

# Reliability Modelling and Simulation of Complex Systems

Yuansheng Lin, Daqing Li\*, Rui Kang

School of Reliability and Systems Engineering, Beihang University, Haidian District, Beijing, China, 100191  
[li.daqing.biu@gmail.com](mailto:li.daqing.biu@gmail.com)

The efficiency of complex technological systems requires the guarantee on their reliability against underlying catastrophes. The complexity encoded in their structure and functions makes cascading failures the main failure mode in complex systems. The development of complex systems' reliability technology relies on the modelling and simulation of the cascading failure. For the complex system, it is difficult to perform macroscopic analysis with tools based on probability theory because of their numerous system states. Meanwhile, it is also difficult to analyse it completely by microscopic system details due to their nonlinear coupling characteristics. The properties of complex systems require systematic analysis in multi-level. As the network science becomes available to model and study the complex system, its underlying concept of statistical physics is suitable to understand the complex system from the relationship between macroscopic properties and microscopic activities. We will review the progress made by network science recently including latest formalism of interdependent network theory, which can be used to understand and study the reliability problem of complex system.

## 1. Introduction

With the complex structure and nonlinear coupling, the study on reliability of complex systems is facing the challenge that can be hardly found in the past [Enrico Zio, 2009]. The emergence of complicated failure behaviour makes it difficult to quantify, analyse and improve the reliability of these complex systems. For example, when most of the instability seems to be negligible in power grid, under certain condition, the failures may spread globally over the grid and extreme events happen more frequently than expected from traditional risk analysis.

Reliability technology of complex systems requires the understanding of the failure behaviour and suitable modelling methods. Due to the intrinsic multi-level coupling in complex systems, cascading failure becomes the main failure mode in many realistic systems including power grid and transportation systems. Cascading failure is a highly correlated failure process in a system of interacted components, where the failure of one component can trigger the failure of the others. When balance of the system is broken, reorganization of system's functions (load distribution) and large scale of cascading failures will emerge [Billinton *et al.*, 1996]. For example, in power grid systems, if one element fails, it will redistribute its load to neighbours [Glanz J. *et al.*, 2003]. For these neighbouring elements, the accumulation of loads may exceed their capacity so that they become overloaded and failures spread recursively. Finally, the system will collapse and large areas of outages happen, which causes damages up to billion [NERC, 2002]. In the following, we will review the progress made in understanding the reliability of complex systems and especially on the research on the cascading failures based on network science.

## 2. Study of complex systems reliability on single scale

The development of complex systems' reliability technology relies strongly on their modelling and simulation. The complex systems have been modelled mainly in two scales: system scale (macroscopic) and individual scale (microscopic). While macroscopic methods study the system as a whole based on probability theory and system dynamics, microscopic analysis begins with the attributes of individuals and their interactions.

## 2.1 Modelling and Simulation in system scale

From the macroscopic point of view, many used mathematical equations to characterize the dynamical behaviour of complex system, to predict its evolution and adaptation under different boundary conditions.

In the study of reliability of power system, so called branching process model, with the generation rate of the failure, is aiming to describe the cascading process by the concept from statistical process [Dobson *et al.*, 2004]. In branching process model, processes of cascading failure have been considered as branching of failures in consecutive steps, where the branching factor controls the criticality between global and local cascades. Above the criticality in the branching process, cascading failures will occur in a system scale, which will break down the whole system. As for the reliability of transportation systems, the traffic flow at macroscopic level has been studied and modelled as physics of fluid streams. Traffic flow on long crowded roads is first studied by Lighthill M. and Whitham G. at the macroscopic level [Lighthill M.*et al.*, 1955], which has been improved by considering the shock-waves on highways [Richards, 1956].

Neglecting system details of structure and functions, macroscopic models are difficult to reflect the system's inherent failure mode including the spatial spreading pattern of failures, and may even ignore system's potentially high-risk failure state [Helbing D., 1996].

