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Dynamics Of Steam Heated Shell And Tube Heat Exchangers: New Insights And Time Domain Solutions

Franco Evangelista
Department of Chemistry, Chemical Engineering and Materials
University of L'Aquila, Italy

The dynamics of steam heated shell and tube heat exchangers are governed by a non-linear system of integro-partial differential algebraic equations (IPDAE). An approximate solution is obtained by the characteristic, Laplace transform and difference equation methods (CLD) which is valid for generic non-zero initial conditions and any combination of stepwise inputs. It compares well with rigorous solutions (FEM). The results obtained are particularly useful in the design of analog and digital controllers.

1. Introduction

Dynamic studies of chemical processes are of paramount importance for their design and the design of their control systems. Heat exchangers have been the subject of such studies since their early developments. The steam heated 1-1 types seem to have been the first models to simulate for their simpler behaviour. However many assumptions are usually made (Tan and Spinner, 1978; Yin and Jensen, 2003; Evangelista, 2005) . One of the assumption, which may lead to wrong results is that of constant temperature in the shell side. Many authors support such hypothesis by the higher value of the heat transfer coefficient and the small heat capacity in the shell side which would lead to smaller time constants in comparison with the tube side.
In real installation this is not true since keeping steam flow rate constant, the temperature of the condensing steam changes with the temperature and flow rate of the heated stream leading to longer responses instead. A system with multiple interacting capacity seems to be the best candidate to simulate this apparatus. Moreover control systems would have bad performance if designed with wrong models. Therefore, it is the purpose of the present paper to develop a dynamic model able to take into account for the earlier described behaviour and for design and validation purposes. However more difficulties are encountered in this case owing to the non linear dependence of the temperature of the condensing steam with flow rates.
Previously I have described (Evangelista, 2005) a dynamic model with extended capabilities considering true inputs the temperature and flow rate of the heated stream and the temperature of the steam. This model has been solved by combining the method of the characteristics with Laplace transform and difference equation method (CLD).
In this paper the same procedure is exploited in order to obtain a dynamic model of the apparatus but considering the steam flow rate as input instead of its temperature.

2. Theory

With reference to the apparatus shown in Fig. 1, no differences are reported for the tube side behaviour in comparison to a previous publication (Evangelista, 2005). Therefore

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the analysis and the mathematical derivation start with the modelling the behaviour of the saturated steam through the integrated vapour pressure equation:

$$\ln\left(\frac{P_V(t)}{P_o}\right)=\left(\frac{A}{T_o}-\frac{A}{T_V(t)}\right) \quad (1)$$

the equation of the ideal gas:

$$\rho_V(t)=\frac{P_V(t)M_W}{RT_V(t)} \quad (2)$$

and the latent heat approximation:

$$\lambda_V(t)=\lambda_o+B\left[T_V(t)-T_o\right] \quad (3)$$

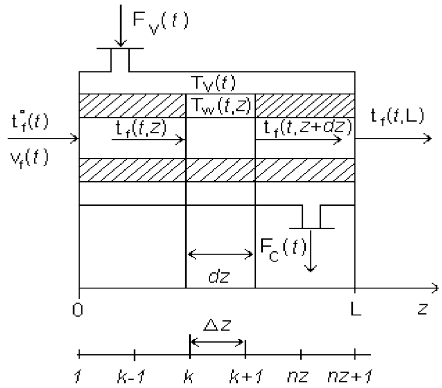

Fig. 1. Apparatus and variables.

