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Extending Process-Network Synthesis Algorithms with Time Bounds for Supply Network Design

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It is highly desirable to develop a software for estimating the quality of supply networks and synthesizing potential alternative realizations to improve the quality of service or to satisfy service level agreements. In the present paper, methodology, algorithm, and software are proposed to improve supply networks where the quality is measured by cost and response time.
The methodology is based on the combinatorial foundations of algorithmic process synthesis. Structure generation and evaluation of a supply scenario are the two major steps of the algorithm elaborated for synthesizing the optimal network structure. The algorithm, besides generating alternative structures, takes into account alternative orders of the activities as well. The structural alternatives are evaluated on the basis of the cost and duration of each individual operation. The cost and response time for a single operation are expressed as concave functions of its volume.

1. Introduction

It has been shown that the P-graph approach to process-network synthesis (PNS) originally conceived for conceptual design of chemical processes (Friedler *et* al. 1992, 1993, 1995, 1996, 1998) provides appropriate tools for generating and analyzing structural alternatives for supply scenarios (Barany *et* al. 2010; Klemeš et al. 2010; Lam *et* al. 2010). However, extension of the original framework to handle constraints specific to supply scenarios may improve the practical applicability of the proposed methodology. To satisfy the deadline is a crucial aspect in supply chain design. In the present paper, time constraints on the availability of the resources, duration of the activities, and deadlines for the final targets are incorporated into the mathematical model as well as into the solution algorithms of PNS.

2. P-graph Framework for Algorithmic Process-Network Synthesis and Optimization

The approach based on the P-graph framework appears to be the only one being capable of executing process-network optimization giving rise to an algorithmically and mathematically proven solution for all steps involved, comprising superstructure generation, construction of the mathematical model, optimization, and the solution interpretation. In the P-graph framework, algorithm MSG produces the maximal structure, i.e. the superstructure, for the PNS problem (Friedler et al. 1992). This maximal structure serves as the input to the generation and solution of the mathematical model by algorithm ABB (Friedler et al. 1996).

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3. Parametric PNS Problem with Fix Charged Linear Cost Functions

The combinatorial components of a PNS problem are given by a triplet (*P*,*R*,*O*) where there exists a set *M* for which $P \subseteq M$ is the set of final targets to be achieved, $R \subseteq M$ is the set of resources, and $O \subseteq \wp(M) \times \wp(M)$ is the set of candidate activities to form a network and reach each of the final targets by deploying any of the available resources. Each activity is defined by its preconditions and outcomes. A precondition can be the availability of a resource or an outcome of another activity. It is assumed that $P \cap R = \emptyset$. A parametric problem definition with a fixed charged linear cost function for the volume of the activities is detailed below (Barany *et* al. 2011).

The lower bound Lp_j on the gross result is greater than zero for each final target m_j, and it is equal to zero for any other entity: $Lp_j = \begin{cases} > 0, \forall m_j \in P \\ 0, otherwise \end{cases}$. The upper bound Up_j on the gross result for each resource m_j is equal to zero, and greater than or equal to Lp_j for any other entity: $Up_j = \begin{cases} 0, \forall m_j \in R \\ \geq Lp_j, otherwise \end{cases}$. The upper bound Uc_j on gross utilization of resource m_j is greater than zero, and equal to zero for any other entity: $Uc_j = \begin{cases} > 0, \forall m_j \in R \\ 0, otherwise \end{cases}$.

The upper bound u_i for the volume of each activity o_i and the price cm_j for each resource or target are also given. The cost of an activity is estimated by a linear function of its volume with a fixed charge. For the cost function of each activity the proportionality constant cp_i and the fixed charge cf_i are defined. The lower bound $L_j = \begin{cases} -Uc_j, \forall m_j \in R \\ Lp_j, otherwise \end{cases}$ and the upper bound $U_j = Up_j$ denote the balance on result vs. utilization of the resources and outcomes of the activities. Furthermore, in the optimal structure let $m \subseteq M$ denote the set of entities and $o \subseteq O$ the set of activities. The relations between the entities and activities are given by parameters a_{ji} denoting the difference of the volumes of entity m_j resulted and utilized by activity o_i. x^* is the vector of the optimal volumes of activities for the problem, and z^* is its objective value. The aim is to determine the network (m^*,o^*,x^*,z^*), which satisfies the following conditions (Eq. 1 – Eq. 6) where z^* is minimal as indicated in Eq. 7.