## 2.2 Modelling and Simulation in individual scale

Modelling in individual scale counts the emergence of system complexity mainly on the microscopic individual states and their interaction mechanisms. The modelling methods include mainly ABMS (agent-based modelling and simulation) [Holland J.H. *et al.*, 1991], especially cellular automata (CA) [Neumann J., 1966]. The agent based modelling is aiming to produce the macroscopic complexity phenomenon out of a set of interacting agents in the individual scale. The principle of interactions between agents is usually "simple and short". Since its earliest model, so called cellular automata, was developed, the agent based modelling combined with Monte Carlo methods has been applied into many fields including sociology, life science, economics and engineering. The nature of these methods depends on the characteristics of learning, adaption and actions embedded in the agents, which is used to capture the essence of the nonlinear coupling and self-organized mechanism of complex systems. While macroscopic approaches analyse complex systems in top-down way, agent based modelling is trying to disintegrate the systems bottom-up and reproduce the emergent system behaviour as a result [Bonabeau E., 2002, Kröger W. *et al.*, 2011].

In transportation systems, for example, these modelling in individual scale consider the traffic flow of vehicles as dynamics of molecules through the modelling of microscopic features of traffic flow include statistics of the position and the velocity of vehicles. Bando *et al.* have introduced the so-called optimal velocity model (OV) [Bando *et al.*, 1995]. Nagel-Schreckenberg model has been developed to explain traffic jams based on cellular automata [Nagel K. *et al.*, 1992].

## 3. Application of network science on the study of complex systems reliability

Although the methods above are successful in describing and modelling the system in some cases, one critical attribute of system is missing: with whom the component (individuals) of system interacts? The structure of complex system can be hardly ignored when different system structures have been found to shape the system behaviour in a large extent. Complex network theory [Albert R. *et al.*, 2002, Boccaletti S. *et al.*, 2006, Cohen *et al.*, 2010, Newman M.E.J, 2003, Pastor-Satorras R. *et al.*, 2001], based on the concept of statistical physics, aims to understand the structure of complex system and can be used to study the reliability of complex system. Cascading failures of complex systems can be analysed and explored by representing the complex system as networks and visualizing the correlation between failures. Complex network theory has proved its validity and potentials in the study of many complex systems such as engineering, social, and biological systems: ' Scientists use networks or graph to represent the complex networks, and this way of abstraction has proved to be useful, and help people to understand the structures of many real-world systems. Meanwhile, the dynamic processes and the interaction between them started to be analysed. We could discover their structural patterns and complexity and to certify the fundamental theories are basically right.' [Science, 2009].

In 1999, since the beginning of network science, error and attack tolerance of complex networks has been studied, showing that scale-free networks are robust to random failure, but vulnerable to intentional attack [Albert *et al.*, 2000, Cohen R. *et al.*, 2000, Cohen R. *et al.*, 2001]. Due to the findings of scale-free network

structure in many realistic engineering system including transportation system and Internet, the robustness properties of scale-free networks have been used to understand and study the reliability of different complex systems. It is suggested that nodes (components) with the highest degrees (hubs) originated from system heterogeneity plays a critical role in integrating the whole system.

However, the hubs could become the vulnerable part of network when cascading failures have to be considered. Motter [Motter *et al.*, 2002] introduced a model studying cascading failures, which could be applied to many realistic flow networks for transmitting energy or information. In this model, an initial load value (betweenness) has been assigned to each node based on routing strategy of shortest path length, with node capacity proportional to  $(1+\alpha)$  times its load. As some node failed, the load of this node will be redistributed to other nodes as a result of rerouting. When the updated load exceeds one's capacity, the failure begins to spread and may continue in a global scale. It has been found that even single removal of node with highest degree or betweenness will make the whole network face the risk of full collapse. This risk has been manifested strongly in scale-free network, while the homogeneous network is less vulnerable for the absence of hub. Furthermore, Crucitti *et al.* [Crucitti *et al.*, 2004] have proposed a special model, which suggests also that the attack on single node is sufficient to break down the entire system. The weighted network is considered in this model, where the weight can be considered as the efficiency of communication link. The model is different from Motter's model in two aspects: overloaded nodes are not removed from the network; the network is characterized by a quantity named efficiency based on the load and capacity value. The damage based on random removal and intentional removal of node with highest load has been studied. These findings could shed some light on the design of network faced with requirement of high reliability.