For moderate temperatures and pressures, the constant A and B can be put equal to $\frac{\lambda_o M_W}{R}$ and $\left(C_{pv}-C_{pf}\right)$ as in the Clapeyron and Kirchoff equations respectively.
For somewhat wider range of operating conditions, but less than 5 bar and 420 °K, the constants can be easily estimated through latent heat and vapour pressure data.
The integro-differential part of the system is given by the transitory mass balance equation in the shell side:

$$V_S\frac{d\rho_V(t)}{dt}=F_V(t)-F_c(t)=F_V(t)-\frac{\pi d_o h_o}{\lambda_V}\int_0^L\left[T_V(t)-T_W(t,z)\right]dz \quad (4)$$

with initial conditions:

$$\text{for} \quad t=0 \quad \rho_V(0)=\rho_{VS} \quad F_c(0)=F_{cs}=\frac{\pi d_o h_{os}}{\lambda_{VS}}\int_0^L\left[T_{VS}-T_{WS}(z)\right]dz \quad (5)$$
$$P_V(0)=P_{VS} \quad T_V(0)=T_{VS} \quad F_V(0)=F_{VS}=F_{cs}$$

To this highly non linear set of equations other two differential equations must be added, one for the wall temperature:

$$\tau(t)\,\frac{\partial Tw(t,z)}{\partial t} = \sigma(t)T_V(t) - Tw(t,z) + [1-\sigma(t)]t_f(t,z) \quad (6)$$

and one for the cold fluid temperature:

$$\frac{\partial t_f(t,z)}{\partial t} + v_f(t)\frac{\partial t_f(t,z)}{\partial z} = \gamma(t)\left[\,Tw(t,z)-t_f(t,z)\right] \quad (7)$$

with boundary conditions:

for $z = 0$ and $t > 0$ $\qquad t_f(t,0) = t_f^o(t)$ (8)

and initial conditions:

for $0 \le z \le L$ and $t = 0$ $\qquad t_f(0,z) = t_{fs}(z) \qquad Tw(0,z) = T_{ws}(z)$ (9)

where:

$$\gamma(t) = \frac{4h_i}{\rho_f C_{pf} d_i} \qquad \sigma(t) = \frac{h_o d_o}{h_o d_o + h_i d_i} \qquad \tau(t) = \frac{C_{Pw}(d_o^2 - d_i^2)\rho_w}{4(h_o d_o + h_i d_i)} \tag{10}$$

From equations (1) – (10) the dependence of the temperature of the condensing steam is derived as function of the other inputs such as the steam flow rate, temperature and flow rate of the cold fluid.
As usual, the steady state conditions are first derived, which also in this case are found analytically. The steady state counter parts of Eqs. (6) and (7) have been integrated (Evangelista, 2005). Substituting the wall temperature result into the steady state counterpart of Eq. (4) the following relationship for the steam temperature is obtained:

$$T_{vs} = \frac{F_{vs}\left(\lambda_o - BT_o\right) + t_{fs}^o \alpha_s}{\alpha_s - BF_{vs}} \tag{11}$$

where:

$$\alpha_s = \pi d_o h_{os}\left(\frac{v_f}{\gamma\sigma}\right)_s \left(1 - \sigma_s\right)\left[1 - e^{-\left(\frac{\gamma\sigma}{v_f}\right)_s L}\right] \tag{12}$$

Therefore fixing the value of cold fluid flow rate and temperature, the flow rate of the steam and calculating the stationary parameters $\gamma_s, \sigma_s, \tau_s$, h_{os}, stationary conditions can by easily calculated from Eqs. (1) – (3) and (11), (12).
Now we could define the deviation variables. But, for brevity, we carry on with absolute variables. No appreciable differences have been noticed between the two derivations in this case.
The system of Eqs. (1) – (10) is highly non-linear that numerical methods are the most suited solutions, such as that derived from Finite Element Methods. However these methods can be time consuming, iterative, and difficult to be included into control system structures. Therefore it is the purpose of the present paper to develop simple marching explicit solutions for the design, performance prediction and validation of control configurations of this distributed parameter system.
In a previous publication I have solved the system of Eqs. (6)- (10) through the (CLD) method taking the steam temperature T_v as independent input, obtaining the following explicit discrete relationships (Evangelista, 2005):

$$t_{f,i,1} = t^{o}_{f(i\Delta t)} \qquad \text{for} \quad i > 1 \text{ and } \quad k = 1$$

