

Robust Control Theory for Time Lag System in Chemical Engineering and Its Application

Mingli Zhou

Beihai Vocational College, Beihai 536000, China
 melix2008@126.com

A series of problems of the time-lag system including robust stability analysis and guaranteed cost controller design are studied mainly based on the Lyapunov stability theory by linear matrix inequality method and time lag partitioning method in this paper. The time lag partitioning method is used to divide the whole time lag interval into N interval parts based on a new Lyapunov-Krasovskii functional for a class of uncertain time lag system in this chapter, besides, some free-weighting matrices are introduced to study time lag-dependent stability, for which sufficient conditions for time lag-dependent stability based on the linear matrix inequality, less conservative compared with previous results, are obtained. After that, by using the Lyapunov stability theorem & linear matrix inequality and combining with the time lag partitioning method to discuss analysis and design of robust H_∞ controller for a class of the Lurie time-lag with existential state and control input at the same time, conditions for asymptotic stability based on linear matrix inequality and with H_∞ performance are obtained and a design method for H_∞ control law with systematic memory state feedback is given. Finally, a simulation example is given to illustrate effectiveness of the proposed method.

1. Introduction

People have long noticed the time lag of biological systems, and later it is found that time lag is often seen in fields of optics, circuits, neural networks, architectural structures, biological environments, and medicine (Ampatzis et al., 2017). In fact, time lag always exists in the actual dynamical system except in ideal cases (Boukal et al., 2017). There are many reasons for the time lag, such as slow chemical reaction process, measurement of system variables, belt transmission and so on which will produce time lags. Besides, the time lag is an important cause of the system instability (Chen et al., 2017). Because of its wide application background, research on the time lag system is widely concerned by many scholars (Chen et al., 2017). Stability analyses of the time lag system and controller design have always been the hot and difficult points in the research of control theory with research methods divided into two categories—time domain and frequency domain (Darivianakis et al., 2017). Background, content and methods researched in this paper are systematically described in this chapter with focuses as basic characteristics of time lag system & robust control and research status (Du et al., 2017). The existence of time lag brings great difficulties to the system stability analysis and controller design (Feng and Wu, 2017). At the same time, the controlled objects are often affected by various uncertain factors such as: interference signals acted on controlled processes, noise measured by sensors (Filho et al., 2017). Robust control is to ensure both the system stability and influence from interference on system performance suppressed to a certain level, i.e. the controlled objects are robust with regard to interference. Therefore, the study of robust control for the time lag system is of great theoretical significance and application value (Hosseinzadeh and Yazdanpanah, 2017).

The time lag phenomenon widely exists in various biological systems and engineering systems, while time lag in the control system adds special difficulties regarding theory analysis and engineering application (Kerdphol et al., 2017). Generally, the time lag is the root cause of the system oscillation and instability, so it is of great theoretical and practical significance to study the time lag system (Kim, 2017).

2. Robust control theory

The most prominent representatives of the robust control theory is H control and μ method. control method in robust control theory is mainly used in this paper, so basis of H control theory is mainly introduced as follows (Lai and Kim, 2017).

H norm reflects the maximum ratio of the output signal to the input signal energy, and also reflects the maximum energy amplification of the system (Loiseau et al., 2017). The basic block diagram of the H standard problem is shown in Figure 1, where G and K are the generalized control object and the controller with the former as the given part of the system and the latter as designed part (Marantos et al., 2017). The signals in the Figure are all vector valued signals. Linear time invariant system in finite dimension and controllers are considered (Nie et al., 2017), where w is external input signal, including sensor noise, interference and reference (instruction) signal; z is the controlled output signal, usually including actuator output, control error and tracking error; u is control input signal; y output signal measured.

