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Redundancy System Fault Sampling Under Imperfect Maintenance

Chao Zhang*, Linhan Guo, Boping Xiao, Rui Kang

School of Reliability and Systems Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China
258325440@qq.com

When conducting simulation for evaluating complex system reliability or availability, method of random sampling is applied to simulate fault occasion of complex system. Current research of fault sampling generally assumes maintenance activity restores systems to “good-as-new” and “bad-as-old” without considering “partly good” situation. However, system usually keeps its function through imperfect maintenance that restores system to “partly good”: the system after maintenance will not as good as new, but younger. A system fault random sampling method under condition of imperfect maintenance must be presented in order to enhance simulation model credibility and correctness. When reliability structure of system is in the form of redundancy, the fault sampling method under condition of imperfect maintenance is difficult because not only redundancy system fault time isn’t exponential distribution but also that the state transition process of redundancy system is hard to model and steady state probability of each component in the redundancy system is hard to determine .So in this paper , a generally repairable parallel system with non-identical components: which is a common form of redundancy is considered, then a fault random sampling method for this kind of redundancy system under condition of imperfect maintenance according to monte carlo simulation principle is presented. The characteristics of this method mentioned above is that Markov chain embedded within this method is employed to model the state transition process of redundancy system and to determine steady state probability of each component in the system. Fault occasions of redundancy system under imperfect maintenance can be simulated and fault components in the system can be determined via the fault sampling method. These are novel contributions made in this paper. Finally, a numerical case using fault sampling method for a redundancy system under imperfect maintenance is given and the validity and feasibility of fault sampling method is verified.

1. Introduction

When conducting simulation for evaluating redundancy system reliability or availability, method of random sampling is applied to simulate fault occasion of system. Redundancy system fault sampling and determination are referred to simulating the fault occasion according to random numbers and the system fault distribution function and determine the component that malfunctioned according to the system structure and component failure rates when system distribution function is known.

Maintenance effect include perfect effect, non-perfect effect and imperfect effect to a component failure. A perfect effect can restore the component to good-as-new, a non-effect to bad-as-old, and an imperfect effect to partly good. Since it is usually assumed that maintenance effect is partly good in reality, so the component fault sampling with imperfect maintenance method is more applicable to engineering system. At the current stage, study mainly focus on the imperfect maintenance model, not referring to the fault sampling method under imperfect maintenance. Imperfect maintenance model is discussed in detail by Doyen and Gaudoin (2004). Jiao and Wang(2006) present a fault sampling method to airplane system whose life is exponential distribution. Su(2006) and Zhao(2006) suppose failure rate of system is time-variant, however, the system reliability structure is simply considered as series ,fault sampling of system with redundancy reliability structure under imperfect maintenance isn’t considered. Guo and Kang(2007) studies the fault sampling method of series system for evaluating system availability, but the life of each

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component is considered as exponential distribution. Zio(2002) conducts the system fault sampling with the consideration that components in system are non-identical, however, the life distribution type is all the same.

In this paper, random fault sampling method for system that has redundancy reliability structure under imperfect maintenance is researched according to Monte Carlo simulation principle. The novelty of the method stems from the facts that not only the maintenance is imperfect, but also the failure rate of components in system is time-variant and the system reliability structure is redundancy. On the basis mentioned above, when conducting fault sampling for redundancy system, determination of fault components becomes difficult, Markov state transition process is used to solve this problem.via the fault sampling method, fault occasions of complex system under imperfect maintenance can be simulated and fault components in the redundancy system can be determined.

2. Component fault sampling under imperfect maintenance

Maintenance effects can be perfect, non-perfect and imperfect. Time-variant functions of component failure rate corresponding to the maintenance effects are different.

Failure rate of component should situate between the level of “good-as-new” and “bad-as-old” after imperfect maintenance, if component fault time are expressed as ($t_1,t_2\cdots t_n$) component failure rate without maintenance is $f(t)$ .Failure rate after imperfect maintenance is $r(t)$, actual age of component after the n-th imperfect maintenance at time of t is:

$$Z(t)=t-\alpha T_n \ , \qquad n=0,1,2.... \tag{1}$$

where α is the recovery coefficient ,$0<\alpha<1$.