$m^* = \psi(o^*)$, i.e., let m^* *be* the set of entities resulting from or utilised by at least one activity in o^*. (1)

$$x^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^T \tag{2}$$

$$0 < x_i^* \leq u_i \Leftrightarrow o_i^* \in o^* \tag{3}$$

$$\forall m_j \in m^* \cap R : -Uc_j \leq \sum_{o_i \in o^*} a_{ji} x_i \leq 0 \tag{4}$$

$$\forall m_j \in m^* \cap P : Lp_j \leq \sum_{o_i \in o^*} a_{ji} x_i \leq Up_j \tag{5}$$

$$\forall m_j \in m^* \setminus \mathrm{R} \setminus P : 0 \leq \sum_{o_i \in o^*} a_{ji} x_i \leq Up_j \tag{6}$$

$$z^* = \sum_{(\alpha,\beta)=o_i \in o^*} \left(cf_i + x_i^* * \left(cp_i - \sum_{m_j \in \alpha \cup \beta} a_{ji} cm_j \right) \right) \qquad (7)$$

4. Time Constrained PNS (TCPNS)

In the time constrained extension of PNS (TCPNS), each target must be achieved in time while the availability of the resources is constrained and the duration of each candidate activity is given as a fix charged linear function of its volume. To the parametric PNS problem with fix charged linear cost functions four additional parameters are given to define time constraints. tf_i is the fixed part; tp_i is the proportionality constant of the function, which estimates the duration of an activity based on its volume, respectively; Ut_j is the deadline for each result: $Ut_j = \begin{cases} \geq 0, \forall m_j \in P \\ \max_{m_j \in P}\{Ut_j\}, otherwise \end{cases}$; and Lt_j is the time of the earliest availability of a resource: $Lt_j = \begin{cases} \geq 0, \forall m_j \in R \\ 0, otherwise \end{cases}$. The aim is to determine the network (m^*,o^*,x^*,z^*) which satisfies the following conditions (Eq. 1 – Eq. 6 and Eq. 8 – Eq. 10) where z^* is minimal as given in Eq. 7.

$$\forall m_j \in M : Lt_j \leq t_{m_j} \leq Ut_j \qquad (8)$$

$o_i = (\alpha_i, \beta_i) \in o, \forall m_j \in \alpha_i : t_{o_i} \geq t_{m_j}$, i.e., the starting time t_{o_i} of an activity o_i cannot precede the time of availability t_{m_j} of any of its precondition m_j; (9)

$o_i = (\alpha_i, \beta_i) \in o, \forall m_j \in \beta_i : t_{m_j} \geq t_{o_i} + tf_i + x_i * tp_i$, i.e., the time of availability t_{m_j} of any outcome of an activity o_i cannot precede the sum of the starting time t_{o_i} and the duration ($tf_i + tp_i * x_i$) of the activity o_i. (10)

5. Structural Examination

In order to introduce time constraints in process synthesis, the structural model needs to be extended to express whether the outcome from an activity and precondition to another activity precede each other or not. As a result, artificial activities are included to the maximal structure; see, e.g., activities Ot1 and Ot2 in Figure 1. Note that including to or excluding from the solution structure these artificial activities clearly represent the decisions on the precedence; see Figure 1.

For the example in Figure 1, the deadline *tb* can only be satisfied if the generation of targets m5 and m6 are independent. It can be achieved by a solution structure where precondition m4 for activity O4 is exclusively provided by activity O2, i.e., O4 does not wait for O1 to generate its results.

