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On-line Updating of Dynamic State-Space Model for Bayesian Filtering through Markov chain Monte Carlo Techniques

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A large number of methodologies dedicated to the continuous monitoring of systems have been developed during the last years. Among these, the model-based Bayesian Filtering methods (e.g. Particle Filters, PF) are able to combine the information provided by a monitoring system with the mathematical models describing the observed phenomena, providing advantages in terms of safety and reliability of the monitored systems. The analytical models of the phenomena are integrated in the Dynamic State Space (DSS) model of the Particle Filter. The DSS model consists of a stochastic evolution equation linking the current state vector with the state vector at the previous (discrete) time step. It is common practice to consider deterministic DSS parameters inside the PF algorithm, with an additional Gaussian or non-Gaussian noise to account for all the system uncertainties and the DSS model remains the same even after the usual procedure of resampling. This often provides a poor description of actual system dynamics. An Adaptive Dynamic State Space model is proposed here in order to overcome this problem. The Adaptive DSS model is built with the prior probability density function of parameters available in literature, and it uses the information provided by the measurement system to update the parameter distributions during the system operation. This distribution updating is obtained through the Markov Chain Monte Carlo (MCMC) techniques for the parameter estimation. The Particle Filtering algorithm based on Adaptive Dynamic State Space model is applied to a Fatigue Crack Growth (FCG) on metallic structures.

1. Introduction

Literature about Bayesian Filtering gains more and more interest inside the field of reliability and availability of complex systems. Even though the number of methods dedicated to fault detection and prognosis recently increases, their applicability is limited because of the difficulties can be encountered in real applications. The modelling of complex systems, the uncertainties of actual cases and the all possible evolutions of the system degradation are not easy to solve outside the simulated environment. The study of numerical methods and especially model-based filtering techniques remains an open field owing to these difficulties. Starting from Kalman filter providing the optimal solution for linear problems subjected to random Gaussian noise, the methods evolved up to advanced filters founded on Bayes' rule and Monte Carlo Sampling (MCS) techniques for highly-nonlinear problems with non-Gaussian noise. Haug (2005) and Arulampalam et al. (2002) have reported a useful explanation of some different tracking techniques based on Bayesian inference, starting from KF up to Particle Filters (PF) for whatever nonlinear problem. In addition, the scientific community handles the problem of parameter estimation of complex systems in parallel. Markov Chain Monte Carlo (MCMC) methods such as the Metropolis-Hastings (MH) algorithm (Hastings, 1970), or the Metropolis Adjusted Langevin Algorithm (MALA) (Roberts et al., 1996) are used to estimate the parameter of a whatever (linear or nonlinear) model. A basic advantage of these methods is the capability to produce the Probability Density Functions (PDFs) of the estimated parameters, without restrictions about the shape of these PDFs. An Adaptive Dynamic State Space model applied in a Particle Filtering algorithm is proposed here thanks to these numerical methods. The DSS model initially built with

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the prior (known) densities of the law parameters (as proposed by the authors, Corbetta et al., 2013), is updated through the information obtained by the observations on the monitored system. The main advantage of the method is the applicability in on-line continuous monitoring systems for prognostic and lifetime prediction. The results will be analysed in terms of prognostic capabilities within a simulated fatigue crack growth tests. Since the mathematical tools contained in this work have been examined in-depth from a theoretical viewpoint by Corbetta et al. (2013), the work is focused on the applicability of these techniques, introducing some characteristics typical of a real environment.

2. Basic theory of Particle Filtering

The literature and the theoretical treatments of Monte Carlo Sampling, Bayesian Filters and MCMC methods are wide and another paper about mathematics of these techniques would be useless. Therefore, only a brief summary of Particle Filtering is given. The interested reader can refers to these authors for the in-depth treatment: Doucet et al. (2001) for Monte Carlo methods, Ochard et al. (2009) for PF algorithm, Roberts and Rosenthal (2001) for MCMC and MH algorithms.

2.1 Summary of Particle Filtering

Particle Filtering is a technique to implement a recursive Bayesian Filter by Monte Carlo simulation and it is based on Sequential Importance Sampling/Resampling (SIS / SIR) technique. The system evolution is represented by a series of possible trajectories or particles $\boldsymbol{x}_{i,k}$ based on the numerical and empirical available knowledge on the system evolution and the current observation on the system $\boldsymbol{z}_k$ (often called the measure). Starting from the weighted particles $\{\boldsymbol{x}_k, w_k\}_i$ it is possible to build the conditioned posterior density of the state given the observations $p(\boldsymbol{x}_k|\boldsymbol{z}_{0:k})$ (Cadini et al. 2009). A brief description of weight calculation is showed in Corbetta et al., 2013; instead see Haugh (2005) for a detailed mathematical dissertation of the method.