Failure rate of component is

$$r(t)=f\left[Z(t)\right]=f\left(t-\alpha T_n\right) \tag{2}$$

Process of component fault sampling is shown as follows:

Function of components under imperfect maintenance cannot restore to “as good as new”. Because fault distribution function of the component is monotonously increasing and the value of fault distribution function of component at t_∞ equals to 1, so inequality (3) can be obtained:

$$\frac{F_{t_n}-F_{t_{n-1}}}{F_{t_\infty}-F_{t_{n-1}}}<1 \tag{3}$$

Failure rate function of component after n-th imperfect maintenance is shown in Eq (4)

$$r_{t_n}=f\left(t_n-\alpha t_n\right) \tag{4}$$

Based on the reliability theory, the relationship between failure rate function and reliability function can be defined as Eq (5):

$$r_{t_n}=\frac{d\ln F_{t_n}}{dt}=\frac{d\ln F\left(t_n-\alpha t_n\right)}{dt} \tag{5}$$

Suppose $p_n=\dfrac{F_{t_n}-F_{t_{n-1}}}{F_{t_\infty}-F_{t_{n-1}}}$,Fault sampling Eq (6) can be obtained from Eq (4) and Eq (5):

$$F\left(t_n-\alpha t_n\right)=p_n+(1-p_n)F\left(t_{n-1}-\alpha t_{n-1}\right) \tag{6}$$

$$t_n=\frac{F^{-1}[(1-p_n)F\left(t_{n-1}-\alpha t_{n-1}\right)]}{1-\alpha}$$

where p_n is a random number from 0 to 1, t_n can be calculated by t_{n-1} and p_n according to equation (6).

3. System fault sampling under imperfect maintenance

3.1 Redundancy System Model

The basic assumptions of redundancy system model are：

- The complex system can be divided into p layers, components in each layer are consisted in parallel. Component is consisted of several sub-components in mentioned relationship above in the next layer
- Suppose system fault is evoked by one sub-component fault for this component is consisted by n subcomponents in parallel.
- Corrective maintenance restores system or components to "partly good" condition.

3.2 Fault sampling method of redundancy system

When conducting fault sampling for redundancy system, the Markov method is used, it's referred to simulate the fault time of system by fault distribution function and Markov chain.

The components malfunction can be determined via combining fault time of the system with random number group and analyzing failure rates relation or fault state probability of components in each layer. The corresponding tree of fault sampling method is shown in Figure 2

According to the fault sampling method given in last chapter, the system fault time can obtained by equation group (7):

$$\begin{cases} t_1 = F^{-1}(\eta_1) \\ t_2 = \dfrac{F^{-1}\left[(1-\eta_2)(1-F(1-\alpha)t_1\right]}{1-\alpha} \\ \cdots \\ t_n = \dfrac{F^{-1}\left[(1-\eta_n)(1-F(1-\alpha)t_{n-1}\right]}{1-\alpha} \end{cases} \tag{7}$$

Suppose a component is combined by N sub-components in parallel, sub-components in the sub-components malfunction can be determined according to sampling result obtained by given random numbers and fault probability at the time of t_n.

The fault state of parallel system can be expressed as {R/F, f(1,2…M-1),r(M…N)}, the state of parallel system is expressed by R or F,F indicates the state that system have faults, R indicates the state that system doesn't have faults, numbers in bracket after f are the subscripts marking the fault components in the parallel system, numbers in bracket after r are the subscripts marking the reliable components, on the basis above, the fault state transition diagram of parallel component with N different sub-components can be shown in Figure 1:

In the steady state, system state probability can be obtained by the following rules: rate of transition from this state to other states multiply the probability of this state equals to sum rates of transition from other states to this state multiply the probability of the corresponding state.

The corresponding fault state probability equation group of parallel component can be shown in equation group(8),in the above equation group, sum of system fault state probability equals to 1, it can be expressed Eq (9).

At time of t_n, according to equation group (8) and Eq (9), equation of system fault state probability can be obtained, Eq (10) is for the state that there is no component, Eq (11) is for the state that there are M-1 fault components in the parallel system.

