Event-triggered Observer-based Robust H ∞ Fault-tolerant Control for Markov Jumping NCSs

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For network control systems (NCSs) with parameter uncertainties, packet loss and time-delays, the design for the observer and robust H[∞] fault-tolerant controller is discussed with the event-triggered mechanism. The random Markov jumping system is used to describe the NCSs with faults. In order to reduce the frequency of data transmission and save network channel resources, the discrete event-triggered mechanism is utilized, which allows the NCS to transmit the signal when the trigger condition is met only. Under this mechanism the system states are estimated by the observer provided, while a robust fault-tolerant controller is designed for actuator failures, so that the NCSs can remain stable in the event of failures. Finally, the proposed method is verified by simulation, the respective results showing its validity and feasibility.

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1. Introduction

Computer technology and network communication technology have developed rapidly. In the development of control theory and control engineering, computer technology and network communication technology have been absorbed and merged, making both control systems more complex and the controlled objects more diversified. This combination of multiple fields has led to the emergence and rapid development of *Network Control Systems* (NCSs) [1–4], which are closed-loop feedback control systems formed over a communication network. NCSs are not only widely used in the fields of complex industrial control, but also in such military fields as guidance and control.

On the one hand, the combination of the control system and the network enables the control field to advance and bring many advantages. On the other hand, due to the diversity and unpredictability of networks, their use also brings new problems to the control systems. For example, network randomness can cause failures such as induced time delays, data packet dropouts, communication constraints, and quantization errors, which can affect system performance and even undo stability [5–9]. In [6], the problem of simultaneous $H\infty$ stability in a large-scale physically interconnected system working in multiple operating modes is studied. The focus is on the design of distributed controllers to make the mean square error stable in the large-scale system. In [7], a robust $H\infty$ control problem is proposed for a class of linear time-varying NCSs with uncertainties and external disturbances, which simultaneously considers both delay and data packet dropouts. A fault-tolerant robust non-fragile $H\infty$ filtering problem for networked control systems with sensor failures is studied in [8].

Network failures make the control system more complicated. In the actual process of the system, components such as sensors and actuators may fail. At the same time, the system will also be affected by external disturbances. These factors will affect the stability of the system. Therefore, when NCSs are discussed, many factors are considered, which makes the NCSs research more challenging.

Among the faults caused by networking, the most common ones are network-induced time delays and data packet dropouts [10–12]. H ∞ output feedback control for NCSs with time delay, data packet dropout, and disorder that occurred in both sensor-to-controller and controller-to-actuator channels, is considered in [11]. In [12], the problem of data-based network control for a class of nonlinear systems is solved. By using a packet-based transmission mechanism and a model-free adaptive control algorithm, a data-based network predictive control method is proposed to actively compensate for the random RTT delay.

Therefore, it is of great research value and practical significance to explore some control methods to minimize or eliminate the impact of the above two faults in system operation. At present, many scholars have studied this and proposed many effective control methods [13–15], here including robust control, random control, event-triggered mechanism, time-delays compensation control, and the like.

In NCSs, the state at the current moment is related to the previous moment, which is very similar to Markov chains. At the same time, network states are random, while the Markov process is also a random process, so this latter can accurately show the states of network systems. Based on the above characteristics, the research of NCSs can be regarded as the research of systems with Markov process [16-20]. Specifically, the Markov jumping system is a random hybrid system with Markov process. This hybrid system can describe several different modes at the same time, and the modes can be transferred and switched through the Markov chain. This feature of the Markov jumping system accurately describes the sudden change for the network states.

In actual engineering applications, it is inevitable that some unpredictable emergencies will occur during system operation, especially in network communication systems. Based on this, the use of the Markov jumping system is an effective method to model complex NCSs. But at present, the research of robust fault-tolerant control (FTC) stability for the Markov jumping system in a networked environment is still not unflawed, and further research is still needed.

For some problems, such as actuator failures and external disturbances in the system, it is necessary to design a robust fault-tolerant control to ensure the stability of the system [21– 25]. Robust fault-tolerant control refers to the design of control methods when there are faults or other uncertain factors in the system so that the system can still operate stably and achieve the desired performance. Therefore, the research of robust fault-tolerant control for NCSs is of great significance. In fact, the research on fault analysis and fault-tolerant control for NCSs with time delay and packet dropouts has attracted many scholars' attention in the recent years. In [21], a new data-based FTC scheme is proposed in a parameter-dependent form. The time-varying parameters are adjusted online based on an adaptive method to automatically compensate the uncertainties, disturbances, and actuator faults. In [22], direct adaptive state feedback control schemes are developed to solve the robust fault-tolerant compensation control problem for linear time-invariant continuous-time systems with actuator failures and external disturbances. The problem of sensor fault estimation and fault-tolerant control for a class of Takagi-Sugeno Markovian jump systems, which are subjected to sensor faults and partially unknown transition rates, is addressed in [23]. This paper describes the design of a fault-tolerant controller with a guaranteed cost for a new network control system with time-varying sensor faults. Based on the time delay of the network transmission environment, a network control system with sensor failure is modeled as a discrete-time system with uncertain parameters in [25].

In NCSs, the traditional time-triggered mechanism has good predictability and realizability. However, the time-triggered mechanism has also disadvantages. The network control system based on the time-triggered mechanism periodically sends sampled signals to the controller, which produces a large number of redundant signals be transmitted over the communication channel. The possibility of data packet collision and network congestion in the communication channel is thus increased. Therefore, the event-triggered mechanism has been gradually applied to NCSs [26–34]. Compared to the time-triggered mechanism, the event-triggered mechanism enables the system to form a path only when the data meets the trigger condition. Time delay and packet dropouts caused by time triggering during network transmission cannot form a path. The event-triggered mechanism can effectively eliminate the impact of time delay and packet dropouts on the system, while also reducing the amount of data transmission in the system and saving network resources. In [26], an event-triggered fuzzy adaptive compensation controller is constructed for uncertain stochastic nonlinear systems with given transient specification and actuator failures. In [27], an event-triggered control problem of uncertain nonlinear systems caused by actuator failures is studied. The failure of the actuator is allowed to be unknown, and the total number of failures can be unlimited. The paper [30] is concerned with event-triggered $H\infty$ control for a class of nonlinear networked control systems: an event-triggered transmission scheme is introduced to select 'necessary' sampled data packets to be transmitted so that precious communication resources can be saved significantly. In [33], a systematic methodology for the design of (distributed) dynamic quantizers is proposed to ensure input-to-state stability of a size-adjustable set around the origin. To save communication resources, the transmission instants of each sensor are determined by event-triggering mechanisms. In [34], the static and dynamic event-triggered control strategies are proposed that aim at reducing the utilization of communication resources while guaranteeing desired stability and performance criteria and a strictly positive lower bound on the inter-event times despite the presence of packet losses.

In summary, the application of the event-triggered mechanism in recent years has attracted the attention of many scholars, whose research has improved the performance of NCSs.

At present, most research results of the event-triggered scheme assume that the states for the controlled object are measurable. However, the NCSs in actual engineering are distributed real-time feedback systems. When technology and other conditions are restricted, it is very difficult to obtain all the states' information for the controlled object, so it is necessary to consider the observer-based control problem. In this paper, for NCSs with parameter uncertainties, time delays, data packet dropouts, and communication constraints, under the event-triggered mechanism, the design for the observer and robust $H\infty$ fault-tolerant controller are discussed. The research object NCSs are described as random Markov jumping systems. The event-triggered mechanism was introduced in order to weaken the influence of network failures. The network system can transmit the signal when the trigger condition is met. Time delays and packet dropouts can be regarded as not satisfying the triggering conditions. By designing the state observer, the operating states of the NCSs are estimated. For random actuators failures, the robust fault-tolerant controller is given to keep the system stable even in cases of failure. The purpose is to make NCSs have fault tolerance and anti- disturbance capabilities. Finally, the effectiveness of the proposed method is verified by simulation and compared with the existing methods.

