CHEMICAL ENGINEERING TRANSACTIONS

VOL. 70, 2018

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Timothy G. Walmsley, Petar S. Varbanov, Rongxin Su, Jiří J. Klemeš

ISBN 978-88-95608-67-9; **ISSN** 2283-9216

DOI: 10.3303/CET1870241

Stochastic Pinch Analysis for Resource Allocation Networks with Multiple Resources

Deepika Arya*, Santanu Bandyopadhyay

Department of Energy Science and Engineering, Indian Institute of Technology Bombay, Powai, Mumbai, 400076, India
16i170004@iitb.ac.in

In Pinch Analysis, flows (e.g., water, hydrogen) from the sources are allocated to different sinks subject to quality constraints (e.g. contaminant concentration) while minimising the requirement of the valuable external resources (e.g. fresh water, external hydrogen). However, in many applications quality parameters associated with the sources are not deterministic due to varying operating and environmental conditions. Consequently, incorporation of these uncertainties is essential for reliable operation of the overall system. In this paper, uncertainties associated with each source quality are modelled as independent normal distributions with known mean and standard deviation. The overall objective of this paper is to optimise the operating cost of a source-sink allocation problem with multiple resources and uncertain qualities through the concept of prioritised cost. Chance constraint programming approach is applied to convert quality constraints associated with sources to a deterministic form. The resultant problem is approximated to a linear programming formulation and solved using Pinch Analysis approach. The proposed method is demonstrated with an illustrative example and results are verified through Monte-Carlo simulation.

1. Introduction

Process Integration emphasises on the efficient utilization of resource by using internally available streams. Pinch Analysis is an efficient tool for developing resource allocation networks in source-sink problems. It started as a thermodynamic approach for energy conservation (Linnhoff et al., 1982) and extended to various applications such as mass exchanger networks (El-Halwagi and Manousiouthakis, 1989), water networks (Wang and Smith, 1994), cooling water systems (Kim and Smith, 2001), hydrogen networks (Alves and Towler, 2002), material synthesis (El-Halwagi et al., 2003), water networks with regeneration (Fan and Liu, 2015) and to many other applications (Klemeš, 2013).These applications of Pinch Analysis are possible due to the development of many techniques, such as Limiting Composite Curve (Wang and Smith, 1994), Material Recovery Pinch Diagram (El-Halwagi et al., 2003), Source Composite Curve (Bandyopadhyay, 2006), etc. to minimise resources and minimising the operating cost of the process. Shenoy and Bandyopadhyay (2007) introduced the concept of prioritised cost in multiple resource problems to minimise the overall operating cost. Tan et al. (2015) recently analysed the effect of cost reduction using Waste Management Pinch Analysis.
The processes involved in these problems considered deterministic parameters (e.g., flow rate, contaminant concentration). However, various factors such as changing environmental conditions and operating conditions lead to uncertainty in these parameters. For overall reliability of the network, uncertainties have to be included at the targeting stage. The approach of mathematical optimisation has been previously used to target problems with parametric uncertainties. Suh and Lee (2002) considered random changes in parameter during a water network operation. Tan et al. (2007) used Monte Carlo simulations to analyse network sensitivity towards changing mass load. Adekola et al. (2013) optimally targeted water and energy requirement by considering time as a variable in flexible schedule. Mathematical optimisation being a complex and time-consuming approach, a simpler methodology is required to address these uncertainties. Consequently, this paper attempts to use physical insight-based methodology of Pinch Analysis to target source-sink problem with uncertain parameter. Chance-constrained programming, introduced by Charnes and Cooper (1959), has been adopted to incorporate uncertainties as it considers the constraints with uncertain parameters to be probabilistic.