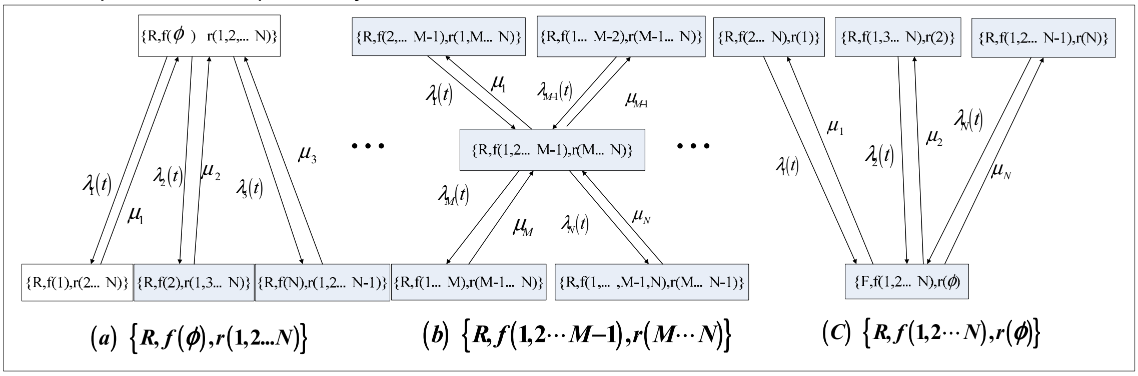

Figure 1 State transition diagram of parallel component with N sub-components

There are 2n fault states for complex system combined by N components in parallel. Suppose sum of fault state probability of the first k states is $P_k = \sum_{i=0}^{k} p_i$ .If the given random number β is between (P_i, P_{i+1}), the parallel system in the i-th state, fault sub-component can be determined by the subscript.

$$\begin{cases} P_{\{R,f(\phi),r(1,2\ldots N)\}} \cdot \sum_{i=1}^{N} \lambda_i(t) = \sum_{i=1}^{N} \left(P_{\{R,f(i),r(1,2\ldots i-1,i+1,N)\}}\right) \\ \ldots \\ P_{\{R,f(1,2\ldots M-1),r(M\ldots N)\}} \cdot \left(\sum_{i=1}^{M-1} \mu_i + \sum_{i=M}^{N} \lambda_i(t)\right) = P_{\{R,f(2\ldots M-1),r(1,M\ldots N)\}} \cdot \lambda_1(t) + \cdots + P_{\{R,f(1,2\ldots M-2),r(M-1\ldots N)\}} \cdot \lambda_{M-1}(t) \\ \qquad + P_{\{R,f(1\ldots M),r(M-1,M+1\ldots N)\}} \cdot \mu_M + \cdots + P_{\{R,f(1\ldots M-1,N),r(M\ldots N-1)\}} \cdot \mu_N \\ \ldots \\ P_{\{F,f(1,2\ldots N),r(\phi)\}} \cdot \sum_{i=1}^{N} \mu_i = \sum_{i=1}^{N} P_{\{R,f(1,2\ldots i-1,i+1,N),r(i)\}} \cdot \lambda_i(t) \end{cases} \tag{8}$$

$$\sum_{i=1}^{2^n} P_i(t) = 1 \tag{9}$$

$$P_{\{R,f(\phi),r(1,2\ldots N)\}} = \frac{\sum_{i=1}^{N} \lambda_i(t_n)}{\sum_{i=1}^{N} \lambda_i(t_n) + \mu_i} = \frac{\sum_{i=1}^{N} \lambda_i\left(\frac{F\left[(1-\eta_n)(1-F(1-\alpha)t_{n-1}\right]}{1-\alpha}\right)}{\sum_{i=1}^{N} \lambda_i\left(\frac{F\left[(1-\eta_n)(1-F(1-\alpha)t_{n-1}\right]}{1-\alpha}\right) + \mu_i} \tag{10}$$

$$\begin{aligned} P_{\{R,f(1,2\ldots M-1),r(M\ldots N)\}} &= P_{\{R,f(\phi),r(1,2\ldots N)\}} \cdot \frac{\sum_{i=1}^{M-1} \lambda_i(t_n)}{\sum_{i=1}^{M-1} \mu_i} = P_{\{R,f(\phi),r(1,2\ldots N)\}} \cdot \frac{\sum_{i=1}^{M-1} \lambda_i\left(\frac{F^{-1}\left[(1-\eta_n)(1-F(1-\alpha)t_{n-1}\right]}{1-\alpha}\right)}{\sum_{i=1}^{M-1} \mu_i} \\ &= P_{\{R,f(\phi),r(1,2\ldots N)\}} \cdot \frac{\sum_{i=1}^{M-1} \lambda_i\left(\frac{F^{-1}\left[(1-\eta_n)(1-F(1-\alpha)t_{n-1}\right]}{1-\alpha}\right)}{\sum_{i=1}^{M-1} \mu_i} \end{aligned} \tag{11}$$

4. Numerical Example

Suppose a redundancy system is designed as two layers. The system is constructed by three LRUs combined in parallel shown in Figure 2.Failure rate functions of system and LRUs can be shown in Table 1:

