CHEMICAL ENGINEERING TRANSACTIONS
VOL. 75, 2019
A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.cetjournal.it
Guest Editors: Sauro Pierucci, Laura Piazza

ISBN 978-88-95608-72-3; ISSN 2283-9216
DOI: 10.3303/CET1975098

Function Projective Synchronization of Chaotic Systems with a New Kind of Scaling Function

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The function projective synchronization of chaotic system with a new kind of scaling function is proposed in this paper. In general, the scaling function factor of function projective synchronization is a function of the time variable. However, in this paper, the scaling function factor we discussed is a function of state variable which imply that this kind of synchronization is more complicated. Via modified active control method, the controller of the proposed synchronization is designed, and successfully applies to the four dimensional energy resources system and a new hyperchaotic Chua system. Numerical simulation is presented to show the validity of the controller and the proposed synchronization.

1. Introduction

Synchronous control of chaotic systems is widely used in motor control and industrial automation. Since Pecora and Carroll(1990) showed that it is possible to synchronize two identical chaotic systems, chaos synchronization has been intensively and extensively studied due to its potential applications in many areas. Various type of chaos synchronization have been investigated such as complete synchronization(Yu and Zhang, 2004), phase synchronization(zhang et al. 2016), lag synchronization(Shahverdiev, 2002), generalized synchronization(Kacarev and Parlitz, 1996), and projective synchronization(Mainieri. and Rehacek, 1999), etc. Among all kinds of chaos synchronizations, projective synchronization is one of the most noticeable one because of the proportionality between its synchronized dynamical states(Zhang and Wang, 2016). This kind of synchronization was first reported by Mainieri and Rehacek(1999) in partially linear systems. Recently, the concept of function projective synchronization (FPS) is introduced by some researchers(Luo, 2008), where the responses of the synchronized dynamical states could be synchronized up to a scaling function factor. In this paper, the FPS is investigated between two different chaotic systems with a new kind of scaling function which have not been discussed in other papers. Via the modified active control(Li and Zhao, 2011), the function projective synchronization between two different chaotic systems is achieved. Then the investigation of a four-dimensional energy resources system(Sun et al., 2009) and a new hyperchaotic Chua system(Paulo, 2009) show the feasibility of the controller.

2. Function projective synchronization and the modified active control

Consider a class of nonlinear chaotic system described by

$$\dot{x} = Ax + Bf(x) \tag{1}$$

where $x = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ is the state vector of the system, $A \in R^{n\times n}$ and $f: R^n \to R^n$ are the linear coefficient matrix and nonlinear part of system (1). $B \in R^{n\times n}$ is an constant matrix of the nonlinear function $f(x)$. We assume the response system as follows:

Paper Received: 7 May 2018; Revised: 9 October 2018; Accepted: 25 December 2018

Please cite this article as: Zeng X., You S., Zhou W., Li Z., Li L., 2019, Function Projective Synchronization of Chaotic Systems with a New Kind of Scaling Function, Chemical Engineering Transactions, 75, 583-588 DOI:10.3303/CET1975098

$$\dot{y} = C\,y + D\,g(y) + U \tag{2}$$

where $y = (y_1, y_2, \cdots, y_n)^{\mathrm{T}}$ is the state vector, $C \in R^{n\times n}$ and $g: R^n \to R^n$ are the linear coefficient matrix and nonlinear part of system (2). $D \in R^{n\times n}$ is an constant matrix of the nonlinear function $g(y)$. $U = (u_1, u_2, \cdots, u_n)^{\mathrm{T}}$ is the controller to be determined. Define the error vector as $e = x - \Gamma(x_i)\,y$, where $\Gamma(x_i)$ is the scaling function factor. Then the error dynamical system can be obtained by subtracting the system (2) from system (1)

$$\dot{e} = E\,e + h(x, y, \Gamma(x_i)) - \dot{\Gamma}(x_i)\,y - \Gamma(x_i)U \tag{3}$$

where $e = (e_1, e_2, \cdots, e_n)$ is the error vector, E is constant matrix, and $h: R^n \to R^n$ is the nonlinear part of system (3).

Remark 1. The scaling function $\Gamma(x_i)$ can be chosen as many kinds of elementary function such as periodic function, polynomial function and etc. $x_i\ (i = 1, 2, \cdots, n)$ is one of the state variables of system (1).