Please cite this article as: Arya D., Bandyopadhyay S., 2018, Stochastic pinch analysis for resource allocation networks with multiple resources , Chemical Engineering Transactions, 70, 1441-1446 DOI:10.3303/CET1870241

In this paper, quality parameter is considered to be uncertain and chance-constrained formulation is applied to this stochastic Pinch Analysis problem involving multiple resources. The problem is first converted to a Linear Pinch Problem through a linear approximation and then the resultant problem is solved using Pinch Analysis techniques for the specified reliability of the overall network. The concept of effective quality is introduced and included in the prioritised cost of resources for minimising the operating cost while maintaining the desired reliability. Applicability of proposed methodology is demonstrated through an illustrative example.

2. Problem statement and mathematical formulation

The considered stochastic problem is a Water Pinch problem with uncertainties in source qualities. A set of N_s internal sources is given which produces a flow F_{si} at an uncertain quality q_{si} with known mean, μ_{qi} and standard deviation, σ_{qi}. It is to be noted that uncertainties in sources are independent of each other. A set of N_d demands is also given which accepts a total flow F_{dj} with a maximum allowable quality, q_{dj}. Internal sources are not always sufficient to meet these demands. A set of N_r resources is given having qualities q_r with mean μ_{qr}, standard deviation σ_{qr} and cost (c_{Rr}). The unutilised flow from the sources is thrown to an external demand called waste. The objective of the problem is to minimise the cost of resources for desired network reliability.
The problem is mathematically formulated by assuming f_{ij} to be the flow from i^{th} source to j^{th} demand, f_{rj} as the flow from r^{th} resource to j^{th} demand and f_{iw} be the flow from i^{th} source to waste. Let α be the reliability of quality load constraint for each demand and the product of reliability of these quality load constraints provides the network reliability. The flow balance for all the internal sources is given by Eq(1). The flow and quality load balance for all the demands are given by Eq(2) and Eq(3). It is to be noted that Eq(3) is considered probabilistic due to the presence of uncertainties in source qualities.

$$\sum_{j=1}^{N_d} f_{ij} + f_{iw} = F_{si} \quad \forall\, i \tag{1}$$

$$\sum_{j=1}^{N_d} f_{ij} + \sum_{r=1}^{N_r} f_{rj} = F_{dj} \quad \forall\, j \tag{2}$$

$$Prob\left(\sum_{j=1}^{N_d} f_{ij} q_{si} + \sum_{r=1}^{N_r} f_{rj} q_r \le F_{dj}\, q_{dj}\right) \ge \propto \quad \forall\, j \tag{3}$$

$$\sum_{r=1}^{N_r} (c_{Rr} \times (\sum_{j=1}^{N_d} f_{rj})) \tag{4}$$

The objective of the problem is to minimise the overall cost given by Eq(4) subject to constraints Eqs(1)-(3).
As constraint in Eq(3) is probabilistic, so the concepts of Pinch Analysis cannot be directly applied. For this reason, chance-constrained programming is used to convert it to a deterministic form in the next section.

3. Mathematical analysis

3.1 Chance-constrained programming

The chance-constrained programming methodology converts the probabilistic constraint to its deterministic equivalent, according to the probability distribution of the uncertain parameter. In this paper, quality is assumed to follow the normal distribution and chance-constrained programming is used to convert probabilistic constraint in Eq(3) to a deterministic one in Eq(5), which is achieved by the inversion of Eq(3). The detailed analysis is not shown due to brevity.

$$\sum_{r=1}^{N_r} f_{rj}\, \mu_{qr} + \sum_{i=1}^{N_s} f_{ij}\, \mu_{qi} + z_\alpha \sqrt{\sum_{r=1}^{N_r} f_{rj}^2 \sigma_{qr}^2 + \sum_{i=1}^{N_s} f_{ij}^2 \sigma_{qi}^2} \le F_{dj} q_{dj} \quad \forall\, j \tag{5}$$

It is to be noted that the deterministic constraint in Eq(5) is non-linear, which leads to non-linear problem formulation, where z_α represents the inverse cumulative distribution function for normal distribution. However, to use the methodology of Pinch Analysis, Eq(5) is linearised to Eq(6) using Cauchy's one-sided inequality.

$$\sum_{r=1}^{N_r} f_{rj}\, (\mu_{qr} + z_\alpha \sigma_{qr}) + \sum_{i=1}^{N_s} f_{ij}\, (\mu_{qi} + z_\alpha \sigma_{qi}) \le F_{dj} q_{dj} \quad \forall\, j \tag{6}$$

As seen in Eq(6), the r^{th} resource quality q_r is replaced with effective resource quality, $q_{Rr}^{eff} = \mu_{qr} + z_\alpha \sigma_{qr}$ and source quality q_{si} is replaced with effective source quality, $\mu_{qi} + z_\alpha \sigma_{qi}$, and linear problem formulation is achieved.

