

# Mathematical Modelling of Wildland Fires Initiation and Spread Using a Coupled Atmosphere-Forest Fire Setting

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The coupled atmosphere/crown fire behaviour model is based on conservation of mass, momentum, species and energy. The system of differential equations for crown canopy was integrated with respect to the vertical coordinate because horizontal sizes of forest are much greater than the heights of trees. As a result, it is used two systems of equations for boundary layer of atmosphere and crown canopy. These equations are solved numerically for turbulent flow with the use of diffusion equations for chemical components and equations of energy conservation for gaseous and condensed phases. The method of finite volume is used to obtain discrete analogies. The boundary-value problem is solved numerically using the method of splitting according to physical processes. It allows investigating dynamics of forest fire initiation and spreading under influence of various external conditions. In this context, a study - mathematical modelling - of the conditions of forest fire spreading that would make it possible to obtain a detailed picture of the change in the temperature and component concentration fields with time and allows determining the total CO and CO<sub>2</sub> emissions into the atmosphere during the spread of forest fire.

## 1. Introduction

The forest fires are very complicated phenomena. At present, fire services can forecast the danger rating of, or the specific weather elements relating to forest fire. There is need to understand and predict forest fire initiation, behaviour and spread. The aim of the present paper is to study the behaviour of crown forest fires propagating through crown canopy and to study the mutual influence of crown forest fires and boundary layer of atmosphere using numerical simulation with a physics-based model and improvement of knowledge on the fundamental physical mechanisms that control forest fire spread. A great deal of work has been done on the theoretical problem of how forest fire spread. Crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism. Crown fires are more difficult to control than surface. The one of the first accepted method for prediction of crown fires was given by Rothermel (1991) and these semi-empirical models allow obtaining a quite good data of the forest fire rate of spread as a function of fuel bulk and moisture, wind velocity and the terrain slope. But these models use data for particular cases and do not give results for general fire conditions. Also crown fires initiation and hazard have been studied and modelled in detail Albini (1985). Conditions for the start and spread of crown fire was studied by Van Wagner (1977). The discussion of the problems of modelling forest fire was provided by a group of co-workers at Tomsk University (Grishin, 1997).

The main results of these studies were presented by Grishin (1997) in his monograph. A mathematical model of forest fires obtained in this work is based on an analysis of known and original experimental data (Konev, 1977), and using concepts and methods from reactive media mechanics. The physical two-phase models used in (Morvan et al., 2004) may be considered as a development and extension of the formulation proposed by Grishin (1997) and continuation of numerical modeling of wildfires initiation (Grishin and Perminov, 1998). Currently, experimental research on the distribution of grass-roots and crown forest fires has been continued by Cruz et al. (2002). However, the investigation of crown fires initiation has been limited mainly to cases without taking into account the interaction of crown forest fires with boundary layer of atmosphere. At present a large amount of research focused on the firebrand generation and transport (Song et al., 2017) at the same time there are many papers related to spread of forest fires because it is an important parameter used in the

evaluation of hazards for fire safety applications. In review (Golner et al., 2017) the problem of flame spread was revisited, with a particular emphasis on the effect of flow and geometry on concurrent flame spread over solid fuels. Despite the diversity of studies related to forest fires, there is currently no data on the dependence of the amount of combustion products emissions on forest characteristics and meteorological data. Typically, measurement data are used to estimate the volume of discarded combustion products, in particular carbon oxides. As a rule, the calculations of CO<sub>2</sub> release were based on the 2006 Intergovernmental Panel on Climate Change (IPCC) guidelines in the Agriculture, Forestry and Other Land Use (AFOLU) sector. For example, GIS was applied as a key tool for implementing the spatial inventory of Carbon Dioxide (CO<sub>2</sub>) emissions and removals (Miphokasap, 2017). At the same time, Mickler et al. (2017) developed a method and approach to estimate aboveground and belowground carbon emissions from a 2008 peatland wildfire by analyzing vegetation carbon losses from field surveys of biomass consumption from the fire and soil carbon losses. In another approach free-burning experimental fires were conducted in a wind tunnel to explore the role of ignition type and thus fire spread mode on the resulting emissions profile from combustion of fine Eucalyptus litter fuels (Surawski et al., 2015). However, these calculations can be used in estimating combustion product emissions for specific regions and in specific non-changing meteorological conditions. It seems more promising to use methods of mathematical modeling that will allow to take into account the dynamics of this process in space and time. In particular, in this paper, an attempt is made to estimate the amount of carbon dioxide and carbon monoxide emissions at crown forest fires spread.

