CHEMICAL ENGINEERING TRANSACTIONS
VOL. 33, 2013
Guest Editors: Enrico Zio, Piero Baraldi

ISBN 978-88-95608-24-2; **ISSN** 1974-9791

A publication of
AIDIC
The Italian Association of Chemical Engineering
www.aidic.it/cet

DOI: 10.3303/CET1333090

Dynamic Reliability Assessment for Multi-State Degraded Systems

Yu Liu[a], Ming J. Zuo*[a,b], Hong-Zhong Huang[a]

[a] School of Mechanical, Electronic, and Industrial Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, P.R. China
[b] Department of Mechanical Engineering, University of Alberta, Edmonton, Alberta, T6G2G8, Canada
ming.zuo@ualberta.ca

With the aim of tracking the deteriorating process of a specific individual multi-state system consisting of more than one multi-state component, a dynamic reliability assessment method which can utilize system-level observation history to dynamically update the reliability function of multi-state degraded systems is developed in this paper. The current and future state probabilities of components in a specific multi-state degraded system can be updated via the proposed recursive Bayesian formula. The effectiveness and accuracy of the proposed method are demonstrated via a water piping system.

1. Introduction

Even though the binary state system reliability theory has long been adopted in engineering practices, as engineered systems designed toward larger scale, more complex, and higher precision, it is extensively observed that systems or components may manifest multiple states from working perfectly to completely failed, during their deterioration process (Lisnianski and Levitin, 2003). Many newly developed methods, like extended decision diagram-based method (Shrestha and Xing, 2008), stochastic process (Li and Pham, 2005), universal generating function (UGF) (Ushakov, 1986), recursive algorithm (Li and Zuo, 2008), etc., have been used to facilitate the reliability and performance evaluation for a variety of MSSs.

However, it is noteworthy that the aforementioned studies are on the basis of traditional time-based reliability models which compute the reliability of an MSS from the “population” or “statics” perspective. Put another way, state transition intensities/probabilities, distribution of sojourn time at a certain state, and many other quantities that characterize stochastic behavior of a system are derived from historical data collected from a large population of identical systems, and the reliability assessment of a system over time is conducted purely based upon these statistic information (Lisnianski et al., 2012). For a specific individual system, if additional useful knowledge or information related to the stochastic (deteriorating) behaviors of the system becomes available, uncertainty associated with the deterioration of this specific system can be further reduced so as to lead a more precise reliability assessment for this system. Bear this general concept in mind, *dynamic reliability* herein is defined *as the reliability function or model of a specific system which can be updated or modified by collecting additional useful knowledge or information related to the stochastic (deteriorating) behaviors of the system after the system is put into use*.

Most reported works on dynamic reliability assessment focus on the situation where knowledge or condition monitoring information collected from a system/component only reflects, either directly or indirectly, the current condition and further evolution of the system/component itself (Wang and Christer, 2000). However, as a majority of engineered systems consist of more than one component and the condition and evolution of a system are eventually determined by its components, tracking and utilizing both component-level and system-level knowledge or condition monitoring information during the system operation stage is expected to improve the accuracy of dynamic reliability assessment for this specific individual system. Our focus in this work is to utilize system-level inspection data of a specific individual MSS to identify the states of its multi-state components and further update the reliability of these

Please cite this article as: Liu Y., Zuo M.J., Huang H.Z., 2013, Dynamic reliability assessment for multi-state deteriorating systems, Chemical Engineering Transactions, 33, 535-540 DOI: 10.3303/CET1333090

components, and then update the reliability function of the system. This issue is very commonly encountered in engineering practices, for example, components' states are oftentimes unobservable, but the system state can be directly observed. However, several combinations of components' states may result in the same system state. In this case, identifying the possible state of each component in the system enable engineers to update the dynamic reliability function of this system and predict system's further behavior and remaining useful life. In addition, it also allows maintaining, as early as possible, the seriously degraded components which have significant contributions to system performance or may cause failure of the entire system. To the best of our knowledge, this issue has rarely been addressed in literature. In this paper, by utilizing all the system-level observation history, a recursive Bayesian method is put forth to recursively identify the possible states of components and further update the reliability function of the system.