2. Problem Statements

2.1. System Modeling

We consider NCSs based on the event-triggered mechanism, as shown in Figure 1, where τ_1 and τ_2 are random time delays. τ_1 is the time delay from the controller to the actuator, τ_2 is the time delay from the sensor to the controller, $0 < \tau_m \le \tau_i \le \tau_M (i = 1, 2)$, where τ_m and τ_M are the minimum and maximum time-delays, respectively.

Given the probability space (Ω, F, P) , where Ω is the sample space, F is the σ algebraic subset of the sample space, and P is the measured probability in F. Assuming that there are network failures such as random time delays, packet dropouts, and communication constraints in the communication of the link in the NCSs, the NCSs model is generally established as the following multi-delays Markov jumping system with parameter uncertainties:

$$\begin{cases} \dot{x}(t) = \left(A\left(r(t)\right) + \Delta A\left(r(t)\right)\right)x(t) + \\ A_d\left(r(t)\right)x\left(t - \tau_1\right) + E\left(r(t)\right)f(t) + \\ \left(B\left(r(t)\right) + \Delta B\left(r(t)\right)\right)u(t) + \\ B_d\left(r(t)\right)u\left(t - \tau_2\right) + D\left(r(t)\right)\omega(t) \\ y(t) = C\left(r(t)\right)x(t). \end{cases}$$
(1)

Where $\{r(t), t \ge 0\}$ is a continuous-time Markov chain defined in the probability space (Ω, F, P) . The value is taken in the finite mode set $S = \{1, 2, ..., s\}$, and the transition probability from mode *i* to mode *j* at time $t + \Delta t$ is

$$P\left\{r\left(t+\Delta t\right)=j:r(t)=i\right\}=$$

$$\begin{cases}\delta_{ij}\Delta t+O\left(\Delta t\right), & j\neq i\\1+\delta_{ij}\Delta t+O\left(\Delta t\right), & j=i\end{cases}$$
(2)

where $\Delta t > 0$, and $\lim_{\Delta t \to 0} O(\Delta t) / \Delta t = 0$, $\delta_{ij} \ge 0$ is the transition probability from state *i* to state *j*. If $j \ne i$ then $\delta_{ij} \ge 0$, otherwise $\delta_{ii} = -\sum_{\substack{j=1, \ j \ne i}}^{S} \delta_{ij}$. In equation (1), $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^p$ is the output vector, $f(t) \in \mathbb{R}^r$ is the unknown actuator fault in the system, and $\omega(t) \in \mathbb{R}^q$ is the external disturbance that satisfies $\omega(t) \in L_2[0, \infty)$. A(r(t)), $A_d(r(t))$, B(r(t)), $B_d(r(t))$, C(r(t)), D(r(t)) and E(r(t)) are the known matrices with appropriate dimensions. $\Delta A(r(t))$ and $\Delta B(r(t))$ are the uncertainties of the parameter matrices in the system. For r(t) = i, $i \in S$, the uncertainty matrices satisfy the following equation

$$\begin{bmatrix} \Delta A_i & \Delta B_i \end{bmatrix} = MF(t)[N_{ai} \quad N_{bi}] \tag{3}$$

where M, N_{ai} and N_{bi} are the known matrices with appropriate dimensions, and F(t) is an unknown time-varying matrix that satisfies $F^{T}(t)F(t) \leq I$. I is the unit matrix with appropriate dimensions. The superscript "T" represents the transpose of a matrix.

Considering the system failures caused by the actuators, the matrix $L = \text{diag}\{l_1, l_2, ..., l_m\}$ is introduced to represent the actuator failures. $l_m = 0$ denotes the complete failure of the *m*-th actuator, while $l_m = 1$ denotes its normal operation. Finally, $l_m \in (0, 1)$ means partial failure of the *m*-th actuator.

In order to facilitate the description of the faults form, assume $E_i = -(B_i + \Delta B_i)$ and Lu(t) = u(t) - f(t), so that the system (1) can be transformed into the following form:

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i) x(t) + A_{di} x(t - \tau_1) + \\ (B_i + \Delta B_i) Lu(t) + \\ B_{di} u(t - \tau_2) + D_i \omega(t) \\ y(t) = C_i x(t) \end{cases}$$
(4)

where A_i , A_{di} , B_i , B_{di} , C_i and D_i , are the known matrices with appropriate dimensions.

2.2. Event-triggered Mechanism

In the process of data transmission through the network systems, the problems of network-induced time-delay and data packet dropouts may



Figure 1. Structure diagram of networked control system based on event triggering.

occur, and useless data may occupy the network channel excessively, which will cause the NCSs to crash. For these network problems, the event-triggered mechanism can be introduced in the system (as shown in Figure 1). A function related to the states at the sampling time is constructed in the event-triggered mechanism, comparing whether the states at the sampling time or the state errors meet the trigger condition, so as to determine whether the state data is transmitted. Data that does not meet the trigger conditions will not trigger the channel to cause data to be transmitted. Therefore, the event-triggered mechanism can eliminate the adverse effects of time delay and packet dropouts network failures, and at the same time, the amount of transmitted data is reduced, so that the network congestion problem is alleviated.

Assumption 1. The system states are completely measurable, driven by events, and the sampling time is h. $i_k h = t_k h + lh$ represents the current sampling time, l is a positive integer ($l \ge 0$), while $t_k h$ is the moment when the data is successfully transmitted.

Assumption 2. The event-triggered mechanism does not affect the detection of fault information in the system, *i.e.*, the whole fault information will be transmitted to the observer at the sampling time.

Remark. Considering system performance requirements and changes in real-time status, in order to make data transmission related only to the status and status error of the system at the sampling moment, an event-triggered communication mechanism (5) based on the sampling moment is created. This makes data transmission a variable cycle.

Definition 1. Considering the limited capacity of the channel, an event-triggered mechanism is adopted to reduce the amount of data transmission in the network. The event-triggered mechanism is defined as follows:

$$t_{k+1} h = t_k h + \min \left\{ lh \left| e_k^T \left(i_k h \right) \sum e_k \left(i_k h \right) \ge \qquad (5) \right. \\ \beta x_k^T \left(t_k h \right) \sum x_k \left(t_k h \right) \right\}$$

where Σ is the event-triggered matrix, and $\beta \in [0, 1)$ is the event-triggered parameter. The state error of transmission is as follows:

$$e_x(i_k h) = x(i_k h) - x(t_k h)$$
(6)

Definition 1 indicates that the release time $t_{k+1}h$ of the next sampling state is obtained by equation (5). According to this event-triggered mechanism, the received current sampling data $x(t_kh + lh)$ will be filtered. The sampled data that meets this condition is sent to the controller, otherwise the current sampled data is discarded.

Definition 2. At the time of transmission, there may be time delays caused by various information transmissions, and the time difference of sampled data in the NCSs is $t - i_k h$. We define the maximum network delay as $\tau(t)$ and $0 < \tau_m \le \tau(t) \le \tau_M$, where τ_m and τ_M are the upper and lower bounds of the time-delay, respectively. Let $\tau_s = \tau_M - \tau_m$.