## 2. Physical and mathematical setting

It is assumed that the forest during a forest fire can be modelled as 1) a multi-phase, multi-storeyed, spatially heterogeneous medium; 2) in the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium; 3) the forest canopy is supposed to be non - deformed medium (trunks, large branches, small twigs and needles), which affects only the magnitude of the force of resistance in the equation of conservation of momentum in the gas phase, i.e., the medium is assumed to be quasi-solid (almost non-deformable during wind gusts); 4) let there be a so-called "ventilated" forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products); 5) the flow has a developed turbulent nature and molecular transfer is neglected; 6) gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound. Let the initial point of the Cartesian system of coordinate is situated at the centre of the surface forest fire source at the height of the roughness level, axis  $Ox_1$  directed parallel to the Earth's surface to the right in the direction of the unperturbed wind speed, axis  $Ox_2$  directed perpendicular to  $Ox_1$  and directed upward. Problem formulated above reduces to the solution of systems of Eq(1) - Eq(8):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho v_j) = Q, \quad j = \overline{1,3}, \quad i = \overline{1,3}; \quad (1)$$

$$\rho \frac{dv_i}{dt} = - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (-\rho \overline{v'_i v'_j}) - \rho s c_d v_i |\vec{v}| - \rho g_i - Q v_i; \quad (2)$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j} (-\rho c_p v'_j \overline{T'}) + q_s R_s - \alpha_v (T - T_s) + k_g (c U_R - 4\sigma T^4); \quad (3)$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j} (-\rho v'_j c'_\alpha) + R_{s\alpha} - Q c_\alpha, \quad \alpha = 1, 2, 3; \quad (4)$$

$$\frac{\partial}{\partial x_j} \left( \frac{c}{3k} \frac{\partial U_R}{\partial x_j} \right) - k c U_R + 4k_s \sigma T_s^4 + 4k_g \sigma T^4 = 0; \quad (5)$$

$$k = k_g + k_s;$$

$$\sum_{i=1}^4 \rho_i c_{pi} \varphi_i \frac{\partial T_S}{\partial t} = q_3 R_3 - q_2 R_2 - k_S (c U_R - 4\sigma T_S^4) + \alpha_v (T - T_S); \quad (6)$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_c R_1 - \frac{M_c}{M_1} R_3, \rho_4 \frac{\partial \varphi_4}{\partial t} = 0; \quad (7)$$

$$\sum_{\alpha=1}^4 c_\alpha = 1, P_e = \rho R T \sum_{\alpha=1}^4 \frac{c_\alpha}{M_\alpha}, \vec{g} = (0, 0, -g). \quad (8)$$

The system of Eq(1) – Eq(8) must be solved taking into account the initial and boundary conditions:

$$t = 0 : v_1 = 0, v_2 = 0, v_3 = 0, T = T_e, c_\alpha = c_{\alpha e}, T_S = T_{se}, \varphi_i = \varphi_{ie}; \quad (9)$$

$$x_1 = 0 : v_1 = V, v_2 = 0, v_3 = 0, T = T_e, c_\alpha = c_{\alpha e}, -\frac{c}{3k} \frac{\partial U_R}{\partial x_1} + \frac{c}{2} U_R = 0; \quad (10)$$

$$x_1 = x_{1e} : \frac{\partial v_1}{\partial x_1} = 0, \frac{\partial v_2}{\partial x_1} = 0, \frac{\partial v_3}{\partial x_1} = 0, \frac{\partial T}{\partial x_1} = 0, \frac{\partial c_\alpha}{\partial x_1} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_1} + \frac{c}{2} U_R = 0; \quad (11)$$