3.1.1 Verification

A network is designed to allocate flows from sources to different demands with modified qualities and its reliability is obtained through Monte Carlo simulation. In this formulation, quality for each source is generated randomly in each simulation of the designed network. The fraction of simulations for which the designed network satisfies all the quality constraints represents the overall reliability of the designed network. The methodology is verified when the reliability obtained from Monte Carlo simulation is higher than the desired network reliability.
As mentioned earlier, the problem considered in this paper consists of multiple resources with uncertain qualities. The concept of Prioritised cost is extended in the next section to incorporate those uncertainties.

3.2 Extension of Prioritised cost

Shenoy and Banyopadhyay (2007) proved that the Prioritised Cost depends on resource quality and Pinch quality for a deterministic problem. However, the problem in this paper considers the qualities to be probabilistic. The expression for Prioritised Cost is modified by replacing the resource quality and Pinch quality with their corresponding effective qualities, q_{Rr}^{eff} and q_p^{eff} as shown in Eq(7).

$$\frac{c_{R1}}{\left(q_p^{eff}-q_{R1}^{eff}\right)} < \frac{c_{R2}}{\left(q_p^{eff}-q_{R2}^{eff}\right)} \quad (7)$$

$$\frac{c_{R2}}{c_{R1}} = \frac{\left(q_p^{eff}-q_{R2}^{eff}\right)}{\left(q_p^{eff}-q_{R1}^{eff}\right)} \quad (8)$$

As observed from Eq(7), for the optimality of the overall problem, the resource with the lowest Prioritised Cost is utilised first and the cost ratio (c_{R2}/c_{R1}) is given by Eq(8).The applicability of these results is demonstrated through an illustrative example in the next section.

4. Illustrative example

The data for this example, shown in Table 1, are adapted from Polley and Polley (2000). In the original problem, the qualities associated with the sources were deterministic. By assuming the standard deviations in qualities (Table 1), the problem is now modified to incorporate uncertainties. In addition, it is also assumed that two fresh water streams, R1 and R2, are available with a mean contaminant concentration of 5 ppm and 25 ppm.

Table 1: Data for the example.

Sources	Quality (ppm)		Flow (t/h)	Demands	Quality (ppm)	Flow (t/h)
	Mean	S.D.				
S1	50	5	50	D1	20	50
S2	100	10	100	D2	50	100
S3	150	15	70	D3	100	80
S4	250	25	60	D4	200	70
R1	5	σ_{R1}				
R2	25	σ_{R2}				

It may be noted that the cost for any of the freshwater streams is not known and are therefore considered to be relative to each other. As the contaminant concentration of R2 is more than R1 (Table 1) and the cost of R1 is considered higher than that of R2 i.e. $c_{R1} > c_{R2}$. Throughout this example, cost of R1 is considered to be 100 units whereas, the cost of R2 is varied.
In order to minimise the overall cost, Prioritised Cost of each resource is considered, which depends on their effective qualities Eq(7) and the effective qualities in turn depends on the standard deviation. In this example, the standard deviations of resources are varied to study the optimal resource requirement at different values of standard deviation. Four cases involving different standard deviation of resources are considered:
(i) Standard deviation of both the resources, R1 and R2, are high,
(ii) Standard deviation of both the resources, R1 and R2, are low,
(iii) Standard deviation of R1 is high and R2 is low,
(iv) Standard deviation of R1 is low and R2 is high.
These ‘high’ and ‘low’ values of standard deviations are defined based on the ratio of standard deviation to mean quality.