$$t_{f,i,k} = \frac{\phi_i t_{f,i-1,k-1} + (1-\phi_i)\left[\varphi_i T_{w,i-1,k} + \sigma_i(1-\varphi_i)T_{v,i}\right]}{1 - (1-\phi_i)(1-\sigma_i)(1-\varphi_i)}$$

$$\text{for} \quad i > 1 \text{ and } \quad 2 \le k \le nz+1 \tag{13}$$

$$T_{w,i,k} = \varphi_i T_{w,i-1,k} + (1-\varphi_i)T_{v,i} + (1-\sigma_i)(1-\varphi_i)t_{f,i,k} \tag{14}$$

where:

$$\phi_i = e^{-\gamma_i \Delta t} \qquad \varphi_i = e^{-\Delta t/\tau_i} \tag{15}$$

One drawback of this method is that, fixed the spatial grid, the time intervals are dependent on the flow rate of cold fluid, and may not be coincident with the sampling time. However continuity of the solutions can be easily recovered and explicitly re-evaluated in a new grid of different size and shape.

Now, turning attention to Eq. (4), we could notice that the vapour hold up in the shell side is not so big in comparison with the other terms. So we can first solve Eq. (4) for T_v assuming quasi-stationary (qs) conditions. Under this hypothesis Eq. (4) becomes an integral equation only and I have exploited the method reported in (Perry and Green, 1998) to solve it. The integral has been approximated by a numerical quadrature with Simpson formulas. Later the wall temperature is evaluated through Eqs. (13) and (14) as function of the wall and fluid temperatures at the previous time step and the current input fluid temperature. At this stage, knowing also the vapour flow rate, the only unknown of Eq. (4) is T_v:

$$T^{qs}_{v,i} = \frac{\left[F_{v,i}(\lambda_o - BT_o) + H_i E_i\right]}{\pi d_o h_o L\left[\delta_i - \sigma_i(1-\varphi_i)\right] - H_i\sigma_i(1-\varphi_i)(1-\delta_i) - \delta_i B F_{v,i}} \tag{15}$$

where:

$$E_i = \varphi_i\left(\delta_i T_{w,i-1,1} + T_{w,i-1,nz+1}\right) + \eta_i\left(\delta_i t^{o}_{f,i} + \phi_i t_{f,i-1,nz}\right) + 2\varphi_i \nu_{i-1} + 2\phi_i \eta_i \psi_{i-1} \tag{16}$$

$$\eta_i = (1-\sigma_i)(1-\varphi_i) \qquad \delta_i = 1 - (1-\phi_i)\eta_i \qquad H_i = \pi d_o h_o \Delta z/3$$

$$\nu_{i-1} = \sum_{k=2}^{nz} T_{w,i-1,k} + \sum_{k=1}^{nz/2} T_{w,i-1,2k} \qquad \psi_{i-1} = \sum_{k=1}^{nz-1} t_{f,i-1,k} + \sum_{k=1}^{nz/2} t_{f,i-1,2k-1}$$

More precise dynamic characteristics of the vapour temperature can be easily accounted for by assuming that its behaviour is of first order type and calculating the time constant through the following relationship derived by linearizing Eq. (4):

$$\tau_V = \frac{V_S M_w \lambda_o P_{vs}\left(M_w \lambda_{vs} - R T_{vs}\right)}{\left\{\pi d_o h_o L - B F_{vs}\right\} R^2 T_{vs}^2} \tag{17}$$

and the vapour temperature calculated with:

$$T_{v,i} = T_{v,i-1} e^{-\Delta t / \tau_{v,i}} + \left(1 - e^{-\Delta t / \tau_{v,i}}\right) T_{v,i}^{qs} \tag{18}$$

Once the condensing temperature has been calculated, the wall and cold fluid temperature can be calculated by Eqs (13) and (14).