Definition 1. For two systems described by system (1) and system (2), we say they are globally function projective synchronous with respect to the scaling function factor $\Gamma(x_i)$ if there exists a vector controller U such that

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|x_m(t) - \Gamma(x_i)\,x_s(t)\| = 0$$

which implies that the error dynamic system (3) is globally asymptotically stable. □

Then we introduce the summary of modified active control to design the controller U.

Definition 2. (see(Li and Zhao, 2011))Define the controller U as

$$\begin{cases} U = U_a + U_b \\ U_a = \Gamma(x_i)^{-1} M\, e \\ U_b = \Gamma(x_i)^{-1} (h(x, y, \Gamma(x_i)) - \dot{\Gamma}(x_i)\,y) \end{cases} \tag{4}$$

where $\Gamma(x_i)$ is the scaling function factor, M is a constant matrix to be determined later. □

Substituting the controller (4) into (3), the error system becomes

$$\dot{e} = (E - M)\,e \tag{5}$$

To make the system (5) asymptotically stable at the origin, we introduce the following lemmas.

Lemma 1. (See (Robinson, 2004)) Let x^* be a fixed point of the equation $\dot{x} = F(x)$, if exist a function V which satisfies (i) $V(x) > V(x^*)$ for all x in a neighborhood U of x^* but distinct from x^* and (ii) $\dot{V}(x) < 0$ for all x in U but distinct from x^*. Then, x^* is asymptotically stable.

Lemma 2. (See (Hu et al., 2010)) The dynamic system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} k_1 a_{11} & k_1 a_{12} & \cdots & k_1 a_{1n} \\ k_2 a_{21} & & \cdots & \\ \vdots & \vdots & \ddots & \\ k_n a_{n1} & & & k_n a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{6}$$

if satisfies the following conditions

(1) $\forall a_{ij} \in R$; (2) $a_{ij} = -a_{ji} (i \neq j)$; (3) $a_{ii} \le 0$ (all a_{ii} are not equal to zero); (4) $\forall k_i > 0$

then the system asymptotically converge to zero. □

Theorem 1. For two systems described by system (1) and system (2), we say they are globally function projective synchronous under the controller (4) provided there is a real constant matrix M, which makes the matrix $E-M$ satisfies the conditions in Lemma 2.

Proof. If the matrix $E-M$ satisfies the conditions in Lemma 2, then the fix point of system (5) is asymptotically stable. Because the system (5) is linear system, the origin is the fix point which means

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|x-\Gamma(x_i)\,y\| = 0 \tag{7}$$

According to Definition 1, the globally generalized projective synchronization between system (1) and system (2) is achieved. □

According to Theorem 1, we know that the synchronization between system (1) and (2) can be transformed into a problem of how to choose the matrix M.

3. FPS between two different systems

Recently, Sun and Tian established a new four-dimensional energy resources system is obtained(Sun et al., 2009). The system can be described as

$$\begin{cases} \dot{x}_1 = a_1\,x_1(1-x_1/M) - a_2\,(y_1+z_1) - d_3\,\omega_1 \\ \dot{y}_1 = -b_1\,y_1 - b_2\,z_1 + b_3\,x_1[N-(x_1-z_1)] \\ \dot{z}_1 = c_1\,z_1(c_2\,x_1 - c_3) \\ \dot{\omega}_1 = d_1\,x_1 - d_2\,\omega_1 \end{cases} \tag{8}$$

when the parameters $a_1 = 0.09$, $a_2 = 0.15$, $b_1 = 0.06$, $b_2 = 0.082$, $b_3 = 0.07$, $c_1 = 0.2$, $c_2 = 0.5$, $c_3 = 0.4$, $d_1 = 0.1$, $d_2 = 0.06$, $d_3 = 0.08$, $M = 1.8$, $N = 1$. In(Paulo, 2009), a new four-dimensional hyperchaotic Chua system was reported. Take the system as the response system and write it as

$$\begin{cases} \dot{x}_2 = \alpha_1(y_2 - a_3\,x_2^3 - (1+c_4)\,x_2) + u_1 \\ \dot{y}_2 = x_2 - y_2 + z_2 + u_2 \\ \dot{z}_2 = -\beta\,y_2 - \gamma\,z_2 + \omega_2 + u_3 \\ \dot{\omega}_2 = -s\,x_2 + y_2\,z_2 + u_4 \end{cases} \tag{9}$$

where u_1, u_2, u_3, u_4 are controllers to be designed. Define the synchronization error vectors as

