

Water Pollution Prevention and Control of Chemical Enterprises Based on Cooperative Game

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Due to the particularity of its production technology and process, chemical enterprises are shouldering important missions in environmental protection and pollution control. For this, in this paper, based on the cooperative game, the pollution of chemical enterprises was analysed firstly; Then, in view of the interactive feature of pollution-emission behaviour and survival in Jilin Chemical Industry Park, the non-cooperative game model and cooperative game model of chemical enterprises were established by considering the factors such as tax revenue, governance cost and pollution loss etc. Finally, taking Jilin Chemical Industry Park as example, the models were verified, including the emission and profits respectively in the non-cooperative game state and cooperative game state. The results indicate that the cooperation situation is conducive to reducing pollution emissions, and the cooperative profit after distribution is greater than non-cooperative one, but more comprehensive countermeasures should be also taken to ensure the effective reduction of pollutant emission load in the park.

1. Introduction

Chemical enterprises are an important driving force for the development of China's national economy. In the new situation, chemical enterprises should do well in prevention and control, reduce emissions, and strengthen environmental protection. They should also pay more attention to the integrated development of environment and social economy, while focusing on their own economic benefits.

The studies for pollution control based on game theory covers almost every domain of environmental science. It used game theory for the first time to establish the sustainable competitive petroleum supply chain (SCPSC) model for pollution minimization and profits maximization (Moradinasab et al., 2018). It adopted a differential game to study the interaction between polluting firms and regulatory agencies (Arguedas et al., 2017). It expanded the agreement through non-cooperative and cooperative game analysis to deepen the climate governance model (Hannam et al., 2017). Taking Jilin Chemical Industry Park as the research object, this paper discusses the pollution-emission strategy, benefits and distribution in the situation of enterprise cooperative game. First of all, in the non-cooperative situation, the non-cooperative game issue of individual enterprise working on its own was explored to solve its Nash equilibrium. Secondly, the coalition game of environmental cooperation among various enterprises in Jilin Chemical Industrial Park was studied, and the distribution of cooperation results was studied. Finally, the model was analyzed with the case verification.

2. Non-cooperative and cooperative game models

2.1 Non-cooperative game

The game in this paper is a complete-information static game. Each pollution emission enterprise in the chemical park is called one player in the game. There is no cooperation agreement between them. The payer's revenue function and alternative strategy are measured by all parties. That is, k players were defined in the enterprise game, and the collection of players was U . The amount of pollutants selected by player x ($k=1,2,\dots,k$) is called the pollution emission strategy, and x has an infinite number of consecutive pollution emission strategies; one specific strategy of x is represented by s_x , and the strategy set is $S_x=\{s_x\}$. If each of k

players chooses one pollution emission strategy, the k-dimensional vector $s=(s_1, \dots, s_i, \dots, s_k)$ is called the strategy combination. P_x is the revenue function of the x-th players and represents the profits that player x obtained. P_x is the function of all player strategies, $p_x=p_x(s_1, \dots, s_i, \dots, s_k)$. Nash equilibrium is the most important concept in non-cooperative game theory. It's defined that in the game $C=(S_1, \dots, S_k; p_1, \dots, p_k)$, if the strategy s_k of any player in the strategy combination $s^k=(s^k_1, \dots, s^k_x, \dots, s^k_k)$ is the best strategy for the other players' strategy combination $s^{k-x}=(s^{k-x}_1, \dots, s^{k-x}_{x-1}, s^{k-x}_{x+1}, \dots, s^{k-x}_k)$,

$$P_x(s^k_x, s^k_{-x}) \geq P_x(s_x, s_{-x}), \quad \forall s_x \in S_x, \quad \forall x \tag{1}$$

Then, $s_x=(s^x_1, \dots, s^x_x, \dots, s^x_k)$ is called as one Nash equilibrium for game C. It's assumed that each enterprise has only one source of pollution and emits one pollutant. The survival of enterprise i is determined by the concentration of certain pollutants in the chemical park:

$$c_x(s_x, s_{-x}) = c_{x0} + \sum_{p \in U} i_{xy} s_p \tag{2}$$

where, i_{xy} is the pollution influence coefficient of p enterprise on x, $i_{xx} > 0$. c_{x0} is the initial cross-section pollutant concentration. The second item on the right side of the formula (2) is the increase value of concentration. Due to the mutual influence of the pollution emission in the chemical industry park, the survival $c_x(s_1, \dots, s_i, \dots, s_k)$ is not only the function of its own pollution emission s_x , but also that of the s_{-x} of the pollution emission -x for the other enterprises. The profit p_x of the player x is determined by the tax revenue $t_x(s_x)$, the pollution governance cost $g_x(s_x)$, and the loss $l_x(c_x)$.