$$x_2 = -x_{2e} : \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, -\frac{c}{3k} \frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0; \quad (12)$$

$$x_2 = x_{2e} : \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0; \quad (13)$$

$$x_3 = 0 : v_1 = 0, v_2 = 0, \rho v_3 = \rho_0 \omega_0, T = T_0, |x_1| \leq x_0, |x_2| \leq x_0, \rho v_3 = 0, T = T_e, |x_1| > x_0, |x_2| > x_0, -\frac{c}{3k} \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0; \quad (14)$$

$$x_3 = x_{3e} : \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial T}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \frac{c}{3k} \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0. \quad (15)$$

Here and above  $\frac{d}{dt}$  is the symbol of the total (substantial) derivative;  $\alpha_v$  is the coefficient of phase exchange;

$$x_3 = 0 : v_1 = 0, v_2 = 0, \rho v_3 = \rho_0 \omega_0, T = T_0, |x_1| \leq x_0, |x_2| \leq x_0, \rho v_3 = 0, T = T_e, |x_1| > x_0, |x_2| > x_0, -\frac{c}{3k} \frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0,$$

$\rho$  - density of gas – dispersed phase,  $t$  is time;  $v_i$  - the velocity components;  $T$ ,  $T_S$  - temperatures of gas and solid phases,  $U_R$  - density of radiation energy,  $k$  - coefficient of radiation attenuation,  $P$  - pressure;  $c_p$  – constant pressure specific heat of the gas phase,  $c_{pi}$ ,  $\rho_i$ ,  $\varphi_i$  – specific heat, density and volume of fraction of condensed phase (1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel),  $R_i$  – the mass rates of chemical reactions,  $q_i$  – thermal effects of chemical reactions;  $k_g$ ,  $k_S$  - radiation absorption coefficients for gas and condensed phases;  $T_e$  – the ambient temperature;  $c_\alpha$  - mass concentrations of  $\alpha$  - component of gas – dispersed medium, index  $\alpha=1,2,3,4$ , where 1 corresponds to the density of oxygen, 2 - to carbon monoxide CO, 3 - to carbon dioxide and 4 - inert components of air;  $R$  – universal gas constant;  $M_\alpha$ ,  $M_c$ , and  $M$  molecular mass of  $\alpha$  -components of the gas phase, carbon and air mixture;  $g$  is the gravity acceleration;  $c_d$  is an empirical coefficient of the resistance of the vegetation,  $s$  is the specific surface of the forest fuel in the given forest stratum. To define source terms which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase, the following formulae were used for the rate of formulation of the gas-dispersed mixture  $Q$ , outflow of oxygen  $R_{51}$ , changing carbon monoxide  $R_{52}$  and carbon dioxide  $R_{53}$ :

$$Q = (1 - \alpha_c) R_1 + R_2 + \frac{M_c}{M_1} R_3, R_{51} = -R_3 - \frac{M_1}{2M_2} R_5, R_{52} = v_g (1 - \alpha_c) R_1 - R_5, R_{53} = R_3 + \frac{M_1}{2M_2} R_5, \quad (16)$$

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s}\right); \quad (17)$$

$$R_2 = k_2 \rho_2 \varphi_2 T_s^{-0.5} \exp\left(-\frac{E_2}{RT_s}\right); \quad (18)$$

$$R_3 = k_3 \rho \varphi_3 s_\sigma c_1 \exp\left(-\frac{E_3}{RT_s}\right); \quad (19)$$

$$R_5 = k_5 M_2 \left(\frac{c_1 M}{M_1}\right)^{0.25} \frac{c_2 M}{M_2} T^{-2.25} \exp\left(-\frac{E_5}{RT}\right). \quad (20)$$

