

Switch Approach of Control Zone and Optimization Zone for Economic Model Predictive Control

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The coordination of control and optimization is proposed for economic model predictive control in the paper. The characteristic is explained briefly with the field theory. The solution of economic model predictive control is gradient descent method. However, economic constraint condition makes the response too long. In order to improve the performance and save respond time, control and optimization should be considered separately in controller. The method is built for coordinating control and optimization, which separate the whole zone into control zone and optimization zone and switch in different zone. The respond time is reduced in control zone, and economic benefit is improved in optimization zone. Rapid response and economic optimization are obtained. The test and verification are made by the case study of Shell Standard Control Problem.

1. Introduction

The algorithm of predictive control is usually expressed by a quadratic programming with a lot of constraint condition that presented by Kothare et al. (1996) and then Abou-Jeyab et al., (2001).. The solution of model predictive control (MPC) is a typical quadratic programming problem (Kuntze et al., 1986). MPC is applied widely to the process with strong interactions and large dead times. In detail, the objective of MPC controllers is achieving the following of reference trajectory. Because long response time will influences economic benefits, a new control strategy had been proposed, which is called economic model predictive control (EMPC) (Ellis et al., 2014). The same constraint conditions are set by EMPC and MPC (Ricker, 1985), but change the cost function (Muller et al., 2014). Economic benefits are considered for cost function of EMPC. The economic benefits are determined by product output and energy consumption. The objective of EMPC is minimizing cost function, which leads to instability of process, so terminal constraint is adopted. However, the coordination of terminal constraint and economic indicator is too hard. General method is tentative searching by offline simulation, which brings some difficult problems on industrial application (Qin et al., 2003).

In this paper, the method is built for coordinating control and optimization, which separate the whole zone into control zone and optimization zone and switch in different zone. The feasibility verification of the method is organized as follows: In section 2, the principle of EMPC is discussed. Further, a detailed description of the method is given in section 3. In section 4, the method is applied to a case of Shell Standard Control Problem.

2. Economic model predictive control

2.1 Typical EMPC

In general, the chemical process is described as difference equations.

$$x(k+1) = f(x(k), u(k)) \quad (1)$$

$$y(k) = g(x(k)) \quad (2)$$

For EMPC, P is prediction horizon, use predictive state sequence $\mathbf{x}(k) = \{x(k|k), x(k+1|k), \dots, x(k+P-1|k)\}$ and control sequence $\mathbf{u}(k) = \{u(k|k), u(k+1|k), \dots, u(k+P-1|k)\}$ to define the cost function $L(\bullet)$.

$$L(\mathbf{x}, \mathbf{u}, k) = \sum_{i=k}^{k+P-1} l(x(i|k), u(i|k)) \quad (3)$$

$$l(x(i|k), u(i|k)) = x^\perp(i|k)R(i)x(i|k) + \Delta u^\perp(i|k)Q(i)\Delta u(i|k) \quad (4)$$

And the $l(\bullet)$ in Eq(3) often define as quadric form like Eq(4), the R and Q are economic cost coefficient. In order to get optimal control sequence, the implicit expression is presented as follow.

$$\mathbf{u} = \arg \min \{L(\mathbf{x}, \mathbf{u}, k)\} \quad (5)$$

$$\text{s.t. } x(k+1+i|k) = f(x(k+i|k), u(k+i|k)) \quad (6)$$

$$x(k+i|k) \in \mathbf{X} \quad (7)$$

$$u(k+i|k) \in \mathbf{U} \quad (8)$$

$$x(k|k) = x(k) \quad (9)$$

$$x(k+P-1|k) \in \mathbf{\Omega} \quad (10)$$

$$i = 0, 1, \dots, P-2 \quad (11)$$

The optimal control sequence $\mathbf{u}^*(k)$ is obtained by solving the Eqs(5) - (11). Then the process is manipulated by the first element in the sequence. The cost function Eq(3) consists of energy consumption coefficient and economic benefit coefficient. In fact, the Eq(5) not includes the information about set point, so the process faces with the risk of instability. Eq(10) is terminal constraint that can drive a state close to the set point by step. Eq(6) is a state update equation for iterating. The x and u must satisfy relevant constraint, shown as Eqs(7) - (8). The initial state is the value of state observer at current time like Eq(9).