2.3. Closed-loop System Based on Event-triggered Mechanism

The state feedback controller based on the event-triggered mechanism is designed as:

$$u(t) = K_i x(t_k h), \ t \in [t_k h + \tau_M, t_{k+1} h \tau_M)$$
(7)

where K_i is the feedback control gain. From Definition 1 and Definition 2, we get:

$$\begin{cases} u(t) = K_i \left(x \left(t - \tau \left(t \right) \right) - e_x \left(i_k h \right) \right) \\ u \left(t - \tau_2 \right) = K_i \left(x \left(t - \tau \left(t \right) \right) - e_x \left(i_k h \right) \right). \end{cases}$$
(8)

Based on the event-triggered mechanism and the output $y(t_k h)$, the server is designed as:

$$\begin{cases} \dot{\hat{x}}(t) = (A_{i} + \Delta A_{i})\hat{x}(t) + A_{di}\hat{x}(t - \tau_{1}) + \\ (B_{i} + \Delta B_{i})u(t) + B_{di}u(t - \tau_{2}) + \\ G_{i}(y(t - \tau) - \hat{y}(t - \tau)) \\ \hat{y}(t) = C_{i}\hat{x}(t) \end{cases}$$
(9)

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimated value of the observation state, $\hat{y}(t) \in \mathbb{R}^p$ is the observation output value, and G_i is the state observer gain matrix. The residual $r_r(t)$, state estimation error e(t) and residual estimation error $r_e(t)$ are respectively defined, as follows:

$$r_r(t) = W\left(y(t) - \hat{y}(t)\right) \tag{10}$$

$$e(t) = x(t) - \hat{x}(t)$$
 (11)

$$r_e(t) = r_r(t) - f(t)$$
 (12)

where *W* is the residual gain matrix.

From equation (11), the following state estimation system error can be obtained:

$$\dot{e}(t) = (A_i + \Delta A_i)e(t) + (A_{di} - G_i C_i)e(t - \tau) - (B_i + \Delta B_i)f(t) + D_i \omega(t)$$
(13)

From equations (10) and (12), the residual estimation error can be obtained:

$$r_{e}(t) = W(y(t) - \hat{y}(t)) - f(t)$$

= WC_i e - f(t) (14)

From equation (4) and (8), and Definition 2, the event-triggered closed-loop control system can be obtained:

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + \\ (A_{di} + K_{bi})x(t - \tau) + \\ D_i \omega(t) - K_{bi} e_x(i_k h) \\ y(t) = C_i x(t) \end{cases}$$
(15)

where $K_{bi} = (B_i + \Delta B_i)LK_i + B_{di}K_i$.

2.4. Several Lemmas

In this section, several related lemmas are given.

Lemma 1 (Schur Complement) [35]. For a given symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix},$$

the following three equations are equivalent:

$$S < 0$$

$$S_{11} < 0, \ S_{22} - S_{12}^{T} S_{11}^{-1} S_{12} < 0 \qquad (16)$$

$$S_{22} < 0, \ S_{11} - S_{12} S_{22}^{-1} S_{12}^{T} < 0.$$

Lemma 2 [36]. For any positive definite symmetric matrix $W \in \mathbb{R}^{n \times n}$, parameter τ ($0 \le \tau \le \tau_M$), and vector function $x: [-\tau_M \quad 0] \to \mathbb{R}^n$, the following integral inequality holds

$$-\tau \int_{-\tau}^{0} \dot{x}^{T} (t+s) W \dot{x} (t+s) ds \leq (17)$$

$$\leq (17) \left[\begin{bmatrix} x^{T}(t) & x^{T} (t-\tau) \end{bmatrix} \begin{bmatrix} -W & W \\ * & -W \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}.$$

Lemma 3 [37]. Assuming that $f_1, f_2, ..., f_N: \mathbb{R}^m \to \mathbb{R}$ have positive values in an open subset D of \mathbb{R}^m , then in D the reciprocal convex combination of $f_i(t)$ satisfies the constraint:

$$\begin{bmatrix} f_i(t) & g_{i,j(t)} \\ g_{j,i(t)} & f_i(t) \end{bmatrix} \ge 0$$
(18)

where $\{g_{i,j}: \mathbb{R}^m \to \mathbb{R}, g_{j,i(t)} = g_{i,j(t)}\}$.

Lemma 4 [38]. Given the matrices *Y*, *U* and *V* with appropriate dimensions, and *Y* that is symmetrical, then:

$$Y + UF(t)V + V^T F^T(t)U^T < 0$$

for all F(t) satisfying $F^{T}(t)F(t) \le I$ (*I* is the identity matrix), if and only if there exists a scalar $\alpha > 0$ such that:

$$Y + \alpha U U^T + \alpha^{-1} V^T V < 0.$$

3. Markov Jump NCSs Performance Analysis Based on Observer

The NCSs in actual engineering are distributed real-time feedback systems. Sometimes it is very difficult to obtain all the state information for the controlled object. Therefore, it is necessary to consider the observer-based control problem.

Theorem 1. For the state estimation error system (13), given positive scalars τ_m , τ_M , τ_s , γ_1 , γ_2 and α , if there are positive definite symmetric matrices P_i , W, Q_i (i = 1, 2) and R_i (i = 1, 2, 3), for which the following linear matrix inequality holds

$$\Phi = \begin{bmatrix} \Phi_a & \Phi_b & \Phi_c \\ * & \Phi_d & \Phi_e \\ * & * & \Phi_f \end{bmatrix} < 0$$
(19)

where

 Φ_1

$$\begin{split} \Phi_{16} &= P_i D_i, \\ \Phi_{17} &= \tau_m A_i^T R_1, \\ \Phi_{18} &= \tau_M A_i^T R_2, \\ \Phi_{19} &= \tau_s A_i^T R_3, \\ \Phi_{110} &= C_i^T W^T, \\ \Phi_{111} &= P_i M, \\ \Phi_{112} &= N_{ai}^T, \\ \Phi_{22} &= -2R_3 - R_3^T - R_3, \\ \Phi_{23} &= 2R_3, \\ \Phi_{24} &= R_3 + R_3^T, \\ \Phi_{27} &= \tau_m \left(A_{di} - G_i C_i\right)^T R_1, \\ \Phi_{28} &= \tau_M \left(A_{di} - G_i C_i\right)^T R_2, \\ \Phi_{29} &= \tau_s \left(A_{di} - G_i C_i\right)^T R_3, \\ \Phi_{33} &= -Q_1 + Q_2 - R_1 - R_3, \\ \Phi_{44} &= -Q_2 - R_2 - R_3, \\ \Phi_{55} &= -\gamma_1^{-2} I, \\ \Phi_{58} &= -\tau_M B_i^T R_2, \\ \Phi_{59} &= -\tau_s B_i^T R_3, \\ \Phi_{512} &= -N_{bi}^T, \\ \Phi_{66} &= -\gamma_2^{-2} I, \\ \Phi_{67} &= \tau_m D_i^T R_2, \\ \Phi_{69} &= \tau_s D_i^T R_3, \\ \end{split}$$

then there is an observer (9), which makes the error dynamic system (13) asymptotically stable and has $H\infty$ performance. The mark is used as an ellipsis for the symmetrically transposed part of the matrix.