The initial values for volume of fractions of condensed phases are determined using the expressions:

$$\varphi_{1e} = \frac{d(1-v_z)}{\rho_1}, \varphi_{2e} = \frac{Wd}{\rho_2}, \varphi_{3e} = \frac{\alpha_c \varphi_{1e} \rho_1}{\rho_3} \quad (21)$$

where  $d$  – bulk density for surface layer,  $v_z$  – coefficient of ashes of forest fuel,  $W$  – forest fuel moisture content. It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is “grey”), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires. To close the system Eq(1)– Eq(8), the components of the tensor of turbulent stresses, and the turbulent heat and mass fluxes are determined using the local-equilibrium model of turbulence (Grishin, 1997). It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy and the space above it, while the appropriate components of the data base are used to calculate the specific properties of the various forest strata and the near-ground layer of atmosphere. This approach substantially simplifies the technology of solving problems of predicting the state of the medium in the fire zone numerically.

The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different type of forest (Grishin, 1997). Because of the horizontal sizes of forest massif more than height of forest, the system of Eq(1) - Eq(15) was integrated between the limits from height of the roughness level - 0 to  $h$  and the problem formulated above is reduced to a solution of two system of equations: 1). for crown and 2). for boundary layer of atmosphere above the forest. It is assumed that heat and mass exchange of fire front and boundary layer of atmosphere are governed by Newton law (Grishin, 1997).

### 3. Numerical solution and results

The boundary-value problem is solved numerically. System of Eq (1) - (7) with the appropriate initial and boundary conditions Eq (8) - (14) for numerical integration is reduced to the discrete form using control volume method (Patankar, 1981). The system of algebraic equations arising in the process of discretization, resolved by using the *SIP* method (Perminov, 1995). The problem associated with the non-linearity in the equation set and the pressure – velocity linkage resolved by adopting an iterative solution strategy such as SIMPLE like algorithm (Patankar, 1981). In order to efficiently solve this problem in a reactive flow the method of splitting according to physical processes was used. The basic idea of this method is based on the information that the physical timescale of the processes is greater than chemical. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. Then the system of ordinary differential equations of chemical kinetics obtained as a result of splitting was then integrated. The time step for integrating each function has to be smaller than the characteristic time of physical process to ensure the convergence of the numerical method. The time step was selected automatically. The accuracy of the program was checked by the method of inserted analytical solutions. Analytical expressions for the unknown functions were substituted in the system of equations and the closure of the equations were calculated. This was then treated as the source in each equation. Next, with the aid of the algorithm described above, the values of the functions used were inferred with an accuracy of not less than 1%. The effect of the dimensions of the control volumes on the solution was studied by diminishing them. Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically. The first stage is related to increasing maximum temperature in the place of ignition with the result that a crown fire source appears. At this process stage over

the fire source a thermal wind is formed a zone of heated forest fire pyrolysis products which are mixed with air, float up and penetrate into the crowns of trees. As a result, forest fuels in the tree crowns are heated, moisture evaporates and gaseous and dispersed pyrolysis products are generated. Ignition of gaseous pyrolysis products of the crown occurs at the next stage, and that of gaseous pyrolysis products in the forest canopy occurs at the last stage. At the moment of ignition, the gas combustible products of pyrolysis burn away, and the concentration of oxygen is rapidly reduced. The isotherms of gas phase components moved in the forest canopy by the action of wind. It is concluded that the forest fire begins to spread. The results of the calculation give an opportunity to consider forest fire spread for different wind velocity, canopy bulk densities and moisture forest fuel. Figures 1a and 1b present the distribution of temperature of gas phase in the crown  $\bar{T}$  ( $\bar{T} = T/T_e, T_e = 300 K$ ) (1- 4, 2 - 3.5, 3 - 3, 4 - 2.6., 5 - 2., 6 - 1.5) for wind velocity  $V_e = 3$  m/s in different instants of time: I -  $t=8$  s, II -  $t=18$  s, III -  $t=28$  s (Figure 1a) and 5 m/s (Figure 1b) (I -  $t=8$  s, II -  $t=18$  s, III -  $t=28$  s). It should be noted that when the wind speed is increased from 3 to 5 m/s, the rate of spread of forest fire is also increased from 2 to 3 m/s. However, in this case the burnt out area decreases from 917 to 830 m<sup>2</sup>. As is known, when burning forest combustible materials in the boundary layer of the atmosphere, pyrolysis and combustion products are released. Figures 2a and Figure 2b show the dynamics of CO<sub>2</sub> and CO emission for the cases under consideration, that is, for wind speeds of 3 and 5 m/s.