$$p_x(s_x, s_{-x}) = t_x(s_x) - g_x(s_x) - l_x(c_x(s_x, s_{-x})) \tag{3}$$

Formula (3) can be written as:

$$p_x = s_x(\alpha_x - 12s_x) - \beta_x s_x - \gamma_x \sum_{p \in U} i_{xy} s_p \tag{4}$$

where, β_x is the cost coefficient of corporate governance pollution. The greater the s_x emissions, the higher the governance cost. γ_x is the constant coefficient.

In the non-cooperative situation, the income of player x is expressed as p^k_x , where the superscript k indicates non-cooperation. The goal pursued by player i in non-cooperative situations is:

$$\max p^k_x \tag{5}$$

Solving this game means to seek for Nash equilibrium strategy of the game. Problem (5) can be solved using the extremum principle. Second-order partial derivatives of s_x was made for p^k_x :

$$\frac{\partial^2 p^k_x}{\partial s^2_x} = -1 < 0 \tag{6}$$

The second order partial derivative is less than 0, and then the maximum value of the function p^k_x , exists. Therefore, from the extreme conditions, the Nash equilibrium strategy for player x is obtained as:

$$s^k_x = \alpha_x - \beta_x - \gamma_x i_{xx} \tag{7}$$

Substituting Formula (7) into (4), the non-cooperative profit p^k_x , of player x under Nash equilibrium can be obtained. Formula (7) shows that in the non-cooperative situation, player x only considers maximizing his own earnings, and the selected pollution emission strategy has nothing to do with others.

2.2 Cooperative game

2.2.1 Form of cooperative game

For the sake of convenience, the empty set is also called the sub-coalition, and k player can form 2k sub-coalition in total. Assume that the cooperative game formed by one certain sub-coalition I is denoted as $\xi(I), I \subset U$. The goal of the grand coalition's cooperative game x is to seek for the pollution emission strategy $(s_x, s_{-x}), x \in U$, in order to maximize the sum of its profits p^g_U :

$$\max p^g_U \tag{8}$$

$$p_x^g = \sum_{x \in U} p_x^g = \sum_{x \in U} \{s_x (\alpha_x - \frac{1}{2} e_x) - \beta_x s_x - \gamma_x \sum_{p \in U} i_{xp} s_p\} \quad (9)$$

where, p_x^g denotes the profit of x in cooperation, and the superscript g denotes cooperation.

The goal of the sub-coalition cooperative game $\xi(I)$ is to seek for pollution emission strategy (s_x, s_{-x}) , $x \in I$, in order to maximize the sum of its profit sum p^g :

$$\max p^g_I \quad (10)$$

$$p^g_I = \sum_{x \in I} p_x^g = \sum_{x \in I} \{s_x (\alpha_x - \frac{1}{2} s_x) - \beta_x s_x - \gamma_x \sum_{p \in U} i_{xp} s_p\} \quad (11)$$

2.2.2 Game Solving

(1) Grand coalition game $\xi(U)$ Solving

p_x^g can be viewed as a multivariate function:

$$p_x^g = p_x^g(s_x, s_{-x}) \quad (12)$$

The first-order partial derivative of the multivariate function to any variable s_x is:

$$\frac{\partial p_x^g}{\partial s_x} = \alpha_x - s_x - \beta_x - \sum_{p \in U} \gamma_p i_{px} \quad (13)$$

Suppose there is a variable s_y different from s_x . its second-order partial derivative is calculated as:

$$\frac{\partial^2 p_x^g}{\partial s_x^2} = -1, \frac{\partial^2 p_x^g}{\partial s_x \partial s_y} = 0, \frac{\partial^2 p_x^g}{\partial s_j^2} = -1, \frac{\partial^2 p_x^g}{\partial s_y \partial s_x} = 0. \quad (14)$$

From the extreme conditions, it can be solved:

$$s_x^g = s_x^k - \sum_{p \in U \setminus x} \gamma_p i_{px} \quad x \in U \quad (15)$$

Therefore, the multivariate function p_x^g has the maximum. Formula (15) is the optimal pollution control strategy under the state of cooperative game in the grand coalition. Substituting formula (15) into (4) and (9), the profits p_x^g ($x \in U$) of any player x and the total profits p^g of the coalition U under the cooperative game $\xi(U)$ state of grand coalition.