Proof: Choose the Lyapunov-Krasovskii function, as follows:

$$V(t) = e^{T}(t) P_{i} e(t) +$$

$$\int_{t-\tau_{m}}^{t} e^{T}(s) Q_{1} e(s) ds +$$

$$\int_{t-\tau_{M}}^{t-\tau_{m}} e^{T}(s) Q_{2} e(s) ds +$$

$$\int_{-\tau_{m}}^{0} \int_{t+\theta}^{t} \tau_{m} \dot{e}^{T}(s) R_{1} \dot{e}(s) ds d\theta +$$

$$\int_{-\tau_{M}}^{0} \int_{t+\theta}^{t} \tau_{M} \dot{e}^{T}(s) R_{2} \dot{e}(s) ds d\theta +$$

$$\int_{-\tau_{M}}^{-\tau_{m}} \int_{t+\theta}^{t} \tau_{s} \dot{e}^{T}(s) R_{3} \dot{e}(s) ds d\theta$$
(20)

Taking the derivative of the function V(t), the results are as follows:

$$\dot{V}(t) = 2e^{T}P_{i}\dot{e} + e^{T}\sum_{j=1}^{S}\delta_{ij}P_{j}e + e^{T}Q_{1}e - e_{\tau_{m}}^{T}Q_{1}e_{\tau_{m}} + e_{\tau_{m}}^{T}Q_{2}e_{\tau_{m}} - e_{\tau_{M}}^{T}Q_{2}e_{\tau_{M}} + \tau_{m}^{2}\dot{e}^{T}R_{1}\dot{e} - \tau_{m}\int_{t-\tau_{m}}^{t}\dot{e}^{T}R_{1}\dot{e}ds + \tau_{M}^{2}\dot{e}^{T}R_{2}\dot{e} - \tau_{M}\int_{t-\tau_{M}}^{t}\dot{e}^{T}R_{2}\dot{e}ds + \tau_{s}^{2}\dot{e}^{T}R_{3}\dot{e} - \tau_{s}\int_{t-\tau_{M}}^{t-\tau_{m}}\dot{e}^{T}R_{3}\dot{e}ds$$

$$(21)$$

From Lemma 2 and Lemma 3, the following inequalities can be obtained:

$$-\tau_{m} \int_{t-\tau_{m}}^{t} \dot{e}^{T} R_{1} \dot{e} \, ds$$

$$\leq \qquad (22)$$

$$e_{r} \quad]^{T} \left[R_{1} - R_{1} \right] \left[e_{r} \right]$$

$$-\begin{bmatrix} e_{\tau} \\ e_{\tau_m} - e_{\tau} \end{bmatrix}^T \begin{bmatrix} R_1 & -R_1 \\ * & R_1 \end{bmatrix} \begin{bmatrix} e_{\tau} \\ e_{\tau_m} - e_{\tau} \end{bmatrix}$$
$$\begin{bmatrix} R_1 & -R_1 \\ * & R_1 \end{bmatrix} \ge 0$$
(23)

$$-\tau_M \int_{t-\tau_M}^t \dot{e}^T R_2 \, \dot{e} \, ds \\ \leq \tag{24}$$

$$-\begin{bmatrix} e_{\tau} \\ e_{\tau_{M}} - e_{\tau} \end{bmatrix}^{T} \begin{bmatrix} R_{2} & -R_{2} \\ * & R_{2} \end{bmatrix} \begin{bmatrix} e_{\tau} \\ e_{\tau_{M}} - e_{\tau} \end{bmatrix}$$
$$\begin{bmatrix} R_{2} & -R_{2} \\ * & R_{2} \end{bmatrix} \ge 0$$
(25)

$$-\tau_{s} \int_{t-\tau_{M}}^{t-\tau_{m}} \dot{e}^{T} R_{3} \dot{e} \, ds \leq (26)$$

$$-\begin{bmatrix} e_{\tau} - e_{\tau_{M}} \\ e_{\tau_{m}} - e_{\tau} \end{bmatrix}^{T} \begin{bmatrix} R_{3} & -R_{3} \\ * & R_{3} \end{bmatrix} \begin{bmatrix} e_{\tau} - e_{\tau_{M}} \\ e_{\tau_{m}} - e_{\tau} \end{bmatrix}$$
$$\begin{bmatrix} R_{3} & -R_{3} \\ * & R_{3} \end{bmatrix} \ge 0.$$
(27)

Let $\dot{e} = \varepsilon \xi(t)$, $r_e(t) = \sigma \xi(t)$, where

$$\mathcal{E} = \begin{bmatrix} \left(A_i + \Delta A_i \right) & A_{di} - G_i C_i & 0 \\ 0 & - \left(B_i + \Delta B_i \right) & D_i \end{bmatrix},$$
$$\sigma = \begin{bmatrix} WC_i & 0 & 0 & 0 & -I & 0 \end{bmatrix},$$
$$(t) = \begin{bmatrix} e^T & e_{\tau}^T & e_{\tau_m}^T & e_{\tau_m}^T & f^T(t) & \omega^T(t) \end{bmatrix}.$$

then the equation (21) can be written as follows:

$$\dot{V}(t) + r_e^T(t)r_e(t) - \gamma_1^2 f^T(t)f(t) - \gamma_2^2 \omega^T(t)\omega(t) \leq \qquad (28)$$

$$\xi^T(t) \Big[\Big(\Big[\tilde{\Phi} + \tau_m^2 \varepsilon^T R_1 \varepsilon + \tau_M^2 \varepsilon^T R_2 \varepsilon + \Big] \Big] \Big] = 0$$

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_{11} \ \Phi_{12} \ R_1 \ R_2 \ \tilde{\Phi}_{15} \ P_i D_i \\ * \ \Phi_{22} \ \Phi_{23} \ \Phi_{24} \ 0 \ 0 \\ * \ * \ \Phi_{33} \ -R_3^T \ 0 \ 0 \\ * \ * \ * \ \Phi_{44} \ 0 \ 0 \\ * \ * \ * \ * \ -\gamma_1^2 I \ 0 \\ * \ * \ * \ * \ * \ -\gamma_2^2 I \end{bmatrix}$$
(29)
$$\tilde{\Phi} < 0$$

 $\tau_s^2 \varepsilon^T R_3 \varepsilon + \sigma^T \sigma \Big] \Big] \xi(t)$

$$\tilde{\Phi}_{11} = P_i \left(A_i + \Delta A_i \right) + \left(A_i + \Delta A_i \right)^T P_i + Q_1 - R_1 - R_2 + \sum_{j=1}^S \delta_{ij} P_j$$
$$\tilde{\Phi}_{15} = -P_i \left(B_i + \Delta B_i \right).$$

Let

 ξ^{T}

$$\tilde{\Phi} + \tau_m^2 \varepsilon^T R_1 \varepsilon + \tau_M^2 \varepsilon^T R_2 \varepsilon + \tau_s^2 \varepsilon^T R_3 \varepsilon + \sigma^T \sigma = \Phi.$$

According to Lemma 4, the uncertain term can be eliminated. From Lemma 1, we can get:

$$\Phi = \hat{\Phi} + \alpha \vec{M}_1 \vec{M}_1^T + \alpha^{-1} \vec{N}_1^T \vec{N}_1$$
 (30)

where

$$\vec{M}_{1}^{T} = \begin{bmatrix} M^{T} P_{i} & 0 & 0 & 0 & 0 \\ 0 & \tau_{m} M^{T} R_{1} & \tau_{M} M^{T} R_{2} & \tau_{s} M^{T} R_{3} & 0 \end{bmatrix},$$

$$\vec{N}_{1} = \begin{bmatrix} N_{ai} & 0 & 0 & 0 & -N_{bi} & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\hat{\Phi} = \begin{bmatrix} \Phi_{a} & \Phi_{b} \\ * & \Phi_{c} \end{bmatrix}.$$
(31)

In summary, the following inequality can be obtained:

$$\dot{V}(t) + r_e^T(t)r_e(t) - \gamma_1^2 f^T(t)f(t) - \gamma_2^2 \omega^T(t)\omega(t) \\
\leq \\ \xi^T(t)\Phi\xi(t) \qquad (32) \\
< \\ 0$$

When f(t) = 0 and $\omega(t) = 0$, the error system is asymptotically stable. For any $\omega(t) \in [0, \infty)$, integrating from t_0 to t on both sides, we can get the following:

$$V(t) - V(t_{0}) < (33)$$

- $\int_{t_{0}}^{t} (r_{e}^{T}(t)r_{e}(t) - \gamma_{1}^{2}f^{T}(t)f(t) - \gamma_{2}^{2}\omega^{T}(t)\omega(t))dt$

In the zero initial state, when $t \rightarrow \infty$, the following inequality holds:

$$\int_{0}^{\infty} r_{e}^{T}(t) r_{e}(t) dt$$

$$< \qquad (34)$$

$$\int_{0}^{\infty} \left(\gamma_{1}^{2} f^{T}(t) f(t) + \gamma_{2}^{2} \omega^{T}(t) \omega(t) \right) dt.$$

It shows that the error system is stable and meets the $H\infty$ performance. The proof is thus complete.