The remainder of this paper is organized as follows. Section 2 introduces the specific problem we study in this work. The details of the proposed recursive Bayesian method along with the formulation of dynamic reliability function, and state probabilities of both system and components are elaborated in Section 3. A numerical example is presented in Section 4 to demonstrate the effectiveness of our proposed method, and it is followed by a brief closure in Section 5.

2. Problem Statement

The MSSs investigated in this paper consist of multiple components. Every component has multiple discrete states that could be distinguished by either performance or level of degradation. For example, a flow transmission component could have a set of states characterized by volume flow rate, say 2.0 tons/min, 3.0tons/min, and 5.0tons/min. Another example is that the health condition of a bearing within a gearbox can be roughly classified into several states from perfectly working to completely failed, say "normal", "medium damaged", "seriously damaged", and "completely failed". The entire system manifests multiple states with different combinations of components' states. Most often, more than one combination of components' states may result in the same system state. A simple water piping system is exemplified here to illustrate this argument. Suppose the water piping system is comprised of three pipes. Units #1 and #2 are connected in parallel and then serially connected with unit #3. The possible flow transmission rates of each component are tabulated in Table 1. Unit #1 has two states while units #2 and #3 each has three states. The transition time of a component from its better state to lower one is assumed to follow homogeneous continuous time Markov process. The transition intensities of components are tabulated in Table 2, and the deterioration processes of these units are statistically independent. The performance G_S of the entire system is completely determined by performances of components G_l ($l = 1,2,3$). Based on the system configuration and property of components' performance, one has $G_S = \min\{G_1 + G_2, G_3\}$. The system states distinguished by the flow transmission rates along with the associated combinations of components' states are given in Table 3. As observed in Table 3, some system states, e.g. states 5, 4, 3, 2, and 1, are caused by multiple combinations of components' states.

Table 1: Flow transmission rates of components (tons/min)

Unit ID	State 1	State 2	State 3
#1	0.0	2.5	/
#2	0.0	2.0	3.5
#3	0.0	4.0	6.0

Table 2: Transition intensities of components in the studied water piping system (month-1)

Unit ID	$\lambda^l_{(3,2)}$	$\lambda^l_{(3,1)}$	$\lambda^l_{(2,1)}$
#1	—	—	0.04
#2	0.05	0.08	0.1
#3	0.035	0.06	0.09

Table 3: The system states and the associated combinations of components' states

System State ID	Performance	Components' State Combination (unit #1, unit #2. unit #3)
7	6.0 tons/min	{(2, 3, 2)}
6	4.5 tons/min	{(2, 2, 3); (2, 3, 2); }
5	4.0 tons/min	{(2,3,2); (2, 2, 2)}
4	3.5 tons/min	{(1, 3, 3); (1, 3, 2)}
3	2.5 tons/min	{(2, 1, 3); (2, 1, 2)}
2	2.0 tons/min	{(1, 2, 3); (1, 2, 2)}
1	0.0 ton/min	{(2, 3, 1); (2, 2, 1); (2, 1, 1);(1, 3, 1); (1, 2, 1); (1, 1, 3);(1, 1, 2); (1, 1, 1)}

In some engineering practices, it might be impossible to observe condition of components, for example it is difficult to setup sensors or devices to detect the status of bearings and gears of a gearbox as they are inside the gearbox, but it is very easy to track and observe the condition of the entire gearbox via critical signals correlated with the system condition, like vibration signals and oil debris. Instead of directly monitoring condition of states of components, the condition monitoring information from system level becomes important to reflect states of components. Once components' states are identified, the reliability function of the entire system can be updated so as to predict the system's future behavior in a more accurate manner. For example, if the system is detected to be at state 4, the unit #1 and unit #2 must be at state 1 and state 3 respectively. The unit #3, however, could be either at its state 2 or at its state 3 which of course lead to different deterioration patterns of both unit #3 and the system in residual lifetime. Identifying the current state of unit #3, therefore, becomes critical to update the reliability function of the system, and this paper severs this purpose.

Before introducing the proposed method to assess the dynamic reliability of a monitored MSS, some assumptions used in this work are summarized as follows:

(1) An MSS consists of M components and has finite number of states $\mathbf{S} = \{S_1, S_2, ..., S_{N_S}\}$, from perfectly working condition to completely failed. N_S denotes the total number of states of the MSS; S_{N_S} and S_1 represent the best and the worst states of the MSS respectively.

