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Sensitivity analysis on a rheological model for the flowability of aerated fine powders

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A new model based on dynamic powder flow properties to predict the torque measured in the mechanically stirred fluid-bed rheometer (msFBR) is proposed. A Schulze shear cell was used to measure the powder flow properties. An inverse procedure to evaluate powder flow properties from torque data is proposed and verified with shear cell data.

1. Introduction

The flow properties of fine powders at very low consolidation levels are relevant to small scale industrial application of powder flow, such as in small process hoppers, or in everyday applications such as toner flow in cartridges. Conventional testers and procedures for powder flow properties however, are not suited for testing powder flow properties at low consolidation values. At University College London (UCL) a mechanically stirred fluid-bed rheometer (msFBR) was developed to study the rheology of fluidized powders (Bruni *et al.*, 2005). This device used with aeration below the fluidization threshold allows to measure powder flow properties at very low powder consolidation levels. In fact, the upwards gas flow produces a vertical gas pressure gradient which acts on the bulk solids as a body force opposed to gravity. The torque necessary to rotate an impeller immersed in a bed of aerated powders was measured and results were compared to those predicted by a powder rheology based model (Bruni *et al.*, 2007a). In this paper a new model based on dynamic powder flow properties is proposed. A more sensitive shear cell was used to measure these flow properties. An inverse procedure to evaluate powder flow properties from torque data is assessed.

2. Experimental apparatus and materials

2.1. Apparatus

The mechanically stirred Fluidized Bed Rheometer (msFBR) was designed, built and commissioned at UCL to carry out rheological experiments in fluidized bed. In the present work, only powders aerated below minimum fluidization were studied. The msFBR, represented in Figure 1, consists of a fluidization unit, an agitating system and a data acquisition unit. The braking torque and the pressure drop across the bed are measured. The experimental procedure for the torque measurements below minimum

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Table 1 Material properties

	Alumina A0	Alumina A5	Alumina A6	Ballotini
d_p [μm] (Sauter)	75.1	49.1	36.0	350
F_{45}	3.2	30.7	30.9	-
%wt 0 – 25 μm	0	8.1	18.8	-
%wt 25 – 45 μm	3.2	22.6	12.1	-
ρ_p [kg m^{-3}]	1730	1730	1730	2500
ϕ_d [°]	34	33	31	26
C_d [Pa]	90	120	158	80
ϕ_w [°]	11.3	13.6	16.5	6.2
A [Pa]	68.7	66.2	68.0	43.3

fluidization condition and the effect of the impeller depth and the aeration are reported by Bruni *et al*. (2007a). Details regarding the effect of the impeller speed and the measurement time are reported by Bruni *et al*. (2005).
Powder flow properties were tested by a Schulze shear cell (Ring Shear Tester RST 01.01) located in the laboratories at the Department of Chemical and Food Engineering of the University of Salerno. Experimental results are subsequently compared with powder flow properties obtained in a previous work with a Peschl shear cell (Bruni *et al*., 2007b).

2.2. Materials

The materials used in these experiments are three alumina powders and a glass ballotini powder whose properties were evaluated by Bruni *et al*. (2007b) and are summarized in Table 1. The alumina A0 is a virtually fines free powder while Alumina A5 and A6 contain the same total amount of fines (F_{45} indicates the weight percentage of fines in the powder) but shifted towards bigger (26-45 μm) or smaller (0-25 μm) size cuts, respectively. Table 1 reports also the angle and the intercept of the dynamic yield locus of the materials (ϕ_d and C_d) and of the wall yield loci (ϕ_w and A) on a plexiglass wall sample linearized in the σ-τ plane, obtained using the Peschl shear cell (Bruni *et al*., 2007b).