Based on the analysis of the $H\infty$ performance of the error system, combined with the idea of congruent transformation and variable replacement, the solution of the observer matrix is given below.

Corollary. For the state estimation error system (13), given positive scalar τ_m , τ_M , τ_s , γ_1 , γ_2 , a, b, c and α if there are positive definite symmetric matrices P_i , $T_i W$, Q_i (i = 1, 2) and R_i (i = 1, 2, 3), the following linear matrix inequality holds:

$$\overline{\Phi} = \begin{bmatrix} \overline{\Phi}_a & \overline{\Phi}_b & \overline{\Phi}_c \\ * & \overline{\Phi}_d & \overline{\Phi}_e \\ * & * & \overline{\Phi}_f \end{bmatrix} < 0$$
(35)

where

$$\begin{split} \overline{\Phi}_{a} = \begin{bmatrix} \Phi_{11} & \overline{\Phi}_{12} & R_{1} & R_{2} & -P_{i}B_{i} & P_{i}D_{i} \\ * & \overline{\Phi}_{22} & \Phi_{23} & \Phi_{24} & 0 & 0 \\ * & * & \Phi_{33} & -R_{3}^{T} & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & * & -\gamma_{1}^{2}I & 0 \\ * & * & * & * & * & -\gamma_{2}^{2}I \end{bmatrix} \\ \overline{\Phi}_{b} = \begin{bmatrix} \overline{\Phi}_{17} & \overline{\Phi}_{18} & \overline{\Phi}_{19} & C_{i}^{T}W^{T} \\ \overline{\Phi}_{27} & \overline{\Phi}_{28} & \overline{\Phi}_{29} & 0 \\ 0 & 0 & 0 & 0 \\ \overline{\Phi}_{57} & \overline{\Phi}_{58} & \overline{\Phi}_{59} & -I \\ \overline{\Phi}_{67} & \overline{\Phi}_{68} & \overline{\Phi}_{69} & 0 \end{bmatrix}, \\ \overline{\Phi}_{c} = \begin{bmatrix} -2aP_{i} + a^{2}R_{1} & 0 & 0 & 0 \\ * & -2bP_{i} + b^{2}R_{2} & 0 & 0 \\ * & * & -2cP_{i} + c^{2}R_{3} & 0 \\ * & * & * & -I \end{bmatrix}, \\ \overline{\Phi}_{d} = \begin{bmatrix} -R_{1} & 0 & 0 & 0 \\ * & -R_{2} & 0 & 0 \\ * & * & -R_{3} & 0 \\ * & * & * & -I \end{bmatrix}, \\ \overline{\Phi}_{d} = \begin{bmatrix} \pi_{n}R_{1}M & 0 \\ \pi_{n}R_{2}M & 0 \\ \tau_{n}R_{3}M & 0 \\ 0 & 0 \end{bmatrix}, \\ \overline{\Phi}_{f} = \begin{bmatrix} -\alpha^{-1}I & 0 \\ * & -\alpha I \end{bmatrix}, \\ \overline{\Phi}_{12} = P_{i}A_{di} - T_{i}C_{i}, \\ \overline{\Phi}_{12} = P_{i}A_{di} - T_{i}C_{i}, \\ \overline{\Phi}_{19} = \tau_{s}A_{i}^{T}P_{i}, \\ \overline{\Phi}_{19} = \tau_{s}A_{i}^{T}P_{i}, \\ \overline{\Phi}_{19} = \tau_{s}A_{i}^{T}P_{i}, \end{bmatrix}$$

$$\begin{split} \overline{\Phi}_{27} &= \tau_m \left(P_i A_{di} - T_i C_i \right)^T, \\ \overline{\Phi}_{28} &= \tau_M \left(P_i A_{di} - T_i C_i \right)^T, \\ \overline{\Phi}_{29} &= \tau_s \left(P_i A_{di} - T_i C_i \right)^T, \\ \overline{\Phi}_{57} &= -\tau_m B_i^T P_i, \\ \overline{\Phi}_{58} &= -\tau_M B_i^T P_i, \\ \overline{\Phi}_{59} &= -\tau_s B_i^T P_i, \\ \overline{\Phi}_{67} &= \tau_m D_i^T P_i, \\ \overline{\Phi}_{68} &= \tau_M D_i^T P_i. \end{split}$$

Then, there is an observer $G_i = P_i^{-1}T_i$, which makes the system asymptotically stable.

Proof: Multiply both ends of Φ by the following diagonal matrix and its transpose

$$diag \left\{ I \quad I \quad I \quad I \quad I \quad I \\ P_i R_1^{-1} \quad P_i R_2^{-1} \quad P_i R_3^{-1} \quad I \quad I \quad I \right\}.$$

From

$$-a^{2}R_{i} + 2aP_{i} - P_{i}R_{i}^{-1}P_{i}$$

$$\leq$$

$$-(aR_{i} - P_{i})R_{i}^{-1}(aR_{i} - P_{i})$$

$$\leq$$

$$0,$$

the following transformation can be obtained

$$\begin{cases} -P_i R_1^{-1} P_i \le -2aP_i + a^2 R_1 \\ -P_i R_2^{-1} P_i \le -2bP_i + b^2 R_2 \\ -P_i R_3^{-1} P_i \le -2cP_i + c^2 R_3 \end{cases}$$
(36)

Let $P_iG_i = T_i$. The matrix inequality $\overline{\Phi}$ can be available. Then, the observer gain matrix is $G_i = P_i^{-1}T_i$. The proof is thus complete.

4. Fault-tolerant Controller Design with Actuator Failure

The robust fault-tolerant control design based on the event-triggered mechanism is proposed in this section. Fault-tolerant control can reduce the effect of actuator failure and keep the closed-loop system asymptotically stable. Combining the ideas of congruent transformation and variable replacement, the gain matrix solution of the fault-tolerant controller is given.