(2) The component l in the MSS could have more than two states denoted as $\mathbf{s}_l = \{s_{l,1}, s_{l,2}, ..., s_{l,n_l}\}$, where n_l the number of possible states of component l; s_{l,n_l} and $s_{l,1}$ are the best and the worst states of component l respectively.

(3) The transition intensities or the deterioration model of component l from its state i to state j ($0 \le j \le i$) is known in advance. This assumption is practical as data collected from either experimental tests or the field for each component can be used to choose deterioration model and estimate unknown parameters in deterioration model.

(4) The structure function $\phi(\cdot)$ of an MSS with respect to states of components is known. The state of an MSS at any time instant t can be, therefore, determined by the combination of components' states at any time instant t as $X_S(t) = \phi\left(X_1(t), X_2(t), ..., X_M(t)\right)$. Tools, like the universal generating function (UGF) (Lisnianski and Levitin, 2003) etc., provide computationally efficient way to find out combinations of components' states with respect to system state.

(5) As multiple combinations of components' states may lead to the same system state, i.e. $N_S < \prod_{l=1}^{M} n_l$. Let $S_{i,j}$ ($1 \le j \le L_i$) indicate the j th combination of components' states when the system is at its state i and L_i denote the total number of combinations of components' states at system state i. $\mathbf{Q}_{i,j}$ is a vector containing components' state of the j th states combination $S_{i,j}$. Hence, one has $\mathbf{Q}_{i,j}(l) \in \mathbf{s}_l$ representing the state of component l in the j th combination of components' state $S_{i,j}$ when the system is at state i.

(6) The states of components of a specific individual MSS at any time instant t after the MSS being put into use are unobservable; whereas the system condition is observable during operation stage by utilizing condition monitoring techniques, and it is assumed in this work that the system state can be immediately and directly identified at any inspection time. The state of the specific individual MSS at the k th inspection time is denoted as $X_S(t_k)$; while $\mathbf{X}_S(\mathbf{t}_k)$ consisting of $\left\{X_S(t_1), X_S(t_2), ..., X_S(t_k)\right\}$ represents a sequence of system states observed at k inspection time instants. The inspection interval could be either periodical or non-periodical.

(7) Assume that there is no maintenance activity intervening the deterioration process of an MSS.

3. Proposed Dynamic Reliability Assessment Method

3.1 Estimation of Current Status

At any inspection time t_k, the current system state $X_S(t_k)$ along with observation history $\mathbf{X}_S(\mathbf{t}_{k-1})$ of a specific individual system is available. The objective here is to identify the states of components at current time instant by using these sequential system-level inspection data. The associated conditional probability is denoted as:

$$\Pr(X_S(t_k) = S_{i,v} \mid \mathbf{X}_S(\mathbf{t}_k)) = \Pr(X_S(t_k) = S_{i,v} \mid X_S(t_k) = S_i, \mathbf{X}_S(\mathbf{t}_{k-1})) \tag{1}$$

where $v \in L_i$, $k \geq 1$. Eq.(1) represents the probability of the inspected system staying at its state i with the v th combination of components' state given the observation history up to t_k. The state of component l at time instant t_k is denoted as $\mathbf{Q}_{i,v}(l)$ where $1 \leq \mathbf{Q}_{i,v}(l) \leq n_l$. $\Pr(X_S(t_k) = S_{i,v} \mid X_S(t_k) = S_i, \mathbf{X}_S(\mathbf{t}_{k-1}))$ separates the observation history into two portions: the inspection data at time instant t_k and these before t_k. By utilizing the following Bayes' rule, the conditional probability of Eq.(1) can be further expanded as:

$$\Pr(X_S(t_k) = S_{i,v} \mid X_S(t_k) = S_i, \mathbf{X}_S(\mathbf{t}_{k-1})) = \frac{\Pr\{X_S(t_k) = S_i \mid X_S(t_k) = S_{i,v}, \mathbf{X}_S(\mathbf{t}_{k-1})\} \cdot \Pr\{X_S(t_k) = S_{i,v} \mid \mathbf{X}_S(\mathbf{t}_{k-1})\}}{\Pr\{X_S(t_k) = S_i \mid \mathbf{X}_S(\mathbf{t}_{k-1})\}}, \quad (2)$$