$e_1 = x_1 - \Gamma(X)\,x_2$, $e_2 = y_1 - \Gamma(X)\,y_2$, $e_3 = z_1 - \Gamma(X)\,z_2$, $e_4 = \omega_1 - \Gamma(X)\,\omega_2$

$\Gamma(X)$ is the scaling function factor. X can be chosen as any state variable of system (8). Then we can obtain the following error dynamic system as follows

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} a_1 & -a_2 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & -\gamma & 0 \\ -s & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} (\alpha_1+\alpha_1 c_4+a_1)\Gamma(X)x_2 - (a_2+\alpha_1)\Gamma(X)y_2 - a_2 z_1 - d_3\,\omega_1 - a_1 x_1^2/M + \Gamma(X)\alpha_1 a_3 x_2^3 \\ (1-b_1)y_1 - (b_2+1)z_1 + (b_3 N - 1)x_1 - b_3 x_1^2 + b_3 x_1 z_1 \\ (\gamma - c_1 c_3)z_1 + c_1 c_2 x_1 z_1 + \Gamma(X)\beta y_2 - \Gamma(X)\omega_2 \\ (d_1+s)x_1 - d_2\,\omega_1 - \Gamma(X)y_2 z_2 \end{bmatrix} - \Gamma(X)U \tag{10}$$

According to **Definition 2**, we assume that

$$U_a = \Gamma(X)^{-1} P e \tag{11}$$

where P is a constant matrix to be determined,

$$U_b = \Gamma(X)^{-1} \begin{bmatrix} (\alpha_1 + \alpha_1 c_4 + a_1)\Gamma(X) x_2 - (a_2 + \alpha_1)\Gamma(X) y_2 - a_2 z_1 - d_3 \omega_1 - a_1 x_1^2 / M + \Gamma(X) \alpha_1 a_3 x_2^3 \\ (1 - b_1) y_1 - (b_2 + 1) z_1 + (b_3 N - 1) x_1 - b_3 x_1^2 + b_3 x_1 z_1 \\ (\gamma - c_1 c_3) z_1 + c_1 c_2 x_1 z_1 + \Gamma(X) \beta y_2 - \Gamma(X) \omega_2 \\ (d_1 + s) x_1 - d_2 \omega_1 - \Gamma(X) y_2 z_2 \end{bmatrix} \tag{12}$$

$$-\Gamma(X)^{-1} (x_2, y_2, z_2, \omega_2)^{\mathrm{T}}$$

Then the controller can be described as

$$U = U_a + U_b \tag{13}$$

Let

$$E = \begin{bmatrix} a_1 & -a_2 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & -\gamma & 0 \\ -s & 0 & 0 & 0 \end{bmatrix}$$

Substituting the controller (13) into the system (10) yield

$$\dot{e} = (E - P) e \tag{14}$$

According to Lemma 2, we choose the matrix P as

$$\begin{bmatrix} k_1(\frac{a_1+1}{k_1}) & k_1(\frac{-a_2+1}{k_1}) & 0 & k_1(\frac{-s}{k_4}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & k_3(\frac{-\gamma+1}{k_3}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting the matrix P into system (14), we obtained the following new error system

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & k_1 \frac{s}{k_4} \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ k_4 \frac{-s}{k_4} & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{15}$$

Let $k_1 = 10$, $k_2 = 20$, $k_3 = 20$ and $k_4 = 50$, then the system (15) satisfies the Theorem 1, which implies that the system (8) and (9) are globally fucntion projective synchronized with respect to the scaling function factor $\Gamma(X)$.