Theorem 2. For the closed-loop control system (15), given positive scalars τ_m , τ_M , τ_s , γ , α and $\beta \in [0, 1)$, if there are positive definite symmetric matrices X_i , Y_i , S, U_i (i = 1, 2) and V_i (i = 1, 2, 3), for any actuator failures, the following linear matrix inequality holds.

$$\Psi = \begin{bmatrix} \Psi_a & \Psi_b & \Psi_c \\ * & \Psi_d & \Psi_e \\ * & * & \Psi_f \end{bmatrix} < 0$$
(37)

where,

$$\begin{split} \Psi_{a} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & D_{i}X_{i} \\ * & \Psi_{22} & -V_{3} & -V_{3} & \Psi_{25} & 0 \\ * & * & \Psi_{33} & \Psi_{34} & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & 0 \\ * & * & * & * & \Psi_{55} & 0 \\ * & * & * & * & \Psi_{55} & 0 \\ * & * & * & * & -\gamma^{2}I \end{bmatrix}, \\ \Psi_{b} = \begin{bmatrix} \tau_{m}XA_{i}^{T} & \tau_{M}XA_{i}^{T} & \tau_{s}XA_{i}^{T} \\ \Psi_{27} & \Psi_{28} & \Psi_{29} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Psi_{57} & \Psi_{58} & \Psi_{59} \\ \tau_{m}D_{i}^{T} & \tau_{M}D_{i}^{T} & \tau_{s}D_{i}^{T} \end{bmatrix}, \\ \Psi_{c} = \begin{bmatrix} M & XN_{ai}^{T} & XC_{i}^{T} \\ 0 & (N_{bi}LY_{i})^{T} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \Psi_{d} = \begin{bmatrix} -V_{1}^{-1} & 0 & 0 \\ * & -V_{2}^{-1} & 0 \\ * & * & -V_{3}^{-1} \end{bmatrix}, \\ \Psi_{e} = \begin{bmatrix} \tau_{m}V_{1}M & 0 & 0 \\ \tau_{M}V_{2}M & 0 & 0 \\ \tau_{M}V_{2}M & 0 & 0 \end{bmatrix}, \end{split}$$

$$\begin{split} \Psi_{f} &= \begin{bmatrix} -\alpha^{-1}I & 0 & 0 \\ * & -\alpha I & 0 \\ * & * & -I \end{bmatrix}, \\ \Psi_{11} &= XA_{i}^{T} + A_{i}X - 2X - U_{1} + V_{1} + V_{2} + \\ &\sum_{j=1}^{S} \delta_{ij}XP_{j}X, \\ \Psi_{12} &= A_{di}X + (B_{i}L + B_{di})Y_{i}, \\ &\Psi_{13} = 2X - V_{1}, \\ &\Psi_{13} = 2X - V_{2}, \\ \Psi_{15} &= -(B_{i}L + B_{di})Y_{i}, \\ &\Psi_{22} = \beta(2X - S) + V_{3}, \\ &\Psi_{25} = -\beta(2X - S), \\ \Psi_{33} &= -4X + U_{1} - U_{2} + V_{1} + V_{3}, \\ &\Psi_{34} = 2X, \\ &\Psi_{44} = -6X + U_{2} + V_{2} + V_{3}, \\ &\Psi_{55} = (\beta - 1)(2X - S), \\ &\Psi_{27} &= \tau_{m} (A_{di}X + B_{i}LY_{i} + B_{di}Y_{i})^{T}, \\ &\Psi_{28} &= \tau_{M} (A_{di}X + B_{i}LY_{i} + B_{di}Y_{i})^{T}, \\ &\Psi_{29} &= \tau_{s} (A_{di}X + B_{i}LY_{i} + B_{di}Y_{i})^{T}, \\ &\Psi_{57} &= -\tau_{m}Y_{i}^{T} (B_{i}L + B_{di})^{T}, \\ &\Psi_{59} &= -\tau_{s}Y_{i}^{T} (B_{i}L + B_{di})^{T}. \end{split}$$

Then the fault-tolerant controller is

$$K_i = Y_i X_i^{-1},$$

which can make the system stable and meet a certain $H\infty$ performance.

Proof: Choose the Lyapunov-Krasovskii function, as follows:

$$V(t) = x^{T}(t)P_{i}x(t) +$$

$$\int_{t-\tau_{m}}^{t} x^{T}(s)Q_{1}x(s) ds +$$

$$\int_{t-\tau_{M}}^{t-\tau_{m}} x^{T}(s)Q_{2}x(s) ds +$$

$$\int_{-\tau_{m}}^{0} \int_{t+\theta}^{t} \tau_{m}\dot{x}^{T}(s)R_{1}\dot{x}(s) ds d\theta +$$

$$\int_{-\tau_{M}}^{0} \int_{t+\theta}^{t} \tau_{M}\dot{x}^{T}(s)R_{2}\dot{x}(s) ds d\theta +$$

$$\int_{-\tau_{M}}^{-\tau_{m}} \int_{t+\theta}^{t} \tau_{s}\dot{x}^{T}(s)R_{3}\dot{x}(s) ds d\theta.$$
(38)

Taking the derivation of V(t) and introducing the event-triggered mechanism, the following expression can be obtained:

$$\dot{V}(t) = 2x^{T}P_{i}\dot{x} + \sum_{j=1}^{S} \delta_{ij}P_{j} + x^{T}Q_{1}x - x_{\tau_{m}}^{T}Q_{1}x_{\tau_{m}} + x_{\tau_{m}}^{T}Q_{2}x_{\tau_{m}} - x_{\tau_{M}}^{T}Q_{2}x_{\tau_{M}} + \tau_{m}^{2}\dot{x}^{T}R_{1}\dot{x} - \tau_{m}\int_{t-\tau_{m}}^{t}\dot{x}^{T}R_{1}\dot{x}ds + \tau_{M}^{2}\dot{x}^{T}R_{2}\dot{x} - (39)$$

$$\tau_{M}\int_{t-\tau_{M}}^{t}\dot{x}^{T}R_{2}\dot{x}ds + \tau_{s}^{2}\dot{x}^{T}R_{3}\dot{x} - \tau_{s}\int_{t-\tau_{M}}^{t-\tau_{m}}\dot{x}^{T}R_{3}\dot{x}ds + e_{x}^{T}(i_{k}h)\Sigma e_{x}(i_{k}h) - e_{x}^{T}(i_{k}h)\Sigma e_{x}(i_{k}h).$$

Let

$$\dot{x}(t) = \delta \eta(t) ,$$

$$\delta = \begin{bmatrix} A_i + \Delta A_i & A_{di} - K_{bi} & 0 & 0 & -K_{bi} & D_i \end{bmatrix},$$

$$\eta^T(t) = \begin{bmatrix} x^T & x_\tau^T & x_{\tau_m}^T & x_{\tau_m}^T & e_x^T & \omega^T \end{bmatrix}.$$

According to Lemma 1, Lemma 2 and the event-triggered mechanism, equation (39) can be written as:

$$\dot{V}(t) + y^{T}(t)y(t) - \gamma^{2}\omega^{T}(t)\omega(t) \leq \eta^{T}(t)\left[\tilde{\Psi} + \tau_{m}^{2}\delta R_{1}\delta + \tau_{M}^{2}\delta R_{2}\delta + \tau_{s}^{2}\delta R_{3}\delta\right]\eta(t)$$
(40)

$$\tilde{\Psi} = \begin{bmatrix} \tilde{\Psi}_{11} & \tilde{\Psi}_{12} & R_1 & R_2 & -P_i K_{bi} & P_i D_i \\ * & \tilde{\Psi}_{22} & 2R_3 & R_3 + R_3^T & -\beta \Sigma & 0 \\ * & * & \tilde{\Psi}_{33} & -R_3^T & 0 & 0 \\ * & * & * & \tilde{\Psi}_{44} & 0 & 0 \\ * & * & * & * & \beta \Sigma - \Sigma & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$
(41)
$$\tilde{\Psi} < 0$$

$$\begin{split} \tilde{\Psi}_{11} &= P_i \left(A_i + \Delta A_i \right) + \left(A_i + \Delta A_i \right)^T P_i + \\ Q_1 - R_1 - R_2 + C_i^T C_i + \sum_{j=1}^S \delta_{ij} P_j, \\ \tilde{\Psi}_{12} &= P_i \left(A_{di} + K_{bi} \right), \\ \tilde{\Psi}_{22} &= -2R_3 - R_3^T - R_3 + \beta \Sigma, \\ \tilde{\Psi}_{33} &= -Q_1 + Q_2 - R_1 - R_3, \\ \tilde{\Psi}_{44} &= -Q_2 - R_2 - R_3. \end{split}$$