Moreno Y. *et al.* [Moreno Y. *et al.*, 2002] studied the robustness of scale-free network based on the fiber bundle model (FBM) [Herrman H.J. *et al.*, 1990, Y. Moreno *et al.*, 2000]. In fibre bundle model, each node has been given an identical load value, while the threshold for each node has been drawn from a given distribution. Different from Motter's model, the overload here will only spread to the neighbours and the cascading of failures depends on the local properties instead of global properties induced in Motter's model. The criticality of model has been deeply explored by measuring the size of giant component and avalanche size distribution. An abrupt phase transition has been found in the study of the size of giant component as a function of node load, which implies difficulties in controlling of cascading failures. It is observed that during the cascading failure process, the nodes with large degree become failed much easier, which signifies the vulnerability of hubs.

Later, Moreno Y. *et al.* [Moreno *et al.*, 2003] proposed so called threshold model aiming at description of instabilities from different overload mechanisms. The overload is modelled on the link instead of node. There are three different mechanisms of load redistribution during overloads: deterministic redistribution, random redistribution and dissipative redistribution. The critical point and cluster distribution in the phase transition process has been studied with these redistribution mechanisms.

Sansavini G, *et al.* [Sansavini G, *et al.*, 2009] has studied the cascading failures induced by addition of load, instead of structural removal. They introduce global efficiency, which is inversely proportional to distance between each pair of nodes. It is found that the maximal global efficiency is not the only goal to peruse for the network provider if reliability has been taken into consider. Lower global efficiency will increase the network resilience against cascading failures.

#### 4. Interdependent network modelling

Network science mostly assumes no interactions between different networks. In reality, many complex systems, like computer networks and power grid, cannot function solely by themselves. Network of networks (NON) theory becomes critical to understand the structure, function and vulnerability of these interconnected systems [Gao J. *et al.*, 2011].

In 2010, noticing catastrophe of the electrical blackout in Italy 2003 [Rosato, V. *et al.*, 2008], Buldyrev *et al.* have proposed a mathematical framework to deal with the cascading failures between interdependent networks [Buldyrev *et al.*, 2010]. In interdependent network, nodes are connected by dependency link between networks and will function if the corresponding node in another network belongs to its giant component. Therefore, failure of some nodes in one network will cause the failures in another network. Cascading failures happen when this process continues recursively as a result of interdependence between networks and percolation inside one network. This framework can solve the system state during and after the cascading failures using generating function. They have found that the interdependent scale-free networks are more fragile than interdependent ER networks under random removal. This finding

broadens the understanding of network reliability, which centered on the assumption that the scale-free network is always more robust than ER network under random removal.

#### **4.1 Network with connectivity links and dependency links**

In the previous modelling, nodes between networks are considered mutually connected only by one type of links, dependency link. Meanwhile, connectivity links exist between many realistic interdependent systems such as interdependent transportation systems. Dependency links bind failures of nodes in one network to failures of corresponding nodes in another network, while connectivity links provide only functional support between nodes without failure correlations. Parshani R. *et al.* [Parshani R. *et al.*, 2010] provide an analytical framework to study the robustness of this type of interdependent networks, by extending the framework developed by Buldyrev. The ratio of dependency links has been found to play an important role in deciding the phase transition type of cascading failures. For high concentration of dependency links, the system undergoes a first-order transition of cascading failures; for low concentration of dependency links, the network disintegrates under a smooth change as a second-order transition during the cascading failures. It is found that a broader degree distribution leads to vulnerability when both connectivity links and dependency links are presented. Bashan *et al.* [Bashan *et al.*, 2011] analytically studied the effect of distribution of dependency links on the robustness of networks. Based on the concept of dependency cluster, Bashan successfully generalizes the known Erdős-Rényi equation and the result of Parshani for different dependency cluster size.