3. Results and Discussion

In this section I will report some results of preliminary calculations performed with the model developed in this work. The calculations obtained by the Finite Element Method (FEM) are also reported for comparison purposes. The apparatus dimensions are reported in Table I, while steady state operating conditions are reported in Table II. For simplicity sake fluid properties have been kept constant for both cold and hot fluids and equal to that of water and steam. The heat transfer coefficients, instead, are let to vary according to literature relationships, in this case the same as that reported in (Tan and Spinner, 1978), that is, dependence only on velocities through the exponent *n* equal to 0.8. Any other dependence from the inputs can be accounted for.

Some sample calculations have been performed varying all inputs at the same time and only the sequence reported in Fig. 2 is presented. The variation of the inlet temperature of the cold fluid has been limited to ± 10 °K, the vapour flow rate ± 33.3 % of steady state value. Bigger variations of the cold fluid flow rate has been allowed as shown by velocity reported in the same figure. Calculations with quasi-steady state approximations are also shown. As can be seen, big variation of steam temperatures can be noticed which makes any constant profile theory not applicable. The outlet temperature follow as much as possible the steam temperature with delays showing constant pattern and proportionate pattern transitions with varying time constants. All three methods agree quite well, with bigger discrepancies in steam temperature.

However in some part of the response the quasi-stationary approximation fails leading to bad prediction of local variables. As usual the FEM method shows higher dispersion in cold fluid temperature.

4. Conclusions

A new method for simulating the dynamic behaviour of a steam heated shell and tube heat exchangers has been developed and some preliminary calculations show a good agreement with numerical methods. Accepts any combination of stepwise inputs. Non-zero generic initial conditions and any dependence of parameters can be easily handled.

Table I. Apparatus dimensions.

D_i = 5.81 $\cdot 10^{-2}$ m	d_0 = 1.55$\cdot 10^{-2}$ m	d_i = 1.25$\cdot 10^{-2}$ m	L = 20 m

Table II. Steady state conditions

t^0_{fs} = 288 °K	T_{vs} = 370 °K
F_{fs} = 2$\cdot 10^{-4}$ m^3 / s	F_{vs} = 0.015 Kg / s
h_{is} = 0.2 Kcal / s m^2 °K	h_{0s} = 1 Kcal / s m^2 °K

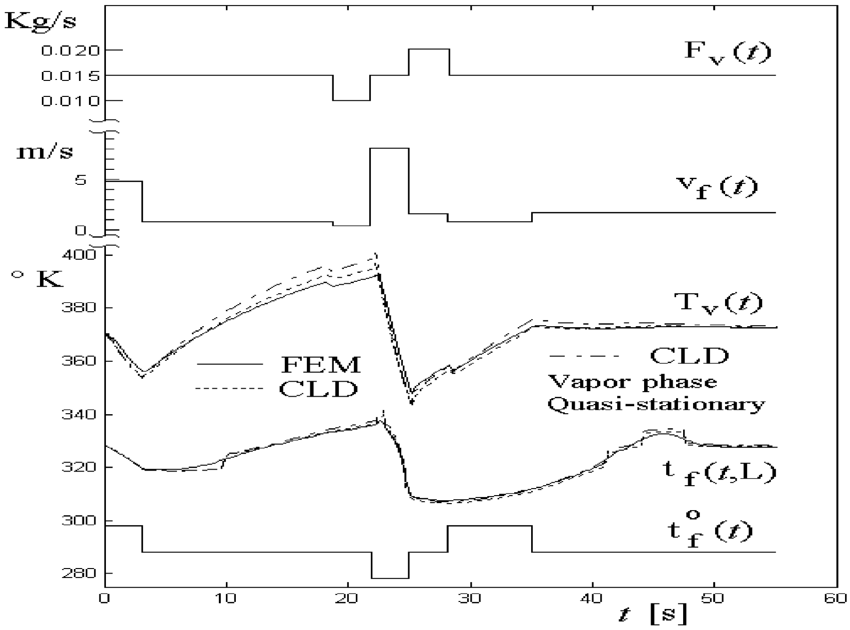

Fig. 2. Steam temperature and outlet temperature of cold fluid as function of time and predetermined sequence of inputs F_v, v_f and t^o_f.

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