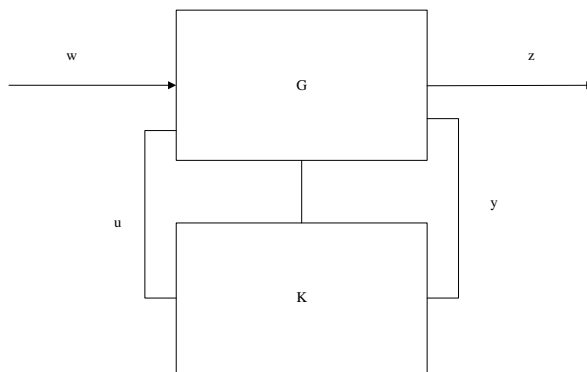


Figure 1: Basic block diagram of standard H control problem

In Figure 1, the actual controlled objects can not be determined as the generalized controlled objects. In case design objectives are different, even if the controlled objects are the same, their generalized controlled objects are also likely to be different. Therefore, two points of H control problem can be obtained:

H optimal control problem is: for the closed-loop control system shown in the figure, a really real rational controller is to be found to ensure internal stability of the closed-loop control system and to minimize H norm of T_{zw} norm in the closed-loop transfer function matrix (Pan et al., 2017).

H suboptimal control problem is: for the closed-loop control system shown in the figure, a really real rational controller K is to be found to ensure internal stability of the closed-loop control system and to minimize H_w norm (which should be less than the given constant $\gamma > 0$) of T_{zw} norm in the closed-loop transfer function matrix (Safa and Abdolmalaki, 2017).

3. Analysis of time lag system

3.1 Stability analysis of the uncertain time lag system based on time lag partitioning method

Uncertainty and time lag, which exist widely in a variety of practical control systems, are often main causes of system instability or performance degradation (Salimifard and Talebi, 2017). In recent decades, research on stability of time lag systems has been very active. In the obtained stability conditions, the time lag dependent stability conditions are less conservative than the time lag independent stability conditions (Sariyildiz et al., 2017). Many scholars have made unremitting efforts to reduce the conservativeness of the obtained results, and obtained a large number of time lag dependent stability criteria for small conservation. In this section, for a class of uncertain time lag system, the time lag interval is divided into N interval parts, and some free weighting matrices are introduced to study the time lag dependent stability problem (Shahbazi et al., 2017). The sufficient conditions for time lag dependent stability based on linear matrix inequalities are obtained (Tajeddini et al., 2017), which are less conservative compared with the previous results. The following class of uncertain linear time lag system is considered:

$$\begin{aligned} \dot{x}(t) &= [A + \Delta A(t)]x(t) + [A + \Delta A_1(t)]x(t - \tau), \\ x(t) &= \varphi(t), t \in [\tau_m, 0] \end{aligned} \quad (1)$$

Where, $x(t) \in R^n$ is the state vector of the system; the matrix $A, A_1 \in R^{n \times n}$ is the dimension constant matrix; $\varphi(t) \in c[-\tau_m, 0]$ is the known continuous initial vector valued function; the constant $0 < \tau < \tau_m$ is state time lag; τ_m is the known constant; $\Delta A, \Delta A_1$ is the uncertain parameter matrix with time-varying characteristics. As shown in Table 1, the maximum upper bound of allowable time lag can be obtained by Equation (1).

Table 1: Maximum asymptotic delay bounds for systems asymptotically stable

Method	Maximum delay bounds
$\tau_1 = \frac{1}{3} \tau$	0.9851
$\tau_1 = \frac{1}{4} \tau, \tau_2 = \frac{3}{4} \tau$	1.0128
$\tau_1 = \frac{1}{2} \tau, \tau_2 = \frac{3}{4} \tau$	1.0162

3.2 Absolute stability of the Lurie time lag system based on time lag partitioning method

Uncertainty and time lag are often the main reasons leading to instability or performance degradation of control systems, so study on absolute stability of the Lurie time lag system has also been concerned by many scholars (Vayeghan and Davari, 2017). Due to the low conservativeness of the time lag dependent conditions, more attention has been paid to research on the time lag dependent stability of the Lurie time lag system. By using the Lyapunov stability theorem & linear matrix inequality and combining with the time lag partitioning method to discuss absolute stability for a class of the Lurie time lag control system (Vromen and Dai, et al, 2017) in this chapter, the absolute stability condition based on linear matrix inequality is obtained. Finally, a simulation example is given to illustrate effectiveness of the proposed method. The following class of Lurie time lag control system is considered:

$$\begin{aligned}
 x(t) &= (A + \Delta A)x(t) + (A_d + \Delta A_d) x(t - \tau) + (D + \Delta D)\omega_1(t), \\
 z(t) &= M_x(t) + C_x(t - \tau) \\
 \omega_1(t) &= \omega_1(t, z(t)) \\
 x(t) &= \varphi(t), t \in [-\tau_m, 0]
 \end{aligned}
 \tag{2}$$

Where, $x(t) \in R^n$ is the system state; A, A_d, D, M, C is the dimension constant matrix; the constant $0 < \tau < \tau_m$ is state time lag, τ_m is the known constant; initial condition $\varphi(t)$ is the initial continuous vector function on $[-\tau_m, 0]$; $\Delta A(t), \Delta A_d(t), \Delta D [-\tau]$ is uncertain parameter matrix with time-varying characteristics. The maximum allowable time lag to ensure absolute stability of the system is listed in Table 2, while the maximum allowable time lag to ensure absolute stability of the system is listed in the Table 3.