This example aims to design a network with the reliability of each internal source as 95 %, which transforms the network reliability to 81.45 %, considering all the sources are independent to each other.

4.1 Standard deviations of both the resources are high

In this case, σ_{R1} and σ_{R2} are considered to be 3 ppm and 15 ppm. In case of linear formulation, the effective qualities of both the resources, R1 and R2 are calculated to be 9.94 ppm and 49.67 ppm and the Pinch quality obtained is 116.45 ppm (using Source Composite Curve). The cost ratio is calculated to be 1.63 using Eq(8). It is observed from Figure 1 that most R1 is replaced by R2 after the cost ratio of 0.63, when only the minimum R1 that satisfies D1 is used. In non-linear formulation, the transition from R1 to R2 is smooth as seen from Figure 1. This smooth transition indicates that combination of both the resources is used in between the cost ratios, 0.54 to 0.8. Below 0.54, it is beneficial to use minimum R1 to meet demand D1, and above 0.8 use only R1, in order to minimise the resource cost.
It is observed from Figure 1 that the resource cost is higher in case of linear formulation as compared to the non-linear formulation. This is due to the fact that the proposed methodology results in conservative solution.

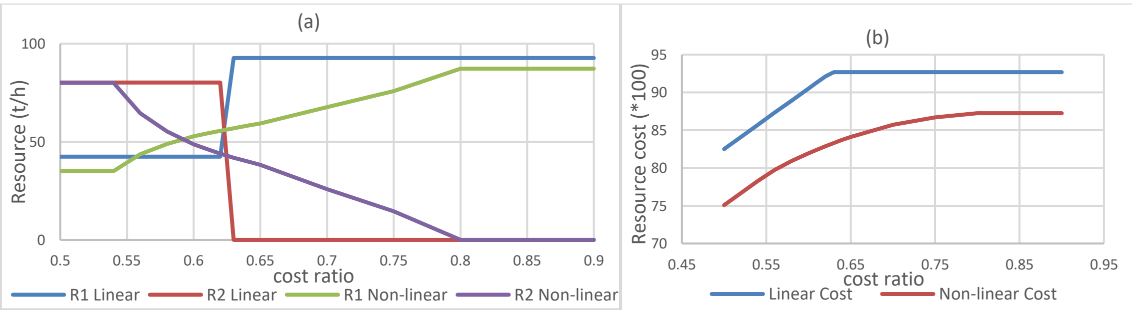

Figure 1: Varying (a) Resource requirement (b) Resource cost with cost ratio (c_{R2}/c_{R1}) for linear and non-linear cases, when standard deviation for both the resources are high.

A network (not shown due to brevity) is designed using effective qualities of internal sources and resources (using only R1) and is verified through Monte Carlo simulation. The overall reliability of the network is achieved to be 94.34 % which is higher than expected reliability of 81.45 %. The methodology delivers conservative results as expected. The reliability (in %) of demands D1, D2, D3 and D4 is obtained as 98.48, 99.96, 95.94 and 99.97.

4.2 Standard deviations of both resources, R1 and R2 are low

In this case, standard deviations, σ_{R1} and σ_{R2} are considered to be 0.5 ppm and 5 ppm. In linear formulation, the effective qualities of resources, R1 and R2 are calculated to be 5.82 ppm and 33.22 ppm and the Pinch quality is determined to be 116.45 ppm. Using Eq(8), the cost ratio is calculated to be 0.75 and as the cost ratio reaches this value, it becomes more beneficial to use R2. These results are consistent with the results depicted in Figure 2, where the transition occurs just after the cost ratio 0.75.