2.2 Disadvantage of EMPC

EMPC can ensure the economic benefits all the time, but it has some problems about control performance. The expression of the cost function is quadratic form like Eq(4). Because the effect of cost function is stronger than terminal constraint, the optimal control sequence $\mathbf{u}^*(k)$ is determined by economic constraint. However, economic optimization action should start at an appropriate neighbourhood of set point. Global economic optimization will sacrifice the control performance when process states are not nearby the set point. In other word, economic optimization should not be considered in controller until the process state moving into an appropriate zone.

The control objective in some petro-chemical and process industries is not a point but a neighbourhood of set point. The economic optimization should be considered once the process state moving into this zone in the controller. Besides, the state should be ensured moving into this zone quickly if current state has been out of the zone. This problem is shown in Figure 1.

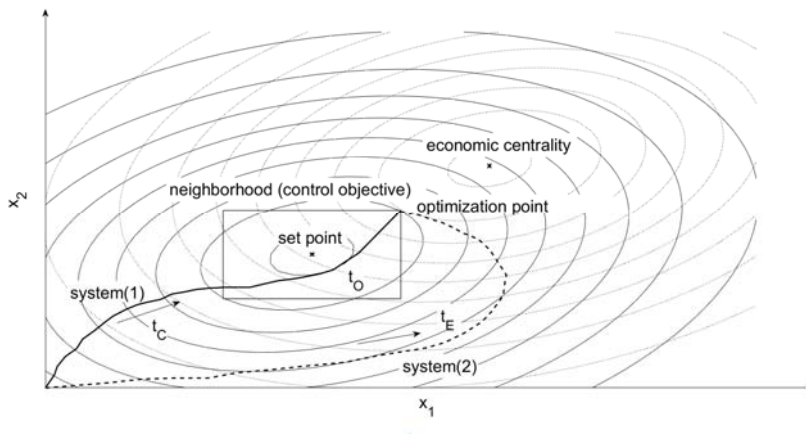


Figure 1: The example for disadvantage of EMPC.

In Figure 1, the whole zone has two central points. One of them is economic centrality, and for explaining clearly, the economic equipotential lines are represented as dotted line. The quadratic programming problem

of EMPC is finding a system trajectory that can get minimal economic potential. Similarly, just consider the control, the equipotential lines is represented as solid line. In practice, the control objective is a neighborhood of set point. The neighborhood, called optimization zone, is defined by $\Gamma(\delta)$ in Eq(12). And the remainder is concave called control zone. From theory of convex optimization, the optimization point is on the boundary of convex.

$$\Gamma(\delta) = \{x_i \mid x_i \in [x_{\text{ref}} - \delta, x_{\text{ref}} + \delta], i = 1, 2, \dots, m\} \quad (12)$$

Generally, when changing the set point, the state should move to the control zone quickly. If process state moves into optimization zone, economic optimization action is applied to improve economic benefit. The state trajectory of system(1) has two stages, firstly it moves to the optimization zone from control zone, and then it starts optimization action to arrive optimization point. The system(2) uses EMPC all the time, but the state trajectory only has one stage: optimization. The system(2) can also arrive optimization point, but the optimizing trajectory is unreasonable because system is not arrive control objective until the system reaches a steady state. In addition, the system takes long time to along with the unsatisfactory trajectory.

Table 1: Time summary

System	Control time	Optimization time	Total time
system(1): proposed	t_c	t_o	$t_c + t_o$
system(2): EMPC	t_E	t_E	t_E

The each part time is shown in Table 1 and Figure 1. The system(1) have two stages, t_c is control time means the time spent in control zone. And t_o , named as optimization time, means the time spent in optimization zone until process arrives optimization point. The EMPC controller is used in the system(2), and that it only has one stage. Control and optimization are considered at the same time in the EMPC strategy, so control time, optimization time, and total time is just the same as t_E .

3. Switch approach of EMPC

3.1 Switch method

In order to improve the performance of controller and make optimizing trajectory more reasonable, this paper proposes a switch method to discuss this problem. In the control zone, MPC controller with small prediction horizon is selected. In the optimization zone, which is related to parameter δ ($\delta \geq 0$), the state will drive by another MPC controller with large prediction horizon and an optimization layer (OPT). The cost function of MPC is Eq(13).

$$J(\mathbf{x}, \mathbf{u}, k) = \sum_{i=k}^{k+P-1} \{ \|x(i|k) - x_{\text{sp}}\|_{\mathbb{R}}^2 + \|\Delta u(i|k)\|_{\mathbb{Q}}^2 \} \quad (13)$$

$$\text{if } (x(k+i|k) \in \Gamma(\delta)) \text{ then } u(k+i|k) = 0 \quad (14)$$

$$\text{if } (x(k+i|k) \notin \Gamma(\delta)) \text{ then } u(k+i|k) = 0 \quad (15)$$