3. Experimental results

In the present work the failure properties of the three alumina sample A0, A5 and A6 and of the glass ballotini powder were measured by a Schulze shear cell. In a previous

Fluidization unit	
Internal diameter *D*	140 mm
Vessel height	300 mm
Agitating system	
Steel shaft height	165 mm
Impeller height *h*	7 mm
Impeller horizontal dimension *d*	36 mm

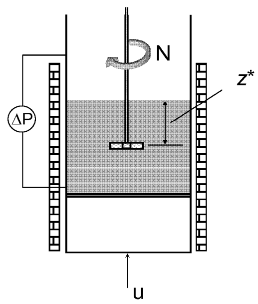

Figure 1 Schematic of the msFBR unit

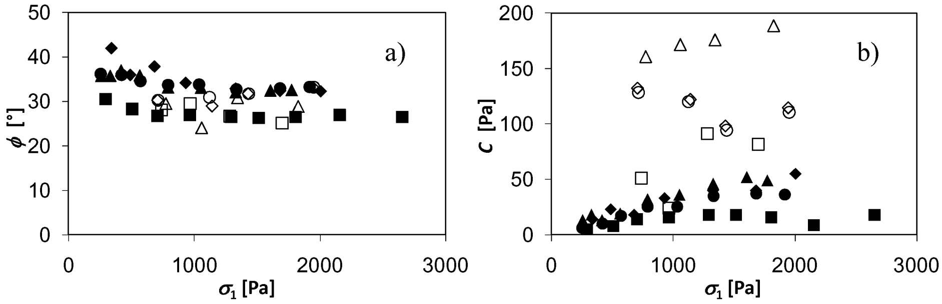

Figure 2 Powder flow properties as a function of the major principal stress: a) static angle of internal friction and b) cohesion. Shear cell: hollow symbols, Peschl; filled symbols, Schulze. ◇◆, Alumina A0; ○●, Alumina A5; △▲, Alumina A6; □■, Ballotini.

analysis (Bruni *et al.* 2007b) the material properties were measured by a rotational Peschl shear cell. However, the ring shear tester allows measurement on larger samples and provides with more reliable results. In Figure 2 the measured values of the static angle of internal friction and of the cohesion are compared to that previously performed. It can be seen that the results obtained with the two different cells are similar for the static angle of internal friction ϕ but they diverge considerably for the cohesion C. As expected, data collected with the Schulze cell show a larger internal consistency.
These experiments confirmed small differences between the angle of internal friction (Figure 2a) of the powders analyzed. The alumina samples A0, A5 and A6 showed very similar angles of internal friction while the ballotini showed lower values of ϕ. These low values of ϕ detected for the ballotini are probably due to their perfectly round shape and low rugosity which make shearing easier. Similar results are found for the powder cohesion (Figure 2b) and the effective angle of internal friction δ (Figure 3) both potted as a function of the consolidation stress. Samples A0, A5 and A6 exhibited very similar behaviors while ballotini showed lower values. This difference can be easily explained by considering the size and shape of the particles. In fact, powders made of bigger and more regular particles are generally characterized by smaller cohesion.

4. Sensitivity analysis on the rheological model

4.1. The rheological model by Bruni *et al.* (2007a)

To determine the stress distribution in the msFBR the original Janssen's analysis for silo stress distribution was generalized to take into account the air flow through the bed. According to this analysis, stresses were assumed uniform across any horizontal section of the material. Furthermore, the axial and radial stress σ_z and σ_r were assumed as principal stresses and the passive state of stress was hypothesized (Bruni *et al.*, 2007a). In this case the stress distribution into the msFBR is described by the equations:

$$\frac{d\sigma_z}{dz} + \alpha\sigma_z = \beta \quad (1)$$

where

$$\alpha = \frac{4}{D}\frac{1+\sin\phi}{1-\sin\phi} \quad (2)$$

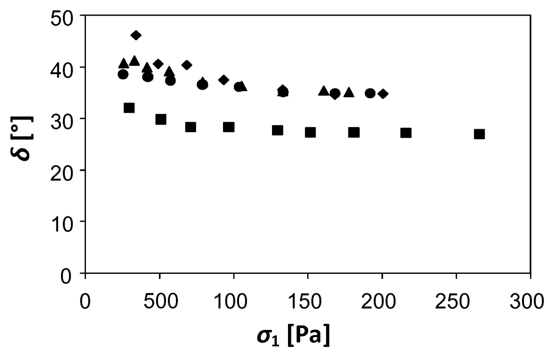

Figure 3 Effective angle of internal friction measured with a Schulze shear cell: ◆, Alumina A0; ●, Alumina A5; ▲, Alumina A6; ■, Ballotini.