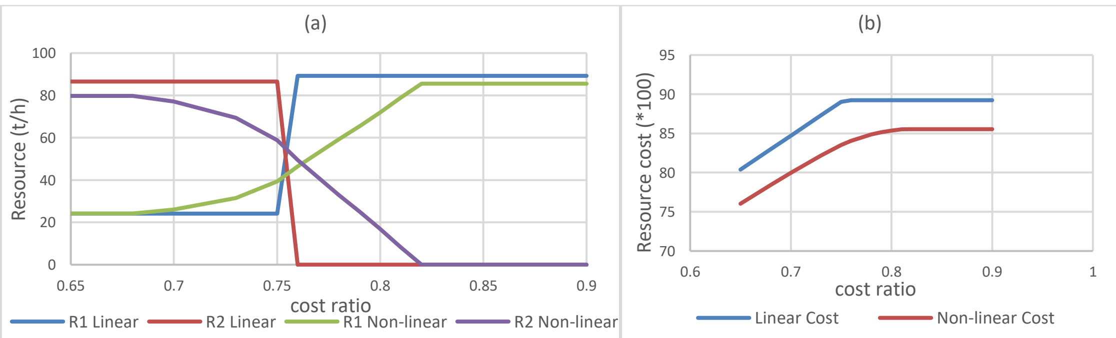

Figure 2: Varying (a) Resource requirement (b) Resource cost with cost ratio (c_{R1}/c_{R2}) for linear and non-linear case, when standard deviation for both the resources are low.

In case of non-linear formulation, again the smooth transition is seen in Figure 2 in the cost ratio range of 0.7 to 0.81, from using minimum R1 to using only R1, and using both resources are optimally in between these cost ratios. It is observed that this transition range is now smaller, compared to the case where standard deviation of both resources were high. Also, the overall cost of resources in non-linear case is lower than the linear case (Figure 2), indicating a conservative solution.
The Monte-Carlo simulation used to verify a designed network provides the network reliability of 96.81 %. The demands D1, D2, D3 and D4 are obtained to have reliability (in %) of 98.17, 99.96, 98.73 and 99.96.

4.3 Standard deviation of R1 is high and R2 is low

In this case, σ_{R1} and σ_{R2} are considered to be 3 ppm and 5 ppm. When linear formulation is considered, the effective qualities of both the resources, R1 and R2 are calculated to be 9.93 ppm and 33.22 ppm and the Pinch quality is determined to be 116.45 ppm. The cost ratio obtained by using Eq(8) is approximately 0.78, which is similar to the results depicted in Figure 3, where just after cost ratio of 0.78, R2 replaces majority of R1.

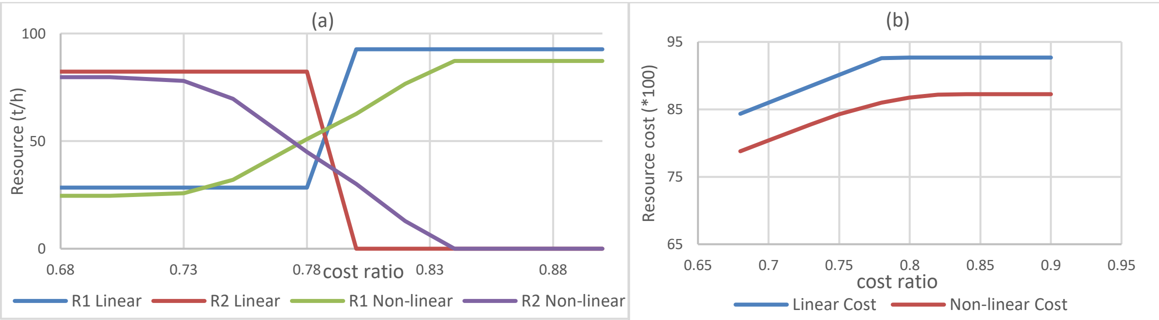

Figure 3: Varying (a) Resource requirement (b) Resource cost with cost ratio for linear and non-linear case, when standard deviation of R1 is high and R2 is low

In non-linear formulation, transition starts after cost ratio of 0.7, and as it reaches near 0.84, mostly R2 is used. Combination of both the resources is required between the cost ratios 0.7 and 0.84, to minimise cost. This transition range is observed to be wider as compared to the case where standard deviations of both resources were low, but narrower as compared to the case where both standard deviations were high. In Figure 3, as the cost of resource is higher for linear case, the overall cost of resources shows the expected trend.
In order to verify the results, reliability of designed network is obtained to be 95.68 % from Monte Carlo simulations and the demands are 98.19 %, 99.99 %, 97.54 % and 99.96 % reliable.