Table 1: Failure distribution of system, LRU1 and LRU2

	System	LRU1	LRU2	LRU3
Distribution Type	Weibull	Weibull	Exp	Exp
Shape Parameter	$m=3, t_0=1$	$m=2, t_0=1$	$\lambda=2$	$\lambda=3$
Failure Rate Function	$f(t)=3t^2$	$f(t)=2t$	$f(t)=2$	$f(t)=3$
Failure Distribution Function	$F(t)=e^{-t^3}$	$F(t)=e^{-t^2}$	$F(t)=e^{-2t}$	$F(t)=e^{-3t}$

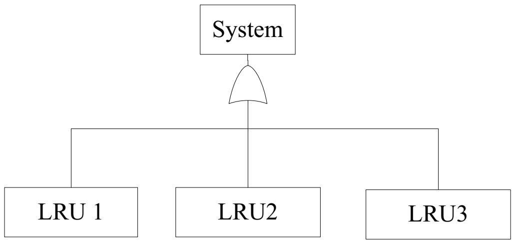

Figure 2 Redundancy system structure

The sampling result of system fault time can be obtained according to Equation group (7) ,suppose the system is under imperfect maintenance and $\alpha = 0.5$, the sampling result can be shown in Table 3:
According to random number group (η_1,η_2,η_3,η_4), the ratio of component failure rate and state transition probability for the third layer , fault components in this system can be determined using top-down method, where repair rate μ_1=3, μ_2=2. The state probability of component 1 shown in Table.4 can be obtained by fault components list which is shown in Fig.4 can be obtained by the equations (12)-(14) given in the 3rd chapter.

Table 2: System fault time

Step	Random Number	Fault Time
	η	*t*
1	0.44	0.833
2	0.71	2.188
3	0.39	2.434
4	0.58	2.775
5	0.26	2.891

Combination of complex system sampling result is shown in Table 2 and Table 4.The result of Table 2 is system fault sampling time, the result of Table 4 is the components malfunction determined by fault state probability of components connected in parallel. For the system is only combined by three parallel LRUs, so when the system malfunctions, the three LRUs must have faults, however, fault components determined by the Markov chain can't be the total LRUs, when the two results contradicts, the fault time can't be true simulation time to be the input data, so this fault time is eliminated, only the fault time which can coordinate the system fault and the three LRUs fault can be the input data, so the system fault time must go over the conformity test. According to the method, fault components of redundancy system are determined and the result which go over conformity test is shown Table 4:

Table 3: Fault state probability of parallel component

STEP	Probability							
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
1	0.07	0.17	0.08	0.04	0.32	0.22	0.01	0.09
2	0.31	0.44	0.01	0.07	0.06	0.05	0.05	0.01
3	0.10	0.08	0.17	0.17	0.09	0.14	0.05	0.2
4	0.16	0.2	0.1	0.08	0.12	0.14	0.06	0.14
5	0.04	0.06	0.1	0.16	0.03	0.01	0.24	0.36

P_1 indicates $P_{\{R,\ f(\phi),r(1,2,3)\ \}}$, P_2 indicates $P_{\{R,\ f(1),r(2,3)\ \}}$, P_3indicates $P_{\{R,\ f(2),r(1,3)\ \}}$, P_4 indicates $P_{\{R,\ f(3),r(1,2)\ \}}$, P_5 indicates $P_{\{F,\ f(1,2)r(3)\}}$, P_6 indicates $P_{\{F,\ f(2,3),r(1)\ \}}$, P_7 indicates $P_{\{R,\ f(1,3),r(2)\ \}}$, P_3 indicates $P_{\{F,\ f(1,2,3),r(\phi)\ \}}$

Table 4: Real fault time

Time	Random Number	Fault component	True result
0.833	0.93	L1,L2,L3	√
2.188	0.40	L1	
2.434	0.88	L1,L2,L3	√
2.755	0.56	L1,L2	
2.891	0.72	L1,L2,L3	√

5. Conclusion

In order to increase the credibility of simulation model when redundancy system reliability and availability are evaluated. this paper presents a fault sampling method for redundancy system under imperfect maintenance. When conducting fault sampling for the redundancy system, Markov state transition process is embedded in the method to determine the fault products. Based on the fault sampling method mentioned above, fault occasions of redundancy system under imperfect maintenance are simulated and fault components in the system are determined, the validity and feasibility of fault sampling method is verified in the numerical example. For adapting to k out of n system, the fault sampling methodology can be extended under imperfect maintenance in future research work.

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