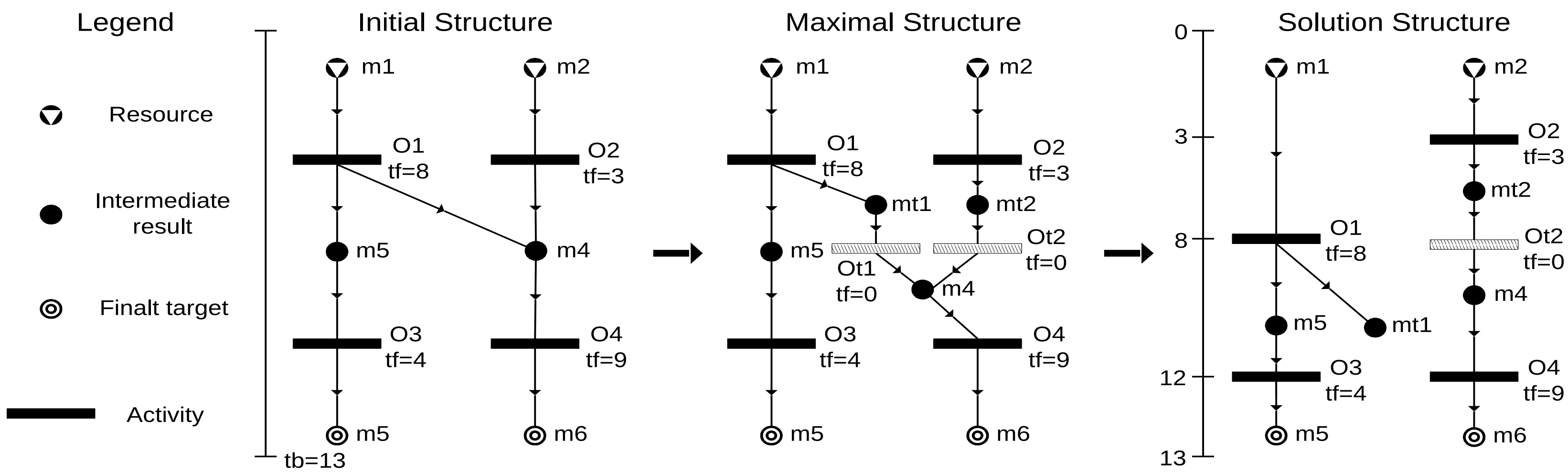

Figure 1: Extension of the Maximal Structure

6. Relaxation of Time Constraints

In the relaxed model of TCPNS, Eq. 10 is relaxed as Eqs. 11 and 12, where *tb* denotes the latest deadline among those upper bounds defined for the final targets; thus,

$$tb = \max_{m_j \in P} \{Ut_j\} \tag{11}$$

$$o_i = (\alpha_i, \beta_i) \in o, \forall m_j \in \beta_i : t_{m_j} \geq t_{o_i} + x_i * tp_i + y_i * (tb + tf_i) - tb \tag{12}$$

$$y_i = 0 \Rightarrow t_{m_j} \geq t_{o_i} - tb \tag{13}$$

$$y_i = 1 \Rightarrow t_{m_j} \geq t_{o_i} + x_i * tp_i + tf_i \tag{14}$$

Note that y_i is a binary variable expressing the existence (y_i = 1) or the absence (y_i = 0) of activity o_i in the structure. In the solution procedure prior to complete decisions on the existence or absence of the activities, the lower bound on the cost and the duration of alternative scenarios are estimated at each step. For the estimation, y_i is relaxed as a continuous value in range [0, 1], and its relation to the volume of the activity is expressed as:

$$x_i \geq u_i * y_i \tag{15}$$

The inequality in Eq. 12 is equivalent to Eq. 13 or Eq. 14 depending on whether y_i is equal to 0 or 1. Note that constraint in Eq. 12 comes into play only if the value of y_i is close to 1 in the relaxed model.

7. Conclusion

The current work presents a methodology to model formally supply chain networks and to algorithmically synthesize optimal supply scenarios by the P-graph framework. Recent extension introduces time constraint in the mathematical model and solution method for PNS. Need for extending the superstructure and the relaxed mathematical model for solving PNS problems with fix charged linear cost function has been satisfied. A method for representing decisions on the precedence of activities by structural decisions has been introduced.

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