)^{\gamma} - cF - dT - eP - \beta^2 (D-T) + \lambda (D-T) + \mu (1-\beta) + \eta\beta$$

or

$$L = (T+F)^{\alpha}(D-T)^{\gamma} - cF - T(d-e) - \beta^2 (D-T) + \lambda (D-T) + \mu (1-\beta) + \eta\beta$$

Since P=D-T and the term eD is a constant which has no relevance on the solution. It has no relevance on the solution even if D was a function of time. In this case, the term D can be just integrated out.

The necessary conditions are:

$$L_F = \alpha(T+F)^{\alpha-1}P^{\gamma} - c = 0 \tag{1}$$

$$L_{\beta}=-2\beta(D-T)^2+\lambda(D-T)-\mu+\eta=0 \tag{2}$$

$$T'=(D-T) \tag{3}$$

$$\lambda'=-(\alpha(T+F)^{\alpha-1}(D-T)^{\gamma}-(D+F)^{\alpha}\gamma(D-T)^{\gamma-1}+2\beta^2(D-T)-(d-e)-\lambda\beta)$$

$$\gamma,\eta,\ \lambda\geq 0 \quad \mu(1-\beta)=0,\ \ \eta\beta=0 \tag{4}$$

These necessary conditions are also sufficient since the Lagrangian is concave in all variables. The necessary and the sufficient conditions for the solution of the optimal control theory problems are detailed in many books including Pontryagin, Bolyantski, Gamkrelidze and Mischenko (1963) and Kamien and Schwartz (1991).

RESULTS

The solution can start with $\beta=0$, or $\beta=1$, or $0<\beta<1$.

Case A: $\beta=0$.

In this starting base, we assume that $\beta=0$ for a finite time 0. In this case, due to equation (5), we have:

$$\mu = 0 \text{ and}; \eta = -\lambda(D-T) \geq 0 \tag{6}$$

which implies that λ should be nonpositive since D is a large positive constant and $T(t)=0$ and remains so due to equation(3) in this period. However, this is contradictory to $\lambda\geq 0$ unless $\lambda=0$ for $0\leq t\leq t_1$. Using $\lambda=0$, equation (5) and equation (1) it is can be easily proved that this is not possible.

Therefore, the solution cannot start with this phase.

Case B: $\beta=1$.

In this case, we have, from equation (1), which states that the marginal contribution of the foreign specialist (the right-hand side) should be equal to their cost c, then we have:

$$(D-T)^{\gamma}/(T+F)^{1-\alpha}=c/\alpha \tag{7}$$

In addition, from equation (3), we have:

$T' = D - T$, which is a linear differential equation, which can be solved easily as:

$$T(t) = D(1 - e^{-t}) \tag{8}$$

with the initial condition $T_0 = 0$ which in turn gives:

$$P(t) = De^{-t} \tag{9}$$

since $P = D - T$. Notice that the value of T will never exceed D except as time goes to infinity, which is not the case here.

Using equation (7), we get:

$$F = (\alpha(P)^{\gamma}/c)^{1/(1-\alpha)} - D(1 - e^{-t}) \tag{10}$$

or

$$F = (\alpha(De^{-t})^{\gamma}/c)^{1/(1-\alpha)} - D(1 - e^{-t})$$

which determines the number of foreign doctors. However, the first term gets smaller while the second gets larger as time increases causing F to reach zero at time $t_{1,}$ which is not infinite. So, the phase where $\beta = 1$ ends at that time, t_1, which can be larger or smaller than the terminal time, T, where we should have:

$\lambda(T) = 0.$

1. Assume $t_1 > T$ the terminal time.

Then, from equation (2), since $\eta = 0$, we have:

$\mu = \lambda(D - T) - 2(D - T)^2 \geq 0$ in this phase. Equivalently, we can write: (11)

$\lambda = 2(D - T) + \mu/(D - T) > 0$

since (D-T) is positive.

This, in turn, implies that λ (T) cannot be zero. Therefore, this phase is not possible.

2. Assume t_1<T

From equation (7) we have:

T (number of local specialists) =a constant for the period after t_1 because F is zero. Then,

T'=0= β(D-T) which can only hold if β=0 since (D-T)>0 at t_1. This implies also that there will be jump in the value of β from 1 to 0 which is possible.

Then, from this phase, the solution may only go into a phase where β=0.