(2) Sub-coalition cooperative game solving

Since the remaining players $o \in U \setminus I$ adopted the non-cooperative Nash equilibrium strategy, c_x can be expressed as:

$$c_x = \sum_{p \in I} i_{xp} s_p + \sum_{o \in U \setminus I} i_{xo} s_o + c_{x0} \quad (16)$$

$$\frac{\partial p_x^g}{\partial s_x} = \alpha_x - s_x - \beta_x - \sum_{p \in I} \gamma_p i_{px} \quad (17)$$

Similar to the solution to the grand coalition game, for the multivariate function $p_x^g(s_x, s_{-x})$, the Hessian matrix H is negative fixed. It's solved by extreme conditions:

$$s_x^g = \alpha_x - \beta_x - \sum_{p \in I} \gamma_p i_{px}, \quad x \in I \quad (18)$$

Substituting formula (18) into (4), the profit p_x^g ($x \in I$) of any player x and the total profit p^g of the coalition I in the state of the coalition cooperative game $\xi(I)$ are obtained. In particular, when coalition I has only one element, $s_x^g = s_x^k$, $p_x^g = p_x^k$. Based on formula (7), formula (18) is written as:

$$s_x^g = s_x^k - \sum_{p \in I \setminus x} \gamma_p i_{px} \quad x \in I \quad (19)$$

Formula (18) shows that in the situation of coalition cooperation, the strategy choice of any player x takes into account the maximization of global benefits, and the selected pollution emission strategy are related to the pollution loss coefficient γ_p of other players, and the players environment influencing coefficient i_{px} for other

players. The difference between the coalition cooperation game strategy and the non-cooperation game strategy lies only in the second item on the right side of formula (19): when the sum of the product of the loss coefficient for other players and the influence coefficient of the player on other players increases, the difference in the pollution emission shall be greater between the cooperative and non-cooperative states. $At_{p_x} = 0$ or $i_{p_x} = 0$, it means that player x has no effect on the overall profits of the coalition game, and its cooperation strategy is the same as the non-cooperation strategy.

3. Distribution plan of cooperative game

According to the actual meaning of the environmental cooperation game, the eigen function of the grand coalition I can be defined as the total surplus obtained through cooperation. It's expressed as:

$$f(U) = \sum_{x \in U} p_x^g - \sum_{x \in U} p_x^k \quad (20)$$

Similarly, for any sub-coalition, its eigen function $f(I)$ refers to the total surplus obtained by Coalition I through the internal cooperation of I .

$$f(I) = \sum_{x \in I} p_x^g - \sum_{x \in I} p_x^k \quad (21)$$

When the coalition I has only one player:

$$f(I) = f(\{x\}) = 0 \quad (22)$$

The distribution solution to the coalition cooperation strategy refers to one distribution plan for the profit $f(U)$ of the overall coalition. $h(f)$ is defined as one distribution plan of the eigen function v :

$$h(f) = (h_1(f), \dots, h_x(f), \dots, h_k(f)) \quad (23)$$

Based on cooperative game theory, the distribution $h_x(f)$ of x should satisfy at least two conditions:

(1) Individual rationality

$$h_x(f) \geq f(\{x\}), \quad x \in U. \quad (24)$$

The surplus that all players have obtained after cooperation should not be lower than the those they earn individually.

(2) Overall rationality

$$\sum_{x \in U} h_x(f) = f(U) \quad (25)$$

The total sum of cooperative surplus that all players are assigned to should be equal to that of cooperation when they all form a coalition. In order to meet the distribution of the above conditions, this paper uses the Shapley value method to make distribution. It's calculated as:

$$h_x(f) = \sum_{I \subset U} z(I) [f(I) - f(I \setminus x)], \quad x \in U \quad (26)$$

$$z(I) = \frac{(k - |I|)! (|I| - 1)!}{k!} \quad (27)$$

$|I|$ is the number of elements in set I . After obtaining the distribution solution to the coalition game, the final profit pex of player x can be further determined:

$$p_x^s = p_x^k + h_x(f) \quad (28)$$

For the distribution of cooperation game $\xi(U)$, the solving steps are as follows: By formula (7), calculate Nash equilibrium in non-cooperative game; By formula (4), calculate non-cooperative game profits p_x^k ; Use Formula (15) and (18) to calculate the player's emissions strategy $s_{x(x \in U)}^g, s_{x(x \in I)}^g$ for the cooperative game $\xi(I)$ and each sub-coalition $\xi(K)$ respectively; use formula (4) to calculate the returns $p_{x(x \in U)}^g, p_{x(x \in I)}^g$ in the case of cooperation game $\xi(I), \xi(K)$; Use Formula (20) and (21) to calculate the eigenvalues $f(U)$ and $f(I)$ according to the definition of the eigen function $f(U), f(I)$; Use Formula (26) and (27) to calculate the distribution $h_x(f)$; Use formula (28) to calculate the final income pex for each player.