Table 2: Maximum asymptotic delay bounds for systems asymptotically stable

Method	Wu	Han	n=2	n=3
Maximum delay bounds (h_{max})	2.3405	3.1604	3.8638	4.0626

Table 3: Maximum asymptotic delay bounds for systems asymptotically stable

Method	Gao	n=2	n=3
Maximum delay bounds (h_{max})	8.0297×10^3	8.0297×10^3	1.0371×10^4

3.3 Guaranteed cost controller design for a class of nonlinear uncertain time lag systems

The nonlinear guaranteed cost control problem for a class of uncertain time lag systems is discussed in this section based on Lyapunov stability theory and linear matrix inequality tools (Wu and Deng, 2017). Different from the previous results, the controller is considered of nonlinear property, while the guaranteed cost controller for nonlinear state feedback obtained can guarantee both local asymptotic stability of the system and the guaranteed cost upper bounds satisfied by the system. Moreover, a simulation example is given to illustrate effectiveness of the proposed method (Xu, 2017).

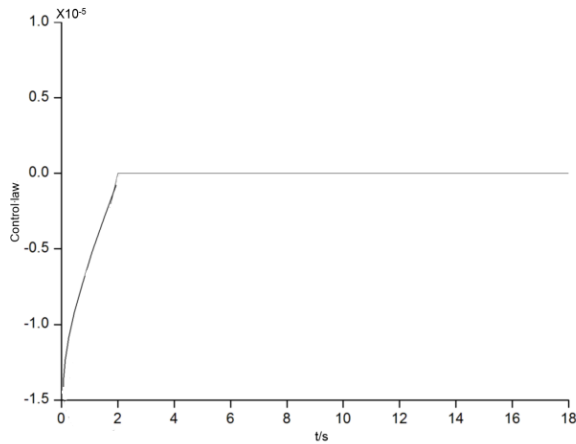


Figure 2: Control law curve

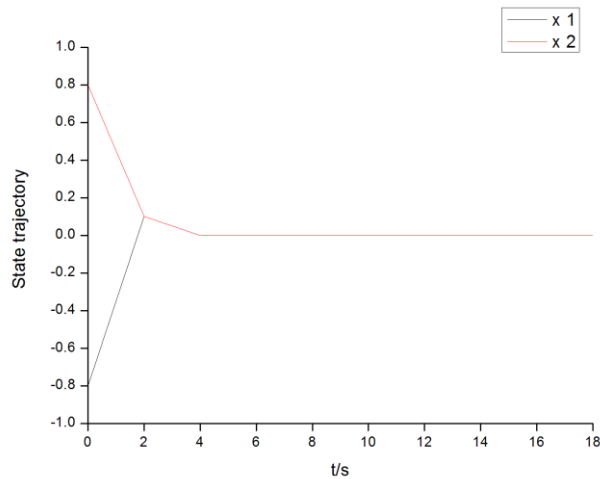


Figure 3: State trajectory of closed loop system

It is learnt from the control law for nonlinear guaranteed cost given in Figure 2 and the state trajectory curve corresponding to the closed-loop system under the control law given in Figure 3 that the state of the closed-loop system tends to zero under function of the nonlinear guaranteed cost controllers, from which that the system is asymptotically stable can be known.

The problem of nonlinear guaranteed cost control under the given performance index is studied in this chapter for a class of uncertain nonlinear time lag control system. The nonlinear controller presented in this paper not only can guarantee the asymptotic stability of the closed-loop system, but also can enable the corresponding closed-loop performance indicators to meet certain requirements of upper bound. Besides, the control function

considered includes the linear function in the usual sense, which has more scope of application than before. Therefore, research on control of such system has good practical significance.