4.4 Standard deviation of R1 is low and R2 is high

In this case, σ_{R1} and σ_{R2} are considered to be 0.2 ppm and 15 ppm. In case of linear formulation, the effective qualities of the resources, R1 and R2 are calculated to be 5.33 ppm and 49.67 ppm, and the Pinch quality is determined to be 116.45 ppm. Using Eq(8), cost ratio is calculated to be 0.6. It is in accordance with the results shown in Figure 4 that resource R2 replaces most R1 near the cost ratio 0.6. The linear cost of resource is higher than the non-linear cost due to conservative results given by methodology.

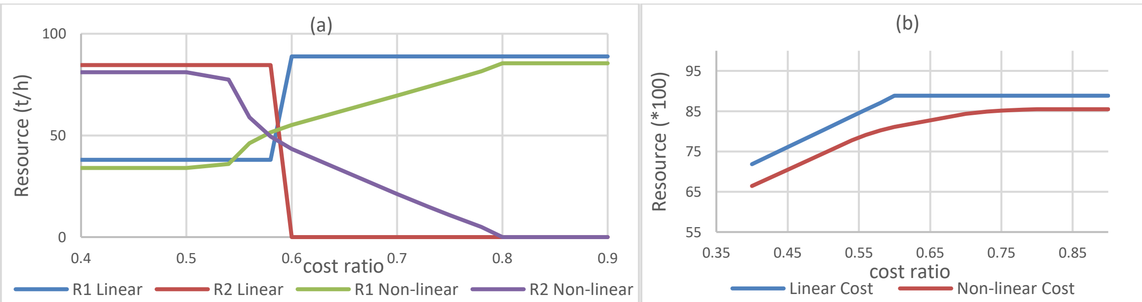

Figure 4: Varying (a) Resource requirement (b) Resource cost with cost ratio for linear and non-linear case, when standard deviations for resource of R1 is low and R2 is high

In case of non-linear formulation, the combination of both the resources is required up to cost ratio of 0.8, and in cost ratios below 0.5, minimum R1 that satisfied D1 is utilised and remaining demands are satisfied by R2. The cost curve shown in Figure 4 depicts the expected behaviour that is, the cost is higher for linear case.
The results obtained from Monte Carlo simulations for this case provides the network reliability of 96.25 %, when the demands are 96.75 %, 99.93 %, 99.62 % and 99.96 % reliable.
From these four cases, it is observed that with the change in values of the standard deviations, the transition range also changes accordingly. The transition range is wider when standard deviations are high and narrows down as the standard deviations get reduced.

5. Conclusions

Pinch Analysis is applicable for source-sink problems with deterministic parameters. Changing operating and environmental conditions may often lead to uncertainty in the quality of the sources and incorporation of these uncertainties is essential for reliable operation of the overall process. In this paper, Pinch Analysis based methodology is extended to problems involving uncertainty in source quality. In the proposed methodology, quality load constraint is considered probabilistic and converted to non-linear deterministic constraint using chance-constrained programming and are then linearised with a linearisation approximation. They have been referred as non-linear and linear formulation. It is observed that the deterministic methodology with linear approximation gives conservative results. The concept of Prioritised Cost is also extended to Stochastic Pinch problems involving multiple resources for selecting the appropriate resource to minimise the cost while maintaining the network reliability. The applicability of the proposed algorithm is demonstrated through an illustrative example. In the linear formulation, another available resource, with higher contaminant concentration, completely replaces the purer resource when its Prioritised Cost is less as compared to purer resource. While in non-linear formulation, a smooth transition from one resource to another is observed, as a combination of resources is required to minimize cost over a range of cost ratios. This range of cost ratio, where combination of resources is utilised, varies with the standard deviation of resources. The future research is focused on bridging the gap between the results of the linear and non-linear formulation.

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