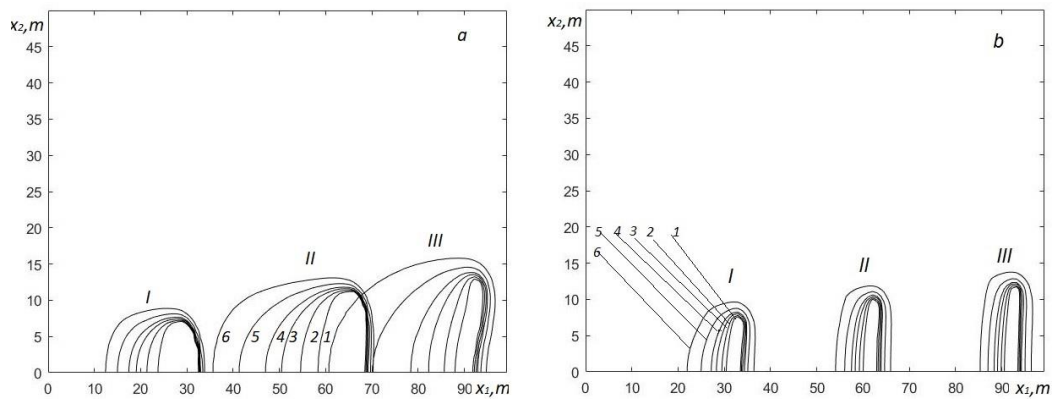


Figure 1: Field of isotherms of the forest fire spread (gas phase) for the wind speeds of (a) 3 m/s and (b) 5 m/s

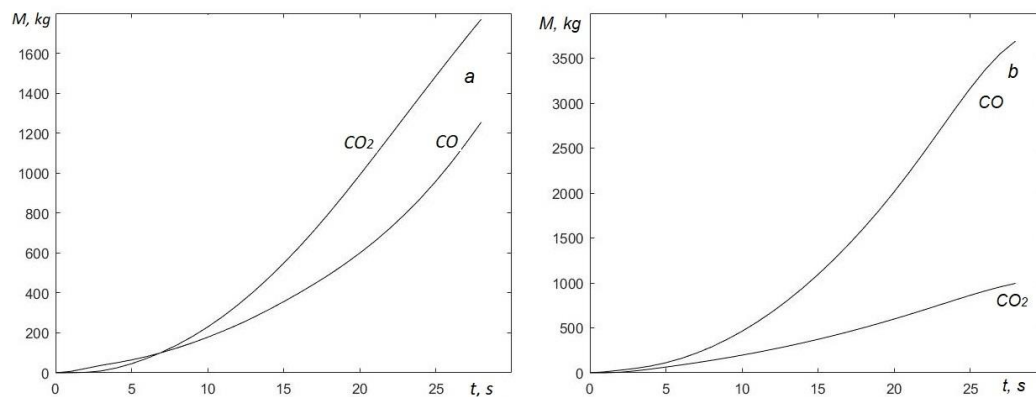


Figure 2: The dynamics of carbon dioxide and carbon monoxide emission for the wind speeds of (a) 3 m/s and (b) 5 m/s

It can be seen from the graphs that in the first case CO<sub>2</sub> emissions prevail, and with an increase in wind speed up to 5 m/s - CO. CO<sub>2</sub> is formed by the combustion of gaseous and condensed pyrolysis products, and CO is released together with pyrolysis products. Obviously, with increasing wind speed, some pyrolysis products do not have time to react and are carried out from the region of increased temperature. Figures 2a and Figure 2b show the total CO and CO<sub>2</sub> emissions to the atmosphere during the spread of forest fire over time.