According to Lemma 1, equation (40) can be written as:

$$\tilde{\Psi} + \tau_m^2 \delta R_1 \delta + \tau_M^2 \delta R_2 \delta + \tau_s^2 \delta R_3 \delta = \overline{\Psi} . \quad (42)$$

According to Lemma 4, the parameter uncertain terms in the matrix can be eliminated. We can then get:

$$\bar{\Psi} = \hat{\Psi} + \alpha \vec{M}_2 \vec{M}_2^T + \alpha^{-1} \vec{N}_2^T \vec{N}_2.$$
 (43)

Let

$$\vec{M}_{2}^{T} = \begin{bmatrix} M^{T}P_{i} & 0 & 0 & 0 & 0 \\ \tau_{m}M^{T}R_{1} & \tau_{M}M^{T}R_{2} & \tau_{s}M^{T}R_{3} \end{bmatrix},$$
$$\vec{N}_{2} = \begin{bmatrix} N_{ai} & (N_{bi}LK_{i})^{T} & 0 & 0 & -(N_{bi}LK_{i})^{T} \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$\hat{\Psi} = \begin{bmatrix} \Psi_a & \Psi_b \\ * & \hat{\Psi}_c \end{bmatrix} < 0$$
(44)

$$\begin{split} \hat{\Psi}_{a} = \begin{bmatrix} \hat{\Psi}_{11} & \hat{\Psi}_{12} & R_{1} & R_{2} & \hat{\Psi}_{15} & P_{t}D_{l} \\ * & \tilde{\Psi}_{22} & 2R_{3} & R_{3} + R_{3}^{T} & -\beta \Sigma & 0 \\ * & * & \tilde{\Psi}_{33} & -R_{3}^{T} & 0 & 0 \\ * & * & * & \tilde{\Psi}_{44} & 0 & 0 \\ * & * & * & * & \beta \Sigma - \Sigma & 0 \\ * & * & * & * & * & -\gamma^{2}I \end{bmatrix}, \\ \hat{\Psi}_{b} = \begin{bmatrix} \hat{\Psi}_{17} & \hat{\Psi}_{18} & \hat{\Psi}_{19} \\ \hat{\Psi}_{27} & \hat{\Psi}_{28} & \hat{\Psi}_{29} \\ 0 & 0 & 0 \\ \hat{\Psi}_{57} & \hat{\Psi}_{58} & \hat{\Psi}_{59} \\ \hat{\Psi}_{67} & \hat{\Psi}_{68} & \hat{\Psi}_{69} \end{bmatrix}, \\ \hat{\Psi}_{c} = \begin{bmatrix} -R_{1} & 0 & 0 \\ * & -R_{2} & 0 \\ * & * & -R_{3} \end{bmatrix}, \\ \hat{\Psi}_{11} = P_{i}A_{i} + A_{i}^{T}P_{i} + Q_{1} - R_{1} - R_{2} + C_{i}^{T}C_{i} + \\ \sum_{j=1}^{S} \delta_{ij}P_{j}, \\ \hat{\Psi}_{12} = P_{i} \left(A_{di} + B_{i}LK_{i} + B_{di}K_{i}\right), \\ \hat{\Psi}_{15} = -P_{i} \left(B_{i}LK_{i} + B_{di}K_{i}\right), \\ \hat{\Psi}_{17} = \tau_{m}A_{i}^{T}R_{1}, \\ \hat{\Psi}_{18} = \tau_{M}A_{i}^{T}R_{2}, \\ \hat{\Psi}_{19} = \tau_{s}A_{i}^{T}R_{3}, \\ \hat{\Psi}_{27} = \tau_{m} \left(A_{di} + B_{i}LK_{i} + B_{di}K_{i}\right)^{T}R_{1}, \\ \hat{\Psi}_{28} = \tau_{M} \left(A_{di} + B_{i}LK_{i} + B_{di}K_{i}\right)^{T}R_{2}, \\ \hat{\Psi}_{69} = \tau_{s}D_{i}^{T}R_{3}, \\ \hat{\Psi}_{29} = \tau_{s} \left(A_{di} + B_{i}LK_{i} + B_{di}K_{i}\right)^{T}R_{1}, \\ \hat{\Psi}_{69} = \tau_{s}D_{i}^{T}R_{2}, \\ \hat{\Psi}_{69} = \tau_{s}D_{i}^{T}R_{2}, \\ \hat{\Psi}_{68} = \tau_{M}D_{i}^{T}R_{2}, \\ \hat{\Psi}_{68} = \tau_{M}D_{i}^{T}R_{2}, \\ \end{bmatrix}$$

$$\hat{\Psi}_{58} = -\tau_M K_i^T (B_i L + B_{di})^T R_2,$$

$$\hat{\Psi}_{59} = -\tau_s K_i^T (B_i L + B_{di})^T R_3,$$

$$\hat{\Psi}_{67} = \tau_m D_i^T R_1.$$

Multiply both sides of $\overline{\Psi}$ by the diagonal matrix

$$diag \left\{ P_i^{-1} \quad P_i^{-1} \quad P_i^{-1} \quad P_i^{-1} \quad P_i^{-1} \\ I \quad R_1^{-1} \quad R_2^{-1} \quad R_3^{-1} \quad I \quad I \right\},$$

and let $X = P_i^{-1}$, $K_i X = Y$, $U_i = Q_i$, $V_i = R_i$, $\Sigma^{-1} = S$.

From

$$-a^{2}R_{i}+2aP_{i}-P_{i}R_{i}^{-1}P_{i}$$

$$\leq$$

$$-(aR_{i}-P_{i})R_{i}^{-1}(aR_{i}-P_{i})$$

$$\leq$$

$$0,$$

the following expressions can be obtained:

$$\begin{cases} P_i^{-1}Q_iP_i^{-1} \ge 2P_i^{-1} - Q_i^{-1} = 2X - U_i \\ P_i^{-1}R_iP_i^{-1} \ge 2P_i^{-1} - R_i^{-1} = 2X - V_i \end{cases}$$
(45)

Therefore, the matrix inequality Ψ can be obtained. In summary, it can be concluded that

$$\dot{V}(t) + y^{T}(t)y(t) - \gamma^{2}\omega^{T}(t)\omega(t) \leq (46)$$

$$\eta^{T}(t)\Psi\eta(t)$$

If $\Psi < 0$ then $\dot{V}(t) < 0$, the error system is asymptotically stable. For any $\omega(t) \in [0, \infty)$, integrating both sides of (46), we can get:

$$V(t) - V(t_0) < (47)$$

$$- \int_{t_0}^t (y^T(t)y(t) - \gamma^2 \omega^T(t)\omega(t)) dt$$

In the zero initial state, when $t \to \infty$, the following inequality holds:

$$\int_0^\infty y^T(t)y(t)\,dt < \int_0^\infty \gamma^2 \omega^T(t)\omega(t)\,dt \qquad (48)$$

That is, the closed-loop system of fault-tolerant control has certain disturbance suppression, and the system can achieve asymptotic stability. The proof is thus complete.

5. Numerical Simulation

In order to verify the effectiveness of the proposed algorithm in the Markov jump NCSs, the corresponding matrices and parameters are chosen, as follows.