4. Empirical Analysis

4.1 Descriptive statistics of variables

Taking the emissions and profits of the non-cooperative game state and cooperative game state of the three enterprises in Jilin Chemical Industry Park as research object, their cooperation profits were distributed accordingly. Non-cooperative Nash equilibrium strategy s^k_x and non-cooperative profits p^k_x were calculated according to steps 1 and 2, as shown in Table 2. According to formula (2), the increase of pollutant concentration on the monitoring section of all enterprises in the fully-cooperated state was calculated, as shown in Table 3.

Table 1 Parameter values of the model

Region	1	2	3
α_i /(Million yuan/t)	6.84	5.35	7.22
i_{xy} /($\text{mg}\cdot\text{L}^{-1}\cdot\text{t}^{-1}\cdot\text{a}^{-1}$)	$i_{11}=0.0038; i_{21}=0.0042;$ $i_{31}=0.0051$	$i_{12}=0.0026; i_{22}=0.0041;$ $i_{32}=0.0039$	$i_{13}=0.0032; i_{23}=0.0036;$ $i_{33}=0.0045$
β_i /(Million yuan/t)	1.02	0.93	1.16
γ_i /(Million yuan· $\text{mg}\cdot\text{L}^{-1}\cdot\text{a}^{-1}$)	54.10	37.00	61.00

Table2 Nash equilibrium strategy s^k_y and payoff p^k_x of non-cooperative Game

Region	1	2	3
s^k_y /(t/a)	2.10	1.08	1.96
p^k_x /(Million yuan/a)	61.35	37.75	59.24

Table3 The added value of pollutant concentration of the water quality monitoring sections

Region	1	2	3
Uncooperative concentration added value/(mg/L)	5.29	7.02	5.81
CO concentration added value/(mg/L)	5.03	6.02	3.55

According to step 3, the emission strategy and benefits of the sub-coalition cooperation game $\xi(I)$ was calculated. The empty set was excluded. The $I(I)$ coalition includes 7 forms such as $\xi(1)$, $\xi(2)$, $\xi(3)$, $\xi(1,2)$, $\xi(1,3)$, $\xi(2,3)$ and $\xi(U)$ (Table 4).

Table4 Emission strategies and payoff of sub-coalition game $\xi(I)$

I	{k}	1U2	1U3	2U3	U
s^{g_1} /(t/a)	1002	921.5	983	1011	814
s^{g_2} /(t/a)	914	902	985	842	816
s^{g_3} /(t/a)	1421	1425	1421	1415	1401
p^{g_1} /(Million yuan/a)	61.35	61.35	61.24	/	60.98
p^{g_2} /(Million yuan/a)	37.75	37.05	/	36.77	37.24
p^{g_3} /(Million yuan/a)	59.24	/	60.13	59.58	61.21

Compared with pre-cooperation state, the emissions of the three enterprises after cooperation decreased by 21.3%, 19.1%, and 10.6% respectively. Cooperation profit increased by 4.55%, 4.14%, and 2.16% respectively. Survival increased by 21.05%, 18.56% and 12.44% respectively. Then, it can be seen that the results of environmental cooperation are much better than those of the individual operation. The distribution plan calculated by the Shapley value method is also feasible.

5. Conclusions

It was found that in the non-cooperative situation, the players in the Jilin Chemical Industry Park only consider maximizing their own returns, and the selected pollution emission strategies have nothing to do with other players. The profits from environmental cooperation is greater than the non-cooperative ones. Therefore, from the perspective of China's current environmental management system, it is necessary to completely change the end control strategy in the chemical industry park and adopt more comprehensive countermeasures to ensure effective reduction of pollutant emission load in the park. It includes: 1) Strengthen the source control,

promote the recycling economy and ecological park construction mechanism; enhance the cleaner production and resource recovery in the park; intensify the rationalization and targeted construction of pipeline network pollution standards; 2) Intensify the separate collection; 3) Strengthen the supervision and early warning of the comprehensive sewage disposal plant and enhance the end-treatment; 4) Enhance the government supervision, public participation, and public opinion supervision; 5) Adopt the mode of plant management approach so as to strictly control the water quality standard of the factory.

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