In the phase where β=0; from equation (2), we have:

$$\eta = -\lambda(D-T) \geq 0$$

Which implies that λ≤0 in that period. However, this is impossible since λ is positive at t_1 and is continuous. Therefore, the period after t_1 cannot be where β=0.

Therefore, in turn, the initial period cannot be where β=1.

The only alternative for the initial phase is when $0 \leq \beta \leq 1$

Case C: $0<\beta<1$

In this case, from equations (2) and (5), we have:

$$\begin{aligned}&\eta, \mu = 0 \text{ and;}\\ &\lambda = 2\beta(D-T)\end{aligned} \tag{12}$$

in addition, from equation (4) we have:

$$\lambda' = -(\alpha(T+F)^{\alpha-1}(D-T)^{\gamma} - \gamma(T+F)^{\alpha}(D-T)^{\gamma-1} - (d-e) - \lambda\beta + 2\beta^2(D-T)) \tag{13}$$

Recalling:

$$f(\beta(D\text{-}T)) = \beta^2(D\text{-}T)^2 \tag{14}$$

We now can rewrite equation (2) as:

$$\begin{aligned}&\lambda(D-T) = 2\beta(D-T)^2 \text{ or,}\\ &\lambda = 2\beta(D\text{-}T)\end{aligned} \tag{15}$$

Differentiating both sides, we get:

$$\lambda' = 2\beta'(D-T) - 2\beta T' \tag{16}$$

Substituting this into equation (13) and using equations (3) and (15), we have:

$$2\beta'(D-T) = 2\beta^2 T' + (d-e) - c - 2\beta^2(D-T) + 2\beta^2(D-T) + \gamma(T+F)^{\alpha}(D-T)^{\gamma-1} \tag{17}$$

Where the term (F+T) can be written in terms of D and T using equation (1) as:

(F+T) =k (D-T) $^{(\Upsilon+\alpha-1)/(1-\alpha)}$ where $k=(c/\alpha)^{\alpha/\alpha-1}$ which is a positive number.
Then:

$$2\beta'(D-T) = 2\beta^2(D-T) + (d-e) - c + k\gamma(D-T)^{(\gamma+\alpha-1)/(1-\alpha)} \tag{18}$$

In addition, we have equation (3):

$$T' = \beta(D-T(t)) \tag{19}$$

Equations (18) and (19) form a system of first order nonlinear, nonhomogeneous differential equations, which cannot be solved explicitly. We will now characterize the solution by Phase Diagram Analysis in (β, T) space.

The loci of points where’= 0 are β=0 and T=D; (20)

Similarly, the loci of points where β’=0 are:

$$2\beta^2(D-T)+(d-e)-c+k\gamma(D-T)^{(\gamma+\alpha-1)/(1-\alpha)}=0 \tag{21}$$

or:

$$2\beta^2(D-T)+k\,\gamma(D-T)^{(\gamma+\alpha-1)/(1-\alpha)} = c+e-d \tag{22}$$

The right-hand side of this equation is positive since salaries for the foreign doctors, c, is assumed greater than the salaries for local specialist doctors, d. The total variation of equation (21) yields:

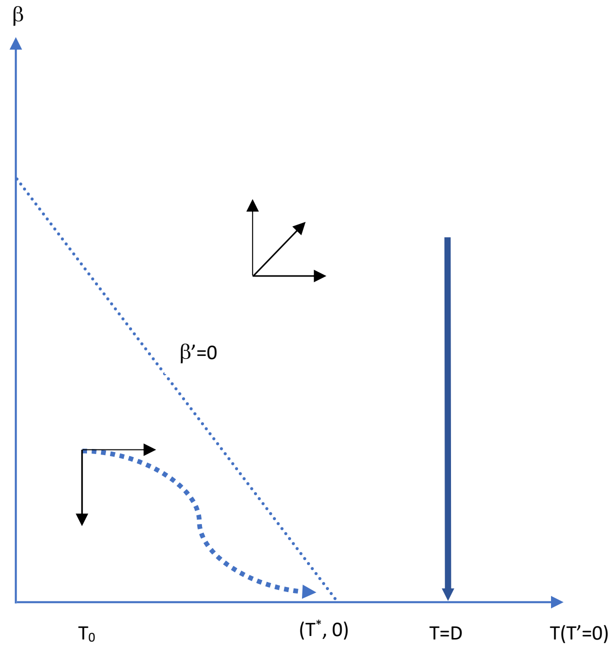
$$4\beta(D-T)\,d\beta + 2\beta^2(-dT) + k\gamma((\gamma+\alpha-1)/(1-\alpha))(D-T)^{m}(-dT) = 0$$

Where $m = (\alpha+\gamma-2)/(1-\alpha) < 0$

which indicates that these loci are a downward sloping line on (T,β) space provided that the term involving kγ(α+γ-1) is negative and greater than $2\beta^2$ which is true because of the concavity requirement, m≤0 and the assumption that D is large enough.