#### 4. Conclusions

Mathematical model gives an opportunity to describe the different conditions of the crown forest fires spread taking account different weather conditions, state of forest combustible materials, which allows applying the given model for prediction and preventing fires. It overestimates the rate of crown forest fire spread that depends on crown properties: bulk density, moisture content of forest fuel, wind velocity and the influence of boundary layer of atmosphere. The model proposed here gives a detailed picture of the change in the temperature and component concentration ( $O_2$ ,  $CO_2$ ,  $CO$  and etc.) fields with time and determine as well as the influence of different conditions on the crown forest fire spreading for the different cases of inhomogeneous of distribution of sources of forest fires initiation. This mathematical model also allows to determine the total  $CO$  and  $CO_2$  emissions into the atmosphere during the spread of forest fire at different times. The results of calculation of the rate of crown forest fires are agreed with the laws of physics and experimental data (Grishin, 1997). The results of numerical calculations show that for different values of the parameters, the quantitative ratio of  $CO$  and  $CO_2$  released during a forest fire varies. This fact is also confirmed by the results of experimental studies (Surawski et al., 2015).

#### Acknowledgments

The paper was supported from RFBR (project code: № 16-41-700022 p\_a) and within the framework of Tomsk Polytechnic University Competitiveness Enhancement Program grant.

#### References

- Albini F.A., Reinhardt E.D., 1985, Modeling ignition and burning rate of large woody natural fuels, *International Journal of Wildland Fire*, 5, 81–91.
- Cruz M.G., Alexander M.E., Wakemoto R.H., 2002, Predicting crown fire behaviour to support forest fire management decision-making, *Proceedings of the IV International conference on forest fire research, Luso-Coimbra, Portugal*. (Ed. D. X. Viegas), 11.
- Golner M.J., Miller C.H., Wei Tanga, Singh A.V., 2017, The effect of flow and geometry on concurrent flame spread, *Fire Safety Journal*, 91, 68–78.
- Grishin A.M., 1997, *Mathematical Modeling Forest Fire and New Methods Fighting Them*, Publishing House of Tomsk University, Tomsk, Russian Federation.
- Grishin A.M., Perminov V.A., 1998, Mathematical modeling of the ignition of tree crowns, *Combustion, Explosion and Shock Waves*, 34, 378–386.
- Konev E.V., 1977, *The physical foundation of vegetative materials combustion*, Nauka, Novosibirsk, USSR.
- Mickler R.A., Welch D.P., Bailey A.D., 2017, Carbon emissions during wildland fire on a North American temperate peatland, *Fire Ecology*, 13, 34–57.
- Miphokasap P., 2017, Spatial inventory of  $CO_2$  emissions and removals from land use and land use changes in Thailand, *Chemical Engineering Transactions*, 56, 13–18.
- Morvan D., Dupuy J.L., 2004, Modelling the propagation of wildfire through a Mediterranean shrub using a multiphase formulation, *Combustion and Flame*, 138, 199–210.
- Patankar S.V., 1981, *Numerical Heat Transfer and Fluid Flow*, Hemisphere Publishing Corporation, New York, USA.
- Perminov V.A., 1995, *Mathematical modeling of crown and mass forest fires initiation with the allowance for the radiative - convective heat and mass transfer and two temperatures of medium*, PhD Thesis, Tomsk State University, Tomsk, Russian Federation.
- Rothermal R.C., 1991, Predicting behaviour and size of crown fires in the Northern Rocky Mountains. In: *Res.Pap. INT-438*. US Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station, 46p, Ogden, UT, United States, DOI: 10.2737/INT-RP-438.
- Song J., Huang X., Liu N., Li H., Zhang L., 2017, The wind effect on the transport and burning of firebrands, *Fire Technology*, 53, 1555-1568.
- Surawski N.S., Sullivan A.L., Meyer C.P., Roxburgh S.H., Polglase P.J., 2015, Greenhouse gas emissions from laboratory-scale fires in wildland fuels depend on fire spread mode and phase of combustion, *Atmospheric Chemistry and Physics*, 15, 5259–5273.
- Van Wagner C.E., 1977, Conditions for the start and spread of crown fire, *Canadian Journal of Forest Research*, 7, 23–34.