The minimum and maximum values of the time delay are $\tau_m = 0.01$ and $\tau_M = 0.03$, the event-triggered parameter is $\beta = 0.2$, while the sampling time is h = 0.1 s.

Consider that the system switches between two modes, $S = \{1, 2\}$. The state probability transition rates matrix and the system parameter matrices of the two modes are given as:

$$\delta = \begin{bmatrix} -0.4 & 0.4 \\ 0.3 & -0.3 \end{bmatrix}$$

Mode 1:

$$A_{1} = \begin{bmatrix} -1.2 & 0.8 \\ 0.7 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} -0.7 & 0.3 \\ 1 & 0 \end{bmatrix},$$
$$B_{1} = \begin{bmatrix} 1 & 0 \\ 0 & -0.8 \end{bmatrix}, B_{d1} = \begin{bmatrix} 0.1 & 0.3 \\ 0 & 0.6 \end{bmatrix},$$
$$C_{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_{1} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}.$$

Mode 2:

$$A_{2} = \begin{bmatrix} -3 & 1 \\ 2 & -1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -2.3 & 1.1 \\ 0.7 & -0.1 \end{bmatrix},$$
$$B_{2} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, B_{d2} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$C_{2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_{2} = \begin{bmatrix} 0 \\ -0.2 \end{bmatrix}.$$

The uncertain term matrices ΔA_i and ΔB_i satisfy the equation (3), where $M = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}^T$, N_{ai}

= $\begin{bmatrix} 0.3 & 0.1 \end{bmatrix}$ and $F(t) = \sin t$ are selected. The external disturbance is $\omega(t) = 0.1 \sin 0.5t$. Other parameters are defined as $\gamma_1 = 1.7$, $\gamma_2 = 2.5$, $\gamma = 1$, $\alpha = 1$, a = 0.1, b = 0.2, c = 0.3.

According to *Corollary*, the observer gain matrix and residual gain matrix can be solved as:

$$G_{1} = \begin{bmatrix} 0.6764 & 0.4810 \\ 0.4235 & 0.1443 \end{bmatrix},$$
$$W_{1} = \begin{bmatrix} 2.1208 & 1.1614 \\ 1.1614 & 1.6549 \end{bmatrix},$$
$$G_{2} = \begin{bmatrix} -3.5578 & 1.1641 \\ 1.6781 & -0.7568 \end{bmatrix},$$
$$W_{2} = \begin{bmatrix} 0.6782 & -0.4783 \\ -0.4783 & -0.8285 \end{bmatrix}.$$

The system states and estimated states are shown in Figure 2 and Figure 3. It can be seen that the state curves observed by the observer are very close to the actual system. This shows that the observer designed in this paper can approximate the real states of the system, which proves that the observer has good performance.

As a comparison, according to the method in the reference [39], the state curves of the observer are shown in Figure 4 and Figure 5. It can be seen that the method in the reference shows a relatively large error for state estimation.

Next, fault tolerance performance is verified through simulation for two cases. Assume that the two types of actuator failure matrix are defined as:

$$L = L_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix}$$
, or $L = L_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

According to Theorem 2, the gain matrices of the fault-tolerant controller and the event-triggered matrices in the case of corresponding faults can be obtained as follows.

When L_1 fails, the matrix of the fault-tolerant controller and the event-triggered matrix are:

$$\begin{split} K_1 &= \begin{bmatrix} -0.0148 & 0.0109 \\ -0.0102 & 0.0102 \end{bmatrix} \\ \Sigma_1 &= \begin{bmatrix} -1.1898 & 0.1318 \\ 0.1318 & -2.0166 \end{bmatrix} \\ K_2 &= \begin{bmatrix} 2.0012 & -0.201 \\ -0.2031 & 0.0207 \end{bmatrix} \\ \Sigma_2 &= \begin{bmatrix} -0.2213 & 0.1228 \\ 0.1228 & -0.4397 \end{bmatrix} \end{split}$$

Now, the system state curves are shown in Figure 6 and Figure 7.

When L_2 fails, the matrix of the fault-tolerant controller and the event-triggered matrix are:



Figure 2. State x1 response curve.



Figure 3. State x2 response curve.



Figure 4. State x1 curves in the reference.



Figure 6. State x1 under L_1 fault condition.

$$\begin{split} K_1 = \begin{bmatrix} -3.2017 & 0.0228\\ 0.0238 & -2.2307 \end{bmatrix} \\ \Sigma_1 = \begin{bmatrix} -2.1995 & 0.0548\\ 0.0548 & -2.1432 \end{bmatrix} \\ K_2 = \begin{bmatrix} -1.0311 & -0.0206\\ 0.0398 & -0.4116 \end{bmatrix} \\ \Sigma_2 = \begin{bmatrix} -0.7627 & 1.2355\\ 1.2355 & -1.2624 \end{bmatrix}. \end{split}$$

At this time, the system state curves are shown in Figure 8 and Figure 9.

It can be seen from Figure 6 to Figure 9 that when the actuators have partial or complete failure, the system state can quickly stabilize



Figure 5. State x2 curves in the reference.



Figure 7. State x2 under L_1 fault condition.

after adopting fault-tolerant control, which shows that the designed robust fault-tolerant controller can effectively reduce the impact of both faults and external disturbances. The effectiveness of the fault-tolerant control method is thus proven.

Under the event-triggered mechanism, the release instants and release interval are shown in Figure 10. If the time-triggered mechanism is adopted, the control task needs to be executed 300 times. By using the event-triggered mechanism described in this paper, the control task is executed 248 times, thus saving 52 communication instances. It can be seen that, based on ensuring the asymptotic stability of the NCS, the event-triggered mechanism is well-applied to reduce the number of data transmissions in the channel, hence saving network resources. The validity of the designed event-triggered mechanism and controller is thus verified.



Figure 8. State x1 under L_2 fault condition.



Figure 10. Release instants and release interval.

As a comparison, and according to the method from [40], the state curves under fault L_1 condition are shown in Figure 11 and Figure 12. It can be seen that the state changes in the case of



Figure 9. State x2 under L_2 fault condition.

failure are more obvious when using the method from the reference.

In summary, the above simulations show that the observer and robust fault-tolerant controller design proposed in this paper are effective.

6. Conclusion

In this paper a kind of parameter uncertainty random time-delay Markov jumping system is used to describe NCSs with time delay, communication constraints and other network faults. In order to eliminate the influence of time delay and communication constraints on NCSs, an event-triggered mechanism is introduced. This event-triggered mechanism enables the transmission signal in NCSs to form a path when the trigger condition is met only, thus ef-



Figure 11. State x1 under L_1 fault condition.



Figure 12. State x2 under L_1 fault condition.

fectively reducing the occupation of network data channels and solving the above communication constraints. Subsequently an observer is designed to estimate the system states, and a robust fault-tolerant controller design is proposed for actuator failures in NCSs. The sufficient condition for the existence of the fault-tolerant controller under the event-triggered mechanism is given, too. Finally, both the effectiveness and feasibility of the proposed method are verified by simulation.

Based on the event-triggered mechanism, the observer and fault-tolerant controller are designed in this paper without considering the saturation of sensors or actuators. However, in actual systems, due to the limitations of the system itself, information security and communication technology, the infinite amplitude signals cannot be provided by sensors or actuators. For this reason, sensor or actuator saturation will occur. Therefore, in this case the control analysis and design research of NCSs based on the event-triggered mechanism are the topics for future work. In addition, the analysis and design of NCSs under multiple triggering strategies are also worthy of attention.

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