Switch strategy includes two MPC controllers. The controller MPC(1) consist of Eq(13), Eqs(6)-(9), Eq(14), and Eq(11). Constraint condition Eq(14) can make sure MPC(1) compute the control sequence when system in control zone. Another controller MPC(2) consist by Eq(13), Eq(6)-(9), Eq(15), and Eq(11). Constraint condition Eq(15) can make sure MPC(2) compute the control sequence when system in optimization zone. The set point of MPC(1) is the central of convex $\Gamma(\delta)$, and small prediction horizon is used for enhancing the speed of respond. Optimization point is calculated by OPT, which is the input of MPC(2). Controller MPC(2) use a large prediction horizon because optimization is a long-term process. There are many methods to solve OPT, and optimization point y_{opt} is calculated by set point y_{sp} and expectation control law u_{opt} (Luo et al., 2014).

3.2 Directions for switch approach of EMPC

The system structure diagram is shown in Figure 2.

In Figure 2, these two controllers are active, and a high selector is used for switching control sequence automatically. The boundary of optimization zone is determined by the parameter δ .

In Table 2, it shows that the two controllers play different roles. MPC(1) is designed in the control zone, while MPC(2) is designed in the optimization zone.

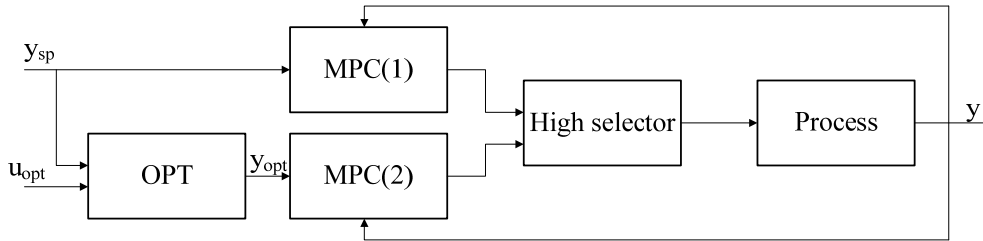


Figure 2: The system structure diagram on switch approach of EMPC.

Table 2: Optimal control sequence in different zone

Controller	Control zone	Optimization zone
MPC(1)	u (for set point)	0
MPC(2)	0	u (for optimization point)

4. Case study

In order to test and verify the feasibility of the method, Shell model is simulated and discussed. The model is shown in Figure 3 and Table 3.

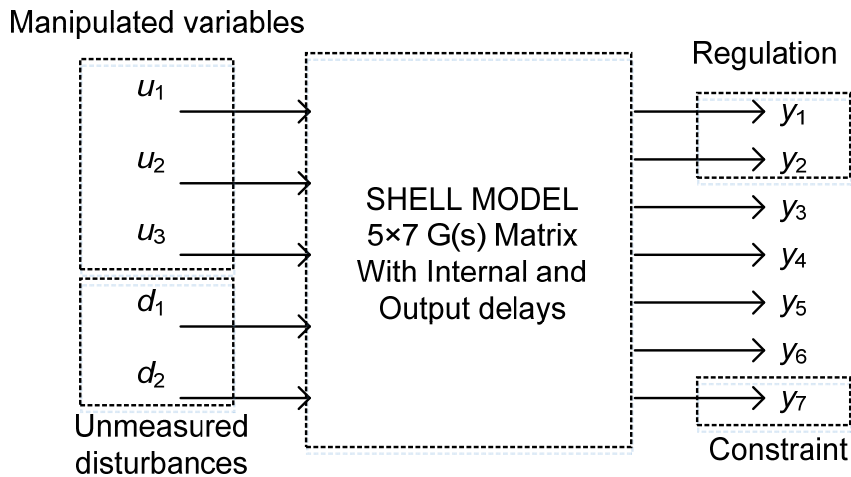


Figure 3: The Shell standard control problem.