$$\beta = \rho_b g - \frac{\mathrm{d}P}{\mathrm{d}z} - \frac{4}{D}\frac{2C\cos\phi}{1-\sin\phi}\tan\phi_w - \frac{4A}{D} \qquad (3)$$

Since ϕ and C depend on the consolidation stress, they are a function of z and, therefore Eq (1) needs to be solved numerically. To evaluate the torque acting on the impeller in the msFBR, the shearing surface around the impeller was assumed to be shaped like the flat cylinder described by the impeller rotation, having height h and diameter d. The resistant torque will be the sum of the contributions of the stresses acting on the upper surface, the lower surface and the lateral surface.

$$T = T_{up} + T_{down} + T_{lateral} \qquad (4)$$

Due to the small impeller height the stress variations in the region around the impeller were neglected. Furthermore, in shearing conditions, the proper relation between the shear and the normal stress is given by the dynamic yield locus. Therefore:

$$T_{up} = T_{down} = \int_0^{d/2} \tau_{z\theta} 2\pi r^2 \mathrm{d}r = \frac{\pi}{12}\tau_{z\theta}(z^*)d^3 = \frac{\pi}{12}\left[\sigma_z(z^*)\tan\phi_d + C_d\right]d^3 \qquad (5)$$

$$T_{lateral} = \frac{\pi}{2}d^2\int_{z^*-h/2}^{z^*+h/2}\tau_{r\theta}(z)\,\mathrm{d}z = \frac{\pi}{2}d^2 h\left\{\left[\sigma_z(z^*)\left(\frac{1+\sin\phi}{1-\sin\phi}\right)+\left(\frac{2C\cos\phi}{1-\sin\phi}\right)\right]\tan\phi_d + C_d\right\} \qquad (6)$$

4.2. The model based on dynamic flow properties

Full mobilization of powder is often assumed to model the stress state in silos or hoppers. The effective yield locus properly describes this condition. Assuming full mobilization for the powder in the msFBR requires a change of the model. In particular, Eqs (2), (3), (5) and (6) turn into the following:

$$\alpha = \frac{4}{D}\frac{1+\sin\delta}{1-\sin\delta} \qquad (7)$$

$$\beta = \rho_b g - \frac{\mathrm{d}P}{\mathrm{d}z} - \frac{4A}{D} \qquad (8)$$

$$T_{up} = T_{down} = \frac{\pi}{12}\left[\sigma_z(z^*)\tan\delta\right]d^3 \qquad (9)$$

$$T_{lateral} = \frac{\pi}{2}d^2 h\left\{\left[\sigma_z(z^*)\left(\frac{1+\sin\delta}{1-\sin\delta}\right)\right]\tan\delta\right\} \qquad (10)$$

As δ depends on the consolidation stress, Eq (1) needs to be solved numerically also in this case. Figure 4 shows the comparison between experiments and the values of the torque estimated with Eqs (5) and (6) and those estimated with Eqs (9) and (10). The rate of aeration was expressed as the ratio of the ΔP across the bed measured during the experiment and the calculated ΔP_c corresponding to full bed support $\Delta P_c = Mg / A$. Inspection of the figure reveals that the predictions obtained with Eqs (9) and (10), accounting for the effective yield locus (EYL), better describe experimental data especially at low impeller depth. These findings tend to support the assumption of full mobilization of powders.