4. Conclusions

A complete set of theoretical system which has become an important part of modern control theory is gradually formed for the research of time lag system. Two systems—time-lag linear system and time-lag nonlinear system are mainly considered in this paper to study the robust stability of a class of uncertain linear time-lag system, guaranteed cost controller design for a class of uncertain nonlinear time-lag system and robust stability & control problems a class of Lurie system. The main research results are summarized as follows: for a class of uncertain linear time-lag system, time lag dependent stability conditions for the system is given, and the time lag interval is divided into N interval parts with different energy functions defined in each sub-space to construct a new Lyapunov functional. Later on, time lag dependent conditions for uncertain linear time lag system is given with a form of linear matrix inequality. For a class of Lurie time-lag systems, robust stability problems are discussed, and Lyapunov stability theory, linear matrix inequality and other tools are used, moreover, the time lag partitioning is used to divide the time lag interval into N interval parts with different energy functions defined in each sub-space to obtain the absolute stability condition based on linear matrix inequality. What makes it different from the previous research is that the controller studied is nonlinear, that the guaranteed cost controller for time lag state feedback in the time lag system obtained with LMI toolbox is able to ensure both local asymptotical stability of the system and guaranteed cost upper bound satisfied by the system.

Reference

- Ampatzis M., Nguyen P.H., Kamphuis I.G., 2017, Robust optimisation for deciding on real-time flexibility of storage-integrated photovoltaic units controlled by intelligent software agents, *IET Renewable Power Generation*, 11, 1527-1533, DOI: 10.1049/iet-rpg.2016.0967
- Boukal Y., Darouach M., Zasadzinski M., Radhy N.E., 2017, Robust H_∞ Observer-Based Control of Fractional-Order Systems With Gain Parametrization, *IEEE Transactions on Automatic Control*, 62, 5710-5723, DOI: 10.1109/TAC.2017.2690140
- Chen X., Hu J., Wu M., Cao W., 2017, T --S Fuzzy Logic Based Modeling and Robust Control for Burning-Through Point in Sintering Process, *IEEE Transactions on Industrial Electronics*, 64, 9378-9388, DOI: 10.1109/TIE.2017.2708004
- Chen C., Liu Z., Xie K., Liu Y., Zhang Y., Chen C.L.P., 2017, Adaptive Fuzzy Asymptotic Control of MIMO Systems With Unknown Input Coefficients Via a Robust Nussbaum Gain-Based Approach, *IEEE Transactions on Fuzzy Systems*, 25, 1252-1263, DOI: 10.1109/TFUZZ.2016.2604848
- Darivianakis G., Georghiou A., Smith R.S., Lygeros J., 2017, The Power of Diversity: Data-Driven Robust Predictive Control for Energy-Efficient Buildings and Districts, *IEEE Transactions on Control Systems Technology*, 1-14, DOI: 10.1109/TCST.2017.2765625
- Du J., Hu X., Sun Y., 2017, Adaptive Robust Nonlinear Control Design for Course Tracking of Ships Subject to External Disturbances and Input Saturation, and *Cybernetics: Systems IEEE Transactions on Systems, Man, 1-10*, DOI: 10.1109/TSMC.2017.2761805
- Feng S., Wu H.N., 2017, Hybrid Robust Boundary and Fuzzy Control for Disturbance Attenuation of Nonlinear Coupled ODE-Beam Systems With Application to a Flexible Spacecraft, *IEEE Transactions on Fuzzy Systems*, 25, 1293-1305, DOI: 10.1109/TFUZZ.2016.2612264
- Filho C.M., Terra M.H., Wolf D.F., 2017, Safe Optimization of Highway Traffic With Robust Model Predictive Control-Based Cooperative Adaptive Cruise Control, *IEEE Transactions on Intelligent Transportation Systems*, 18, 3193-3203, DOI: 10.1109/TITS.2017.2679098
- Hosseinzadeh M., Yazdanpanah M.J., 2017, Robust adaptive passivity-based control of open-loop unstable affine non-linear systems subject to actuator saturation, *IET Control Theory Applications*, 11, 2731-2742, DOI: 10.1049/iet-cta.2017.0459
- Kerdphol T., Rahman F.S., Mitani Y., Watanabe M., Küfeoğlu S., 2017, Robust Virtual Inertia Control of an Islanded Microgrid Considering High Penetration of Renewable Energy, *IEEE Access*, 1, DOI: 10.1109/ACCESS.2017.2773486
- Kim C.H., 2017, Robust Control of Magnetic Levitation Systems Considering Disturbance Force by LSM Propulsion Systems, *IEEE Transactions on Magnetics*, 53, 1-5, DOI: 10.1109/TMAG.2017.2728810
- Lai N.B., Kim K.H., 2017, Robust Control Scheme for Three-phase Grid-connected Inverters with LCL-filter under Unbalanced and Distorted Grid Conditions, *IEEE Transactions on Energy Conversion*, 1, DOI: 10.1109/TEC.2017.2757042