It is obvious that the first term in the numerator of Eq.(2) equals to one as event $X_S(t_k) = S_i$ contains $X_S(t_k) = S_{i,v}$. The second term in the numerator of Eq.(2) can be expanded as:

$$\Pr\{X_S(t_k) = S_{i,v} \mid \mathbf{X}_S(\mathbf{t}_{k-1})\} = \sum_{m=1}^{L_j} \Pr\{X_S(t_k) = S_{i,v} \mid X_S(t_{k-1}) = S_{j,m}\} \cdot \Pr\{X_S(t_{k-1}) = S_{j,m} \mid \mathbf{X}_S(\mathbf{t}_{k-1})\}, \quad (3)$$

where $X_S(t_{k-1}) = S_{j,m}$ indicates that at the $(k-1)$ th inspection time, the system is at state j with the m th combination of components' state. The denominator of Eq.(2) can be written as:

$$\Pr\{X_S(t_k) = S_i \mid \mathbf{X}_S(\mathbf{t}_{k-1})\} = \sum_{n=1}^{L_i} \sum_{m=1}^{L_j} \Pr\{X_S(t_k) = S_{i,n} \mid X_S(t_{k-1}) = S_{j,m}\} \cdot \Pr\{X_S(t_{k-1}) = S_{j,m} \mid \mathbf{X}_S(\mathbf{t}_{k-1})\}. \quad (4)$$

By plugging Eqs.(3) and (4) into Eqs.(2) and (1), one has:

$$\Pr(X_S(t_k) = S_{i,v} \mid \mathbf{X}_S(\mathbf{t}_k)) = \frac{\sum_{m=1}^{L_j} \Pr\{X_S(t_k) = S_{i,v} \mid X_S(t_{k-1}) = S_{j,m}\} \cdot \Pr\{X_S(t_{k-1}) = S_{j,m} \mid \mathbf{X}_S(\mathbf{t}_{k-1})\}}{\sum_{n=1}^{L_i} \sum_{m=1}^{L_j} \Pr\{X_S(t_k) = S_{i,n} \mid X_S(t_{k-1}) = S_{j,m}\} \cdot \Pr\{X_S(t_{k-1}) = S_{j,m} \mid \mathbf{X}_S(\mathbf{t}_{k-1})\}}. \quad (5)$$

It is noted that Eq.(5) is a recursive Bayesian formulation in which the probability of the system staying at its state i with the v th combination of components' state at inspection time t_k is a function of the probability of the system staying at its state j with the m th combination of components' state at inspection time t_{k-1} and the probability of the system transiting from the state $S_{j,m}$ to $S_{i,n}$ within the inspection interval $[t_{k-1}, t_k]$. Since we have assumed that at the beginning of operation stage, the system and its components are in brand new condition, the initial condition of Eq.(5) for the recursive process is set to $\Pr\{X_S(0) = S_{N_S, L_{N_S}}\} = 1$.

With the assumption that state transitions of components possess Markov property, that is, the future states which a component will degrade to is only dependent with the present state, but independent with the past states. $\Pr\{X_S(t_k) = S_{i,n} \mid X_S(t_{k-1}) = S_{j,m}\}$ is, therefore, statistically independent with $\Pr\{X_S(t_{k-1}) = S_{j,m} \mid \mathbf{X}_S(\mathbf{t}_{k-1})\}$. If the stochastic behaviors of components are also statistically independent of one another, one has:

$$\Pr\{X_S(t_k) = S_{i,v} \mid X_S(t_{k-1}) = S_{j,m}\} = \prod_{l=1}^{M} \Pr\{X_l(t_k) = \mathbf{Q}_{i,v}(l) \mid X_l(t_{k-1}) = \mathbf{Q}_{i,m}(l)\}, \quad (6)$$

where $X_l(t_k)$ is the state of component l at time instant t_k; $\mathbf{Q}_{i,v}(l)$ is the state of component l in the v th combination of components' states $S_{i,v}$ when the system is at its state i. Obviously, the state transition probability of system is transformed into a product of state transition probabilities of components.

To compute the probability of state transitions of a component, many well-established stochastic models, e.g. Markov model, Petri net etc., can be used. We only briefly review homogenous Markov model which have been extensively adopted in MSS modelling (Lisnianski and Levitin, 2003). If the degradation of components is governed by homogenous Markov model, the transition time between any pair of states of a component is assumed to be exponentially distributed. The probability $\Pr\{X_l(t_k) = \mathbf{Q}_{i,n}(l) \mid X_l(t_{k-1}) = \mathbf{Q}_{i,m}(l)\}$ of component l can be derived by solving the corresponding a set of Kolmogorov differential equations.