#### **4.2 Network of Networks**

Complex systems are forming the interdependency in the manner of network such as the dependency network composed of water or energy supply system, communication system, and transportation system. Since each complex system can be modeled by a network, the interaction of these complex systems can be understood by the formulism of “network of network” (NON) developed by Gao J. *et al.* [Gao J. *et al.*, 2011]. For a NON system composed of  $n$  coupled randomly connected networks, they extend and generalize the classical percolation result for single network and result of Parshani *et al.* and show that for  $n = 1$  the percolation transition is second order as in the classical case, for  $n > 1$  first order transition occurs with the presence of cascading failures. Their results have proved that the classical percolation theory is the extreme case of general theory for NON. It is found that the network vulnerability will significantly increase with number of networks coupled increases. A minimum degree has been found critical for the robustness of network of ER networks, below which the system will collapse under removal of any finite number of nodes. Dynamics of cascading failures induced by initial failure in the NON systems have been studied and compared [Gao, J. *et al.*, 2011], where Random Regular NON is found to be more robust than ER NON. Specifically, for any  $n > 1$ , cascades of failures will occur as well as the first order transition compared to a second order transition in the classical percolation of single network. Cascading failures, increasing with number of coupled networks  $n$ , will make the NON systems vulnerable significantly.

## **Conclusion**

Although cascading failures, as the pervasive and harmful failure mode in complex systems, have been studied in many model and realistic systems, the reliability technology of complex systems can be hardly built with it. Based on the understanding of cascading failures from network theory, how to quantify, calculate and design the reliability of complex system remain fundamental questions. As the interdependency increased as a result of modern ICT technology, further steps towards the development of reliability technology on specific complex systems have to be made.

## **Acknowledgement**

Daqing Li is supported by National Basic Research Program of China (Grant No. 2012CB725404) and National Natural Science Foundation of China (Grant No. 61104144).