The β intercept for equation (21) will yield two roots of opposite signs for β because the second term on the left of equation (22) is small compared to the term on the right-hand side when T=0). This intercept will be denoted by β^*. We will use the positive root for analysis. The T intercept is calculated by letting T=0 in equation (22) and solving for this intercept will be denoted by T^* which will be positive if the number of doctors, D, is not very small.

Figure 2. Phase Diagram Analysis of the System of Differential Equations (18, 19)



Source: constructed by the author.

The phase diagram will appear as below.

It is easy to see that above the loci T'=0, we have:

T'>0 indicating that T will increase in that region. Below that curve we have T'<0 indicating that it decreases in that region. These directions (directional arrows) are indicated in the Phase Diagram.

Similarly, to the right of the loci β'=0, we have β'>0 indicating that it is increasing in that region and conversely it will be decreasing below that loci. These directions are indicated on the Phase diagram also (note that the directional arrows are not indicated in regions where T<0. The intersection of these loci is called the equilibrium point, the point at which the system will stabilize if certain conditions are met. It is denoted as (T^*,β^*) on the Diagram.

The optimal strategy is, therefore, to start with a high β and gradually to decrease it while increasing the number of local specialist doctors. This implies that the government should invest heavily in the specialization of local doctor while decreasing the number of foreign doctors.

We showed in Appendix that the equilibrium point is stable.

It is clear from the Phase Diagram that if the initial number of local specialists, T_0, is larger than the equilibrium number of specialists, there is no stable equilibrium. This phase is the only solution to the whole problem since necessary conditions and the transversality conditions are all met for the entire planning horizon.

Comparative Statics and Dynamic Analysis:

- Some parameters like the total number of doctors, salaries of different types of doctors are all assumed constants. However, different values of these parameters will affect the steady state values of control or state variables. Comparative statics analysis allows us to see this impact.
- Comparative dynamics analysis, which involve analysis of changes in the entire optimal path with respect to a change in a parameter, not just the steady state. We will do both the comparative analysis and the comparative dynamic analysis only on the total number of doctors, D, to observe the impact of a change in D both on the equilibrium point(comparative statics) and the optimal path leading to the equilibrium point (comparative dynamics).

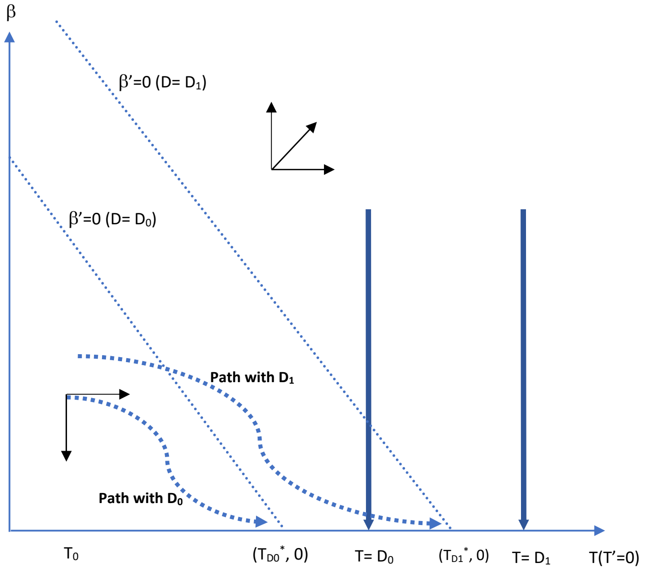
The Comparative Statistics Analysis:

- It is quite easy to show, using equation (22), that the T intercept (β) is larger for a larger D. This is true for β intercept also (T) also. Thus, a larger T shifts equation (22) to the right. This implies that for larger number of total doctors that we will have larger number of local specialist doctors at the equilibrium point.

Comparative Dynamics Analysis:

- Redrawing the phase diagram above both with smaller (D_0) and larger D, (D_1), we have figure 3 (skipping directional arrows):

Figure 3. Phase Diagram Analysis of the System of Differential Equations (18, 19) with both smaller and larger



Source: constructed by the author.