Table 3: The description of variables in shell model

Manipulated variables	Description	Model output	Description
u_1	Top draw	y_1	Top end point
u_2	Side draw	y_2	Side end point
u_3	Bottoms reflux duty	y_3	Top temperature
Unmeasured disturbances	Description	y_4	Upper reflux temperature
d_1	Inter. Reflux duty	y_5	Side draw temperature
d_2	Upper reflux duty	y_6	Inter. reflux temperature
		y_7	Bottoms reflux temperature

The main purpose is control y_1 and y_2 to set point, and constraint of y_7 is satisfied at the same time. The system has strong interactions and large dead times, so the MPC control strategy is better than PID control. The essential parameters are given as follow: $\delta=0.05$, $y_1=0.3$, $y_2=0.3$, $R=1$, $Q=1$. In addition, prediction horizon of MPC(1) is 60, MPC(2) is 180, and EMPC is 120.

The result of simulation is as follow:

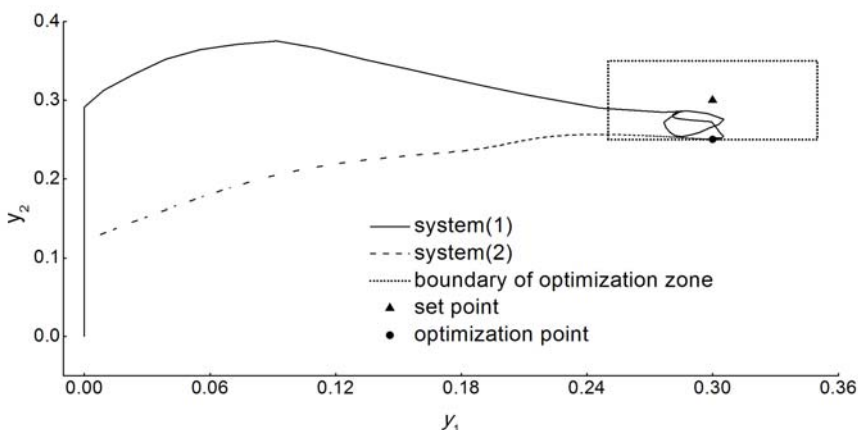


Figure 4: Simulation result: state trajectory (system(1) use switch strategy, system(2) use EMPC).

Table 3: Time summary

System	Control time	Optimization time	Total time
system(1): proposed	43 min	125 min	168 min
system(2): EMPC	218 min	218 min	218 min

Switch method is applied to system(1). MPC(1) save the running time in control zone, but the aggressiveness of MPC(1) leads to the overshoot in optimization zone, shown in Figure 4. The hysteretic aggressiveness is eliminated by the large prediction horizon of MPC(2). At the same time, this aggressiveness also assists the improvement of rapidity, shown in Table 4. System(2) uses EMPC, although system has strong robustness, it takes long time to arrive same cost value, shown in Figure 5.

For qualitative, the switch method shorten the transition time and reduce waste production caused by changing set point. For quantitative, transition time is decreased from 100 minutes to 43 minutes in SHELL model. In other word, the method saves more than half a waste production in transition process.

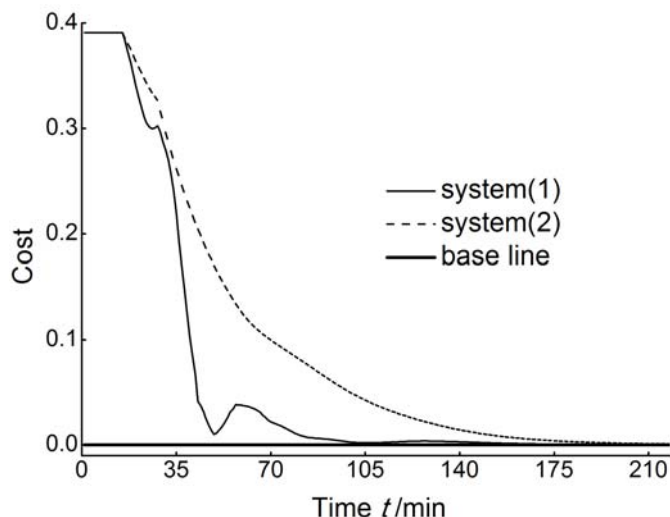


Figure 5: Simulation result: the value of cost function (system(1) use switch strategy, system(2) use EMPC).

5. Conclusions

In this paper, a switch method of EMPC is proposed to improve economic benefits. Typical EMPC consider the control and optimization simultaneously, and the respond time is increased by the interaction between the two tasks. This paper gives a brief overview of EMPC search principle, and the reason why EMPC have long respond time is found. The way to overcome this problem is to separate the whole zone into control zone and optimization zone, and different control method is applied in different zones. The result shows that the switch method has significant effect on enhancing economic benefit and saving response time.

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