## References

- Albert R., Barabási A. L., 2002, *Statistical Mechanics of Complex Networks*, the American Physical Society, Rev. Mod. Phys. 74, 47–97.
- Albert R., Jeong H., Barabási A. L., 2000, Error and attack tolerance of complex networks, *Nature* 406, (378-382).
- Bando M., Hasebe K., 1995, Dynamical Model of traffic congestion and numerical simulation, *Phys. Rev. E* 51, 1035-1042.
- Bashan A., Parshani R., Havlin S., 2011, Percolation in networks composed of connectivity and dependency links, *Physical Review* 83, 051127.
- Billinton R., Allan R.N., 1996, *Reliability evaluation of power systems* (2nd ed.), New York: Plenum Press.
- Boccaletti S., Latora V., Moreno Y., and Chavez M., Hwang D-U., 2006, *Complex networks: structure and dynamics*, *Physics Reports*, 424, (175-308).
- Bonabeau E., 2002, Agent-based modeling: Methods and techniques for simulating human systems, *PNAS*, doi: 10.1073/pnas.082080899, vol.99 no. Suppl 3 7280-7287.
- Buldyrev S. V., Parshani R., Paul G., Stanley H. E., Havlin S., 2010, Catastrophic cascade of failures in interdependent networks, *Nature* 464, (08932).
- Cohen R., Erez K., ben-Avraham D., Havlin S., 2000, Resilience of the Internet to random breakdown, *Physical Review letters* 85,4626.
- Cohen R., Erez K., ben-Avraham D., Havlin S., 2001, Breakdown of the Internet under intentional attack, *Physical Review letters* 86,3682.
- Cohen R., Havlin S., 2010, *Complex Networks: Structures, Robustness and Function*, Cambridge University Press.
- Complex Systems and Networks*, 2009, *Science*, 325, (357-504).
- Crucitti P., Latora V., Marchiori M., 2004, Model for cascading failures in complex networks, *Phys. Rev. E* 69, 045104(R).
- Dobson I., Carreras B.A., Newman D.E., January 2004, A branching process approximation to cascading load-dependent system failure, 37th Hawaii international conference on system sciences, IEEE.
- Enrico Zio, 2009, Reliability engineering: Old problems and new challenges, *Rel. Eng. & Sys. Safety* 94(2): 125-141.
- Gao J., Buldyrev S.V., Havlin S., Stanley H. E., 2011, Networks formed interdependent networks, *Nature Physics*, DOI: 10.1038/nphys2180.
- Gao J., Buldyrev S.V., Havlin S., Stanley H. E., 2011, Robustness of a network of networks, *Phys. Rev. Lett.* 107, 195701.
- Gao J., Buldyrev, S.V., Havlin, S., Stanley H. E., 2011, Robustness of a Tree-like Network of Interdependent Networks, *Phys. Rev. E* 85, 066134.
- Glanz J., Perez-Pena R., 2003, 90 Seconds That Left Tens of Millions of People in the Dark, *New York Times*.
- Helbing D., 1996, Gas-kinetic derivation of Nacier-Stokes-like traffic equations, *Physical Reviews*, Vol. E53, no.3, 2266-2381.
- Herrman H.J., Roux S.(Eds.), 1990, *Statistical Models for the Fracture of Disordered Media*, North-Holland, Amsterdam.
- Holland J.H., Miller J.H., 1991, Artificial Adaptive Agents in Economic Theory, *American Economic Review* 81 (2): 365–71.
- Kröger W., Zio E., 2011, *Vulnerable Systems*, Springer-Verlag (London) .
- Lighthill M., Whitham G., 1955, on On kinematic waves II: A theory of traffic flow on long, crowded roads, *proceedings of the Royal Society of London Ser. A* 229, 317-345.
- Moreno Y., Gomez J.B., Pacheco A.F., 2000, Fracture and Second- Order Phase Transitions, *Physical Review Letters* 85, 2865.
- Moreno Y., Gomez J.B., Pacheco A.F., 2002, Instability of scale-free networks under node-breaking avalanches , *Euro-physics Letters* 58(4), (630).
- Moreno Y., Pastor-Satorras R., Vazquez A., Vespignani A., 2003, Critical load and congestion instabilities in scale-free networks , *Euro-physics Letters* .62, 292.
- Motter A.E., Lai Y.C., 2002, Cascade- based attacks on complex networks, *Phys. Rev.* E66, 065102.
- Nagel K., Schreckenberg M., 1992, A cellular automaton model for freeway traffic, *J.Phys.I France* 2, 2221-2229.
- NERC (North American Electric Reliability Council), 2002, 1996 system disturbances, NERC, Princeton, USA.
- Neumann J.V, 1966, *The Theory of Self-reproducing Automata*, Univ. of Illinois Press, Urbana And London.

- Newman M.E.J, 2003, The structure and function of complex networks. , SIAM Rev.45, 167–256
- Parshani R., Buldyrev S., Havlin S., 2010, Critical effect of dependency groups on the function of networks, Proc. Natl. Acad. Sci. USA 108, 1007.
- Pastor-Satorras R., Vespignani A., 2002, Epidemic dynamics in finite size scale-free networks, Phys.Rev.E 65, 035108(R).
- Richards P.I., 1956, Shock waves on the highway, Operations Research 4, 42–51.
- Rosato V., Issacharoff L., Tiraticco F., Meloni S., Porcellinis S., Setola R., 2008, Modelling interdependent infrastructures using interacting dynamical models, Int. J. of Critical Infrastructure Vol.4, 63–79.
- Sansavini G., Hajj M,R,, Puri I,K,, Zio E., 2009, A Deterministic Representation of Cascade Spreading in Complex Networks. , Europhysics Letters, 87: 48004.