Suppose the optimal path leading to the equilibrium with D_1 intersects the optimal path leading to the equilibrium with D_0. We will show that this is not possible.

At the intersection point (T_2, β_2), the tangent of the path with a (D_1) is larger (less negative) than the path with (D_0). First, dividing equations (17) and (19) side by side, we must have:

$$d\beta / dT = \beta + (d - e - c) / 2\beta (D - T)^2 + (1/2) k\gamma (D - T)^{(\gamma+\alpha-1)/(1-\alpha)-2}$$

representing the tangent of the optimal path generally. This tangent evaluated at the intersection point with D_1 must be larger than the tangent at the intersection point with D_0. Thus:

$$\beta+(d-e-c)/2\beta(D_1-T)^2+(1/2)k\gamma(D_1-T)^{(\gamma+\alpha-1)/(1-\alpha)-2}>\beta+(d-e-c)/2\beta(D_0-T)^2+$$

$$+(1/2)k\gamma(D_0-T)^{(\gamma+\alpha-1)/(1-\alpha)-2}$$

At the intersection, T and β are the same for both sides of the inequality above, we have:

$$(d-e-c)(1/(D_1-T)^2-1/(D_0-T)^2)+(1/2)\,k\,\gamma((D_1-T)^{(\gamma+\alpha-1)/(1-\alpha)-2}-(D_0-T)^{(\gamma+\alpha-1)/(1-\alpha)-2})>0$$

However, this is not possible since $D_1>D_0$ and since the power on the second term is negative making the left- hand side of the above inequality negative. Hence, the optimal paths cannot intersect. This implies that larger the total number of doctors higher will be the time path of β meaning that higher number of practitioner doctors will be allowed to be specialist. For larger number of doctors.

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

The main result of the paper is that all countries with few numbers of local specialists at the beginning of the planning horizon should hire foreign specialists immediately and begin investing in local specialists heavily (high β which is not above $\beta'=0$ loci) decreasing the number of foreign specialists accordingly. At the equilibrium point, the total number of local specialist (all local, no foreign doctors) will be less than the total number of doctors, implying that the total number of practitioners will always be positive. Investment in education of practitioner doctors for specialization will be higher (high β) if the total number of doctors (D) is high and the number of local specialists at the beginning of the planning horizon is low.

Recommendations for Further Research:

- Another extension of the model is that the governments can optimally determine the number of new doctors. In this case the model will be:

$$Max\int_0^{T_1}((T(t)+F(t))^{\alpha}P(t)^{\gamma}-cF(t)-dT(t)-eP(t)-f(\beta(D-T(t))-g(s))dt$$

$st.$

$D'=s-\sigma D$

$T'=\beta(D-T(t))$

$T(0)=T_0$, a small number or zero.

With 0<β<1 assumption.

- Where s and g(s) represent the number of new graduates and the cost of educating them respectively. This extension of the problem is also difficult to solve and will be the subject of another paper.

Appendix: Stability analysis of the solution

The relevant differential equation system is:

$$2\beta'(D-T)=2\beta^2(D-T)+(d-e)-c+Ak\gamma(D-T)^{(\gamma+\alpha-1)/(1-\alpha)} \qquad \text{A1}$$

$$T'=\beta\ (D-T) \qquad \text{A2}$$

The Taylor's expansion of this system around the equilibrium point (T^*, 0) is;

$$\beta' = \left[\frac{(d-e-c)}{2(D-T)^2}+k\gamma\left((D-T)^{(\varepsilon-2)}-\frac{(\alpha+\gamma-1)(D-T)^{(2-\varepsilon)}}{(1-\alpha)}\right)\right](T-T^*)$$

$$T' = (D-T^*)\beta$$

where, $\varepsilon = (\alpha+\lambda-1)/(1-\alpha)$

The terms multiplying *(T – T *)* and β in the differential equation system above are evaluated at the equilibrium point and they are positive. The eigenvalues of this system are real and have opposite signs. Therefore *(T *, 0)* is a saddle point equilibrium.

References

Babcock, J.M., Babcock, B.D., & Schwartz, M.Z. (2013). Maintaining sufficient and quality physician's workforce: the role of for-profit medical schools. *Health Services Insight*, 6, 87-98. http://dx.doi.org/10.4137/HSI.S10462.

Batbata, E., Dorjdagva, J., Livsannyam, A., Savino, M.M., & Amneta, P. (2017). Determinants of patient satisfaction. A systematic review*. Perspectives on Public Health,* 137(2), 89-101. http://dx.doi.org/10.1177/1757913916634136.