- Loiseau P., Chevrel P., Yagoubi M., Duffal J.M., 2017, Investigating Achievable Performances for Robust Broadband Active Noise Control in an Enclosure, *IEEE Transactions on Control Systems Technology*, 1-8, DOI: 10.1109/TCST.2017.2769020
- Marantos P., Bechlioulis C.P., Kyriakopoulos K.J., 2017, Robust Trajectory Tracking Control for Small-Scale Unmanned Helicopters With Model Uncertainties, *IEEE Transactions on Control Systems Technology*, 25, 2010-2021, DOI: 10.1109/TCST.2016.2642160
- Nie H., Sang H., Li P., 2017, Robust H_{∞} control for a class of non-linear discrete time-delay systems with controller failure: a probability-dependent method, *IET Control Theory Applications*, 11, 3211-3220, DOI: 10.1049/iet-cta.2017.0703
- Pan S., Zhao X., Liang Y.C., 2017, Robust Power Allocation for OFDM-Based Cognitive Radio Networks: A Switched Affine Based Control Approach, *IEEE Access*, 5, 18778-18792, DOI: 10.1109/ACCESS.2017.2751565
- Safa A., Abdolmalaki R.Y., 2017, Robust Output Feedback Tracking Control for Inertially Stabilized Platforms With Matched and Unmatched Uncertainties, *IEEE Transactions on Control Systems Technology*, 1-14, DOI: 10.1109/TCST.2017.2761324
- Salimifard M., Talebi H.A., 2017, Robust output feedback fault-tolerant control of non-linear multi-agent systems based on wavelet neural networks, *IET Control Theory Applications*, 11, 3004-3015, DOI: 10.1049/iet-cta.2016.1645
- Sariyildiz E., Chen G., Yu H., 2017, Robust Trajectory Tracking Control of Multimass Resonant Systems in State Space, *IEEE Transactions on Industrial Electronics*, 64, 9366-9377, DOI: 10.1109/TIE.2017.2708001
- Shahbazi B., Malekzadeh M., Koofgar H.R., 2017, Robust Constrained Attitude Control of Spacecraft Formation Flying in the Presence of Disturbances, *IEEE Transactions on Aerospace and Electronic Systems*, 53, 2534-2543, DOI: 10.1109/TAES.2017.2704160
- Tajeddini M.A., Kebriaei H., Glielmo L., 2017, Robust decentralised mean field control in leader following multi-agent systems, *IET Control Theory Applications*, 11, 2707-2715, DOI: 10.1049/iet-cta.2016.1516
- Vayeghan M.M., Davari S.A., 2017, Torque ripple reduction of DFIG by a new and robust predictive torque control method, *IET Renewable Power Generation*, 11, 1345-1352, DOI: 10.1049/iet-rpg.2016.0695
- Vromen T., Dai C.H., van de Wouw N., Oomen T., Astrid P., Doris A., Nijmeijer H., 2017, Mitigation of Torsional Vibrations in Drilling Systems: A Robust Control Approach, *IEEE Transactions on Control Systems Technology*, 1-17, DOI: 10.1109/TCST.2017.2762645
- Wu Y., Deng M., 2017, Experimental Study on Robust Nonlinear Forced Vibration Control for an L-Shaped Arm with Reduced Control Inputs, *IEEE/ASME Transactions on Mechatronics*, 22, 2186-2195, DOI: 10.1109/TMECH.2017.2734696
- Xu Y., 2017, Robust Finite-Time Control for Autonomous Operation of an Inverter-Based Microgrid, *IEEE Transactions on Industrial Informatics*, 13, 2717-2725, DOI: 10.1109/TII.2017.2693233