By substituting Eq.(6) into (5), the probability of components' states at the current inspection time t_k can be derived by utilizing all the system-level observation history up until t_k. It should be noted that the results will vary from system to system even if they pass through the same states but with different transition time instants. This is the unique feature of our proposed method which is able to track the deterioration of every specific individual system rather than a population of identical systems.

3.2 Prediction of Future Status

If the possible states of components and their associated probability are known at the latest inspection time t_k, the behavior of the components and the specific individual system in the future residual lifetime can be more precisely predicted. This is achieved by considering the component state vector making transition from the current state to the next. If the system is observed at state i at the current inspection time, the probability of the system being in state j ($j \in \{1,2,...,i\}, i \in N_S$) at the end of a specified time span t' can be computed by:

$$\Pr\{X_S(t'+t_k)=S_j \mid \mathbf{X}_S(\mathbf{t}_k)\}=\sum_{m=1}^{L_j}\sum_{n=1}^{L_i}\Pr\left\{X_S(t'+t_k)=S_{j,m} \mid X_S(t_k)=S_{i,n}\right\}\cdot\Pr\left\{X_S(t_k)=S_{i,n} \mid \mathbf{X}_S(\mathbf{t}_k)\right\}. \quad (7)$$

where t' is the time elapsed after t_k. The probability $\Pr\left\{X_S(t'+t_k)=S_{j,m} \mid X_S(t_k)=S_{i,n}\right\}$ can be computed in the same manner as Eq.(6), but the only difference worth noting is to replace t_k and t_{k-1} of Eq.(6) by $t'+t_k$ and t_k, respectively. The probability of component l staying at its state j ($j \in \{1,2,..., \max\limits_{n\in\{1,2,...,L_i\}}(\mathbf{Q}_{i,n}(l))\}$) in the rest of lifetime can be computed by:

$$\Pr\{X_l(t'+t_k)=s_{l,j} \mid \mathbf{X}_S(\mathbf{t}_k)\}=\sum_{n=1}^{L_i}\Pr\left\{X_l(t'+t_k)=s_{l,j} \mid X_l(t_k)=\mathbf{Q}_{i,n}(l)\right\}\cdot\Pr\left\{X_S(t_k)=S_{i,n} \mid \mathbf{X}_S(\mathbf{t}_k)\right\}. \quad (8)$$

where $\max\limits_{n\in\{1,2,...,L_i\}}(\mathbf{Q}_{i,n}(l))$ indicates the highest state of component l among all the combinations of components' states when the system is at state i at inspection time t_k.

Hence, the dynamic reliability function of the system in the future lifetime is expressed as:

$$R(t')=\sum_{j=n_{SF}}^{X_S(t_k)}\Pr\{X_S(t'+t_k)=S_j \mid \mathbf{X}_S(\mathbf{t}_k)\}\,, \quad (9)$$

where n_{SF} is the threshold state that the system getting into a state lower than n_{SF} can be viewed as failure.

4. Numerical Example

The illustrative example of a water piping system introduced earlier is presented here to demonstrate the use of the proposed dynamic reliability assessment method. By using the Markov model and universal generating function (Lisnianski and Levitin, 2003), the system state probabilities over time are computed and depicted in Figure 1 by the dot lines with marks. It should be noted that to avoid confusion, only states 1 to 4 are presented here and will be compared with updated state probabilities after inspection.

Per the proposed method in this work, both the state probabilities of components and systems can be updated dynamically for an individual specific system if the system state could be observed during the lifetime of the system. Suppose a specific individual system is observed in its state 4 at $t_1=0.8$ month after being put into use. However, there are two possible combinations of components' states as shown in Table 2. Based on Eq.(5), one has: $\Pr(X_S(t_1)=S_{4,1} \mid X_S(t_1)=S_4)=0.7778$ and $\Pr(X_S(t_1)=S_{4,2} \mid X_S(t_1)=S_4)$ $=0.2222$, indicating that unit #3 is more likely in its state 2 than state 3 at this moment. The updated state probabilities of the system in the residual lifetime right after $t_1=0.8$ month can be derived via Eq.(7). The updated state probabilities of the system in the residual lifetime are readily assessed and delineated in Figure 1 by the dash lines with marks. If state 1 of the system is viewed as the failure state, i.e. $n_{SF}=1$, the reliability of the system equals to the summation of probabilities of the states greater than state 1. The updated reliability curve of the specific system after incorporating the inspection information at t_1 is plotted in Figure 2 by the dash line.