Boquiran, V.M., Hack, T., Beaver, K. , & Williamson, S. (2015). What do measures of patient satisfaction with the doctor tell us? *Patient Educating and Counseling*, 98(12), 1465-1473. http://dx.doi.org/10.1016/j.pec.2015.05.020.

Chojnicki, X., & Moullan, Y. (2018). Is there a "pig cycle" in the labor supply of doctors? How training and immigration policies respond to physician shortages. *Social Science* Medicine, 200, 227-237. http://dx.doi.org/10.1016/j.socscimed.2018.01.038.

Cooper, R.A. (2009). States with more physicians have better health care. *Health Affairs*, 28(1), 91-102. http://dx.doi.org/10.1377/hlthaff.28.1.w91.

Crimson (2017). Top 7 medical Schools Producing the Best Doctors in Australia and New Zealand, https://www.crimsoneduaction.org/za/blog/topmedical-school-australia/Newzealand.Published (accessed: 24.07.2019).

Eurostat (2018). Statistics explained, Health care personnel-Physicians.pp.1, https://ec.europa.eu/eurostat/statistics-explained/pdfscache/37382.pdf (accessed: 31.07.2019).

Faezipor, M., & Ferreira, S. (2013). A systems dynamic point of view of patient satisfaction in health care. *Procedia Computer Science*, 16(1), 148-156. http://dx.doi.org/10.1016/j.procs.2013.01.016.

Farahani, M., Subramanian, S.V., & Canning, D. (2009). The effects of changes in health resources on infant mortality in short- and long-run: A longitudinal econometric analysis. *Social Science* & *Medicine*, 68(11), 1918-1925*.* http://dx.doi.org/10.1016/j.socscimed.2009.03.023.

Grover, A. (2006). Critical care workforce: a policy perspective. *Critical Care Medicine*, 34(3), 7-11.

Hansen, E. (2011). *Applications of optimal control to infectious disease modelling*. Ph.D. Thesis. Kingston, Ontario, Canada: Queens University.

Hockey, P.M., & Marshall, M.N. (2009). Doctors and quality improvement. *Journal of the Royal Society of Medicine*, 102(5), 173-176. http://dx.doi.org/10.1258/jrsm.2009.090065.

Kamien, M.I, & Schwartz, N.L. (1991). *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management 2nd Edition*. Amsterdam: Elsevier.

Kerr, E. (2019). 10 Most Expensive Private Medical Schools. US News and World Report, https://www.usnews.com/education/best-graduate-schools/the-short-list-grad-school/articles/most-expensive private-medical-schools (accessed: 13.01.2019).

Lin, F., Muthuraman, K., & Lawley, M. (2010). An optimal control theory approach to non-pharmaceutical interventions. *BMC Infectious Diseases*, 10(32), 1-13. http://dx.doi.org/10.1186/1471-2334-10-32.

Merkley, A., & Bickmore, A.M. (2017). The Top Five Recommendations for Improving the Patient Experience. *Health Catalyst*, 1-7.

Momoh, A.A., & Fügenschuh, A. (2018). Optimal control of intervention strategies and cost effectiveness analysis for a Zika virus model. *Operations Research for Health Care*, 18, 99-111. http://dx.doi.org/10.1016/j.orhc.2017.08.004.

Nylenna, M., Bjertnaes, Ø.A., Saunes, I.S., & Lindahl, A.K. (2015). What is Good Quality of Health Care? *Professions and Professionalism*: *Special Issue: Professional Satisfaction and the Quality of Medical Care*, 5 (1), 1-15. http://dx.doi.org/10.7577/pp.909.

OECD (2017). *Health at a Glance 2017: OECD Indicators*. Paris: OECD Publishing. http://dx.doi.org/10.1787/19991312.

OECD Data (2017). Doctors, https:/data.oecd.org/healthres/doctors.htm (accessed: 31.07.2019).

OECD (2019a). Health Statistics Nov.2019 – Frequently Requested Data, https://www.oecd.org/health/OECD-Health-Statistics-2019-Frequently-Requested-Data.xls (accessed: 13.01.2019).

OECD (2019b). GDP, volume – annual growth rates in percentage, https://stats.oecd.org/index.aspx?queryid=60703 (accessed: 13.01.2019).