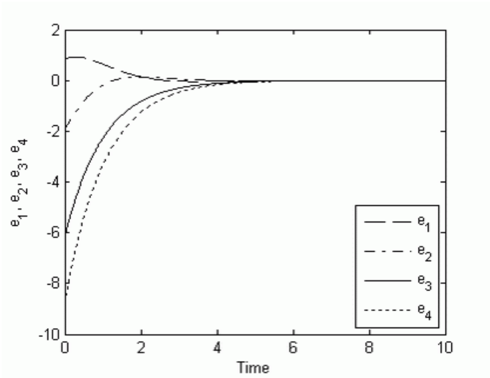

Fig. 1 Time response of synchronization errors

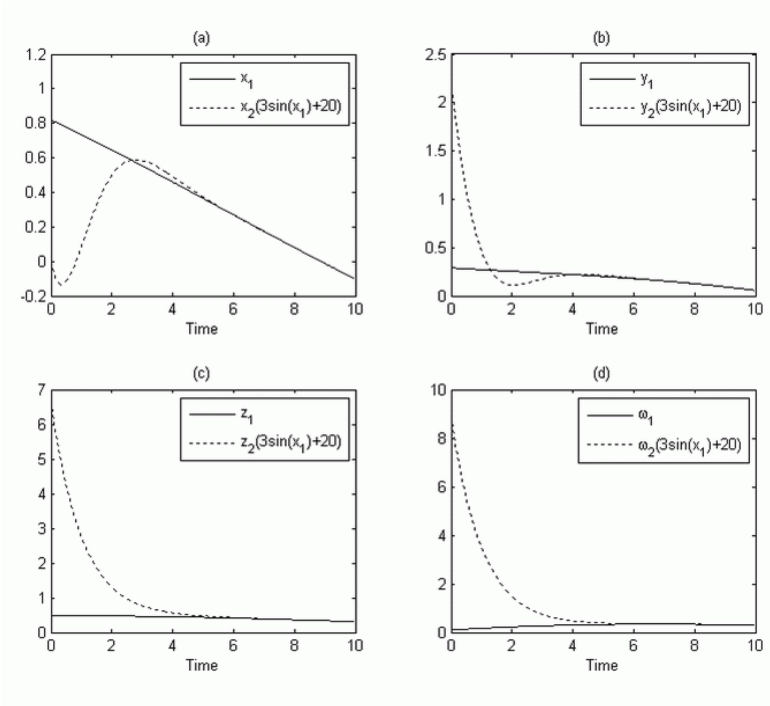

Fig.2 The synchronization result of state variables

4. Numerical simulation

In what follows we would like to use the numerical simulation to verify the validity of the result obtained in last section. The initial values of the drive system (8) and response system (9) are taken as $x_1(0)=0.82$, $y_1(0)=0.29$, $z_1(0)=0.48$, $\omega_1(0)=0.1$, $x_2(0)=0.0$, $y_2(0)=0.1$, $z_2(0)=0.3$, $\omega_2(0)=0.4$, respectively. The scaling function factor $\Gamma(X)$ is chosen as

$$\Gamma(X)=d_1\sin(x_1)+d_2 \tag{16}$$

where x_1 is one of the state variable of system (8). So we can get that

$$\dot{\Gamma}(X)=d_1\,\dot{x}_1\cos(x_1) \tag{17}$$

Substituting equation (16) and (17) into the controller (13) with $d_1=3$ and $d_2=20$, then we can obtain the simulation result. Fig. 1 displays the time response of synchronization errors. As expected, the errors converge to the zero as time goes to infinite. Fig. 2 shows the state variables of drive system and response system synchronize to the scaling function. In general, the scaling function factor of FPS is a function of the time variable. However, in this paper, the scaling function factor of FPS we discussed is a function of state variable which imply that this kind of scaling function factor is complicated than the general one.

5. Conclusions

The function projective synchronization between two different chaotic systems has been investigated. A modified active control for achieving the synchronization is proposed. Then the investigation of a four-dimensional energy resources system and a new hyperchaotic Chua system show the feasibility of the controller. In numerical simulation, we discuss the FPS with a new kind of scaling function which has not been discussed in other papers. The simulation results are shown in corresponding figures which imply that the synchronization we discussed in this paper is feasible.

Acknowledgments

The authors acknowledge support by the National Natural Science Foundation of China (No. 51375416,11172093, 11372102), the Project of Science Research of Department of Education of Hunan Province(No. 15K131, 16C1366) and the Hunan Provincial Innovation Foundation for Postgraduate (No. CX2012B159).

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