5. Estimation of the flow properties with the msFBR

In order to estimate the flow properties of the materials from the torque measurements with the msFBR, as further assumptions we made the hypothesis that wall adhesion A is negligible and that the effective angle of internal friction does not change with consolidation. With these hypotheses, δ and ϕ_w are constant for each material. The model provides with a simplified set of equations in which two characteristic parameters can be identified: T_∞ is the asymptotic value of the torque at large depth values; z_c is the characteristic depth for which torque attains the 63% of T_∞. The expressions of these two parameters are given by the following:

$$T_\infty = \left[\frac{\pi}{6}d^3 + \left(\frac{1+\sin\delta}{1-\sin\delta}\right)\frac{\pi}{2}d^2 h\right]\tan\delta \frac{\left(\rho_b g - \frac{dP}{dz}\right)D}{4\left(\frac{1+\sin\delta}{1-\sin\delta}\right)\tan\phi_w} \tag{11}$$

$$z_c = \frac{D}{4\left(\frac{1+\sin\delta}{1-\sin\delta}\right)\tan\phi_w} \tag{12}$$

Each experimental torque profile for a certain material at a certain aeration condition can be used to provide with a couple of T_∞ and z_c values. In particular, T_∞ was derived by averaging torque values measured at largest impeller depths and z_c was estimated by curve fitting experimental torque profiles to point out the depth at which $T = 0.63T_\infty$.

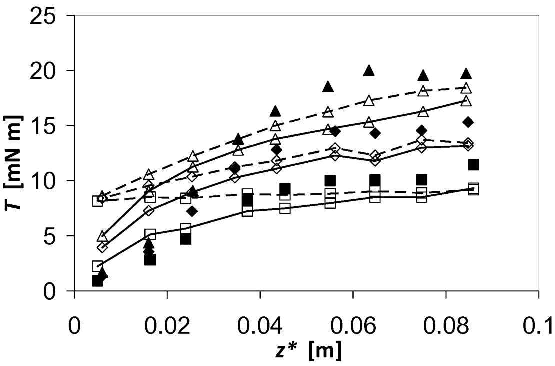

Figure 4 Comparison between experiments and estimated torques for Alumina A0 for different values of $\Delta P/\Delta P_c$*: △▲, 0; ◇◆, 0.25; □■, 0.46. Filled symbols, experimental data. Hollow symbols, model eq (4) plus: — —, eq (5) and (6); ——, eq (9) and (10).*

Table 2 Estimated properties of materials with the msFBR

	Alumina A0	Alumina A5	Alumina A6	Ballotini
T_∞ [mN m]	19.7 (0.00)	18.4 (0.00)	18.7 (0.00)	17.5 (0.00)
$(\Delta P/\Delta P_c)$	14.7 (0.25)	13.1 (0.33)	14.6 (0.24)	13.3 (0.24)
	10.4 (0.46)	7.2 (0.69)	11.4 (0.48)	9.7 (0.48)
z_c [m]	0.024	0.020	0.018	0.028
δ [°]	40.9	42.6	43.5	31.8
ϕ_w [°]	16.9	18.7	19.4	21.4

While each aeration condition provided with a different value of T_∞, it was observed a limited variation of z_c. Table 2 reports for each material the values of T_∞ as a function of the aeration condition (in terms of $\Delta P/\Delta P_c$) and averages of z_c. Table 2 reports also values of the effective angle of internal friction δ and the angle of wall friction ϕ_w providing the best fit of eqs (11) and (12) to experimental values of T_∞ and z_c. The values of δ appearing in Table 2 can be compared with those reported in Figure 3. These latter were measured with a standard shear cell as a function of the consolidation stress σ_1. For each material, Figure 3 reports significant changes only within the low consolidation stress range, that is the most relevant for this comparison. Values of δ in this range are fairly close to those reported in Table 2 and materials are classified by these values in the same order. Instead, comparison of the values of ϕ_w appearing in Table 2 with those reported in Table 1 and measured with a Peschl shear cell does not show the same consistency of data. In particular, the comparison for ϕ_w alumina powders indicates a certain overestimation by the msFBR technique but a similar classification of materials. The discrepancy between the results obtained with the two techniques is much larger for ballotini powder. However, it has to be recalled here that wall shear stress measurement may bear large differences between the measurement conditions and the process operating conditions. In fact, these differences are still a subject of technical debate.
In general it can be concluded that these very preliminary results of the application of the procedure developed to derive powder flow properties from msFBR measurements are promising and encourage further studies.

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