OECD (2019c). Better Policies for Better Health. Health Work Force policies in OECD countries, https://www.oecd.org/health/health-systems/Health-workforce-policies-in-oecd-countries-Policy-brief.pdf. (accessed: 31.07.2019).

OECD (2019d). Life expectancy, https://stats.oecd.org/index.aspx?queryid=30114. (accessed: 13.01.2019).

OLCDB (2018). *Overview of the cost of training health professional. Canadian Ministry of Health*.

Pando, C. (2016). The influence of number of physicians on infant mortality across nations. *Lehigh Review*, 24, 61-71.

Pontryagin, L.S., Bolyantski, V.G., Gamkrelidze, R.V., & Mischenko, E.F. (1963). *The Mathematical Theory of Optimal Processes*. New York: Wiley Science. http://dx.doi.org/10.1002/zamm.19630431023.

Rowthorn, R., & Toxvaerd, F. (2012). The optimal control of infectious diseases via prevention and treatment. *CEPR Discussion Paper*, DP8925.

Sargen, M., Hooker, R.S., & Cooper, R.A. (2011). Gaps in supply of advance practice nurses and physicians' assistants. *Journal of American College of Surgeons*, 212(6), 991-999. http://dx.doi.org/10.1016/j.jamcollsurg.2011.03.005.

Scheffler, R.M., & Arnold, D.R. (2019). Shortages and surpluses of doctors and nurses in the OECD: What looms ahead? *Health Economics, Policy and Law*, 14(29), 274-290. http://dx.doi.org/10.1017/S174413311700055X.

Scheffler, R.M., Liu, J.X., Kinfu, Y., & Poz, M.R.D. (2008). Forecasting the global shortage of physicians: An economic-and needs based approach. *Bulleting of World Health Organization*, 86(7), 516-523.

Senić, V., & Marinković, V. (2012). Patient care, satisfaction and service quality in health care. *International Journal of Consumer Studies*, 37(3), 312-319. http://dx.doi.org/10.1111/j.1470-6431.2012.01132.x.

Sharomi, O., & Malik, T. (2017). Optimal control in Epidemiology. *Annals of Operations Research*, 251(1-2), 55-71. http://dx.doi.org/10.1007/s10479-015-1834-4.

Shetty, A., & Shetty, S. (2014). The correlation between physicians to population (1000) ratio and life expectancy. *Journal of Medicine and Clinical Research*, 2(4), 699-706.

Tehrani, A.B., Feldman, S.R., Camacho, F., Rajesh, F.T., & Balakrishnan, R. (2011). Patient Satisfaction with Outpatient Medical Care in the United States. *Health Outcomes Research in Medicine,* 2(4), 197-202. http://dx.doi.org/10.1016/j.ehrm.2011.09.001.

Vernon, J.A., & Hughen, W.K. (2006). A primer on dynamic optimization and optimal control in pharma economics. *Value in Health*, 9(2), 106-113. http://dx.doi.org/10.1111/j.1524-4733.2006.00088.x.

Vuković, M., Gvozdenović, B.S., Gajić, T., Gajić, B.S., Jakovljević, M., & McCormick, B.P. (2012). Validation of a patient satisfaction questionnaire in primary health care. *Public Health*, 126(8), 710-718. http://dx.doi.org/10.1016/j.puhe.2012.03.008.

World Bank (2017). World Bank Data, https://data.worldbank.org/indicator/SP.DYN.IMRT.IN (accessed: 01.08.2019).

World Health Organization (2019). Density of physicians (total number per 1000 population, latest available year), https://www.who.int/gho/health_workforce/physicians_density/en (accessed: 01.08.2019).

World Health Organization (2018). Progress towards the SDGs: A selection of data from World Health Statistics, https://www.who.int/gho/publications/world_health_statistics/2018/EN_WHS2018_SDGhighlights.pdf?ua=:2018. (accessed: 13.01.2019).

World Health Organization (2018). World health statistics 2018: monitoring health for the SDGs, sustainable development goals. CC BY-NC-SA 3.0 IGO.5., https://apps.who.int/iris/handle/10665/272596 (accessed: 31.07.2019).

World Health Organization, World Bank, & OECD (2018). *Delivering Quality Health Services*: *A Global Imperative for Universal Health Coverage*. Genève: World Health Organization.

Xesfingi, S., & Vozikis, A. (2016). Patient satisfaction with the healthcare system: Assessing the impact of socio-economic and healthcare provision factors. *BMC Health Services Research*, 16, 94. http://dxd.doi.org/10.1186/s12913-016-1327-4.