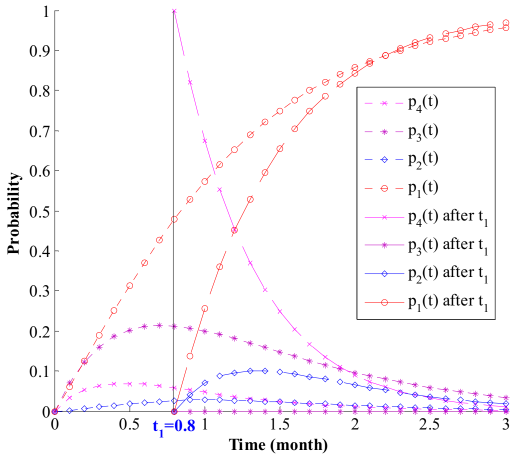

Figure 1: The original and updated state probabilities

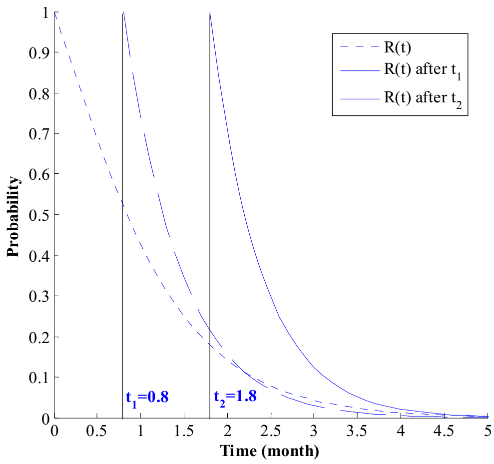

Figure 2: The original and updated reliability

As observed in Figure 2, the updated reliability of the system is greater than the reliability assessed without utilizing the system-level observation over the time span $[0.8, 2.2]$ months; whereas the updated reliability possesses a relatively faster declining trend after 2.2 months. By utilizing the ensuing system-level inspection data, the reliability of the specific system can be further updated in the same manner. Suppose the system is observed in its state 2 at $t_2 = 1.8$ month, the updated reliability of the specific system is plotted in Figure 2 by the solid line.

5. Conclusion

In order to utilize system-level observation history of a specific system to improve the accuracy of reliability assessment, a dynamic reliability assessment method for MSSs is developed in this paper. A Bayesian formula is put forth to recursively identify the possible states of components at inspection time, and further update the reliability function of the system. As demonstrated in our numerical example, by utilizing the system-level observations of a specific system, the reliability function of the system can be dynamically updated so as to provide a more accurate prediction of the remaining useful life of the specific system than that of traditional time-based reliability assessment methods.

Acknowledgement

The authors greatly acknowledge grant supports from the Natural Sciences and Engineering Research Council of Canada (NSERC), the National Natural Science Foundation of China under contract number 71101017, the Specialized Research Fund for the Doctoral Program of Higher Education of China under contract number 20110185120014, and the Fundamental Research Funds for the Central Universities under contract number ZYGX2011J084.

References

Li W., Zuo M.J., 2008, Reliability evaluation of multi-state weighted k-out-of-n systems. Reliability Engineering and System Safety, 93(1): 160-167.

Lisnianski A., Elmakias D., Laredo D., Haim H.B., 2012, A multi-state Markov model for a short-term reliability analysis of a power generating unit. Reliability Engineering and System Safety, 98(1), 1-6.

Lisnianski A., Levitin G., 2003, Multi-state System Reliability Assessment, Optimization, Application. World Scientific, Singapore.

Shrestha A., Xing L., 2008, A logarithmic binary decision diagram-based method for multistate system analysis. IEEE Transactions on Reliability, 57(4), 595-606.

Ushakov I.A., 1986, Universal generating function. Soviet Journal of Computer and Systems Sciences, 24(5), 118-129.

Wang W., Christer A.H., 2000, Towards a general condition based maintenance model for a stochastic dynamic system. Journal of the Operational Research Society, 51(2), 145-155.