2.2 Stochastic Dynamic State Space model for model-based filtering

Consider a system state indicated in the previous subsection as $\boldsymbol{x}_k$ evolves in time observing a particular mathematical law of the phenomenon. This law depends on parameters $\boldsymbol{\vartheta} = [\vartheta_1, \vartheta_2, \dots, \vartheta_n]$. The DSS model links the state $\boldsymbol{x}_{k-1}$ with the subsequent $\boldsymbol{x}_k$ thanks to the physic law and an artificial-added random noise needed to produce a stochastic process. In the PF literature the equations describing the phenomenon are built with deterministic parameters. However, a Stochastic DSS (SDSS) is proposed by the authors in Corbetta et al. (2013). The SDSS is defined with a statistical description of the parameters governing the model equation. In particular, the prior knowledge of model parameter PDFs is inserted inside the PF algorithm. The aim of this technique is to involve the prior knowledge about the uncertainties of the observed quantity within the standard DSS model. In particular, the outperforming of this technique with respect to the traditional DSS for lifetime predictions is emphasized in Corbetta et al., (2013). The Stochastic DSS is shown below:

1. Starting from step 0 (assuming known the quantities $\mathbf{x}_{-1}$):
   - Initialize $\mathrm{N_s}$ particles $\mathbf{x}_{\mathrm{i,0}} \sim \mathrm{IDF}\left(\mathrm{E}(\mathbf{x}_{\mathrm{i,0}}), \mathrm{V}(\mathbf{x}_{\mathrm{i,0}})\right)$ and their weights $\mathrm{w_{i,0}} = \frac{1}{\mathrm{N_s}}\ \forall\, \mathrm{i} = 1 : \mathrm{N_s}$
   - Draw $\mathrm{N_s}$ samples from multivariate PDF of parameters $\boldsymbol{\vartheta}_\mathrm{i} \sim \mathrm{MVPDF}(\mu_{\boldsymbol{\vartheta}}, \mathrm{COV}(\boldsymbol{\vartheta}))$
   - Assign at each particle one parameter sample $\mathbf{x}_{\mathrm{i,0}} \rightarrow \mathbf{x}_{\mathrm{i,0}}(\boldsymbol{\vartheta}_\mathrm{i})$ and the weights $\mathrm{w_{i,0}} = \mathrm{p}(\boldsymbol{\vartheta}_\mathrm{i})$, $\forall\, \mathrm{i} = 1 : \mathrm{N_s}$
2. Run PF algorithm:
   - Sequential estimation of $\mathrm{p}(\mathbf{x}_\mathrm{k}|\mathbf{z}_{0:\mathrm{k}})$ according to normalized weights (equation 7, 8 and 9).
3. If resampling is required at general $\mathrm{k^{th}}$ step:
   - $\mathrm{N_s}$ particle resampling $\mathbf{x}_{\mathrm{i,k}} \sim \mathrm{IDF}\left(\mathrm{E}\left(\mathbf{x}_{\mathrm{i,k}}(\boldsymbol{\vartheta}_\mathrm{i})\right), \mathrm{V}\left(\mathbf{x}_{\mathrm{i,k}}(\boldsymbol{\vartheta}_\mathrm{i})\right)\right)$ according to some resampling method
   - Re-draw $\mathrm{N_s}$ samples from multivariate PDF of parameters $\boldsymbol{\vartheta}_\mathrm{i} \sim \mathrm{MVPDF}(\mu_{\boldsymbol{\vartheta}}, \mathrm{COV}(\boldsymbol{\vartheta}))$
   - Re-assign parameter samples at each particle $\mathbf{x}_{\mathrm{i,0}} \rightarrow \mathbf{x}_{\mathrm{i,0}}(\boldsymbol{\vartheta}_\mathrm{i})$ and the weights $\mathrm{w_{i,0}} = \mathrm{p}(\boldsymbol{\vartheta}_\mathrm{i})$, $\forall\, \mathrm{i} = 1 : \mathrm{N_s}$
   - Go to step 2.

The SDSS describing the evolution of the system for the i^{th} sample at general k^{th} time step assumes the notation in (3). The parameter dependence persists because the parameters of the i^{th} particle remain the same during all the particle life. Thus the evolution of the general i^{th} particle is modified according to the noise statistics only.

$$\boldsymbol{x}_{i,k}(\boldsymbol{\vartheta}_i) = f_k\left(\boldsymbol{x}_{i,k-1}(\boldsymbol{\vartheta}_i), \boldsymbol{v}_{k-1}\right) \tag{3}$$

This novel approach is at the basis of the present work. As a matter of fact, the prior knowledge of parameter PDFs will be updated thanks to the observation vector $\boldsymbol{z}_{0:k}$ through a particular MCMC algorithm.

3. Updating of DSS model

A series of measurements are collected during the system operations. MCMC techniques allow estimating the parameter of whatever mathematical model with whatever distribution by the sequence of noisy observations. Moreover, MCMC techniques are able to estimate the noise associated to these measures. Metropolis-Hasting (MH) algorithm is one of the widely adopted methods to estimate model parameters. However, it has been shown that the variance of the proposal distribution from which draw samples affects the performance of the chain (Roberts et al., 2001). This is a large limitation for the on-line application of the algorithm, because a good variance for the proposal distribution has to be previously selected. Thus, an Adaptive Proposal Metropolis-Hastings (APMH) algorithm developed by Haario et al. (1998) is proposed here to overcome the problem. As a matter of fact, the basic idea of the APMH algorithm is to adapt the variance of the proposal PDF according to the residuals of the chain. Since this technique introduces a correlation among the different samples of the chain the obtained PDF is slightly biased, but this bias can be neglected in most cases (Haario et al., 1998). Moreover, Haario et al. (2001) demonstrate the ergodicity of the chain.

3.1 Adaptive Proposal Metropolis-Hastings

Let us consider at least H points of the chain sampled at time t during the MH operation. It is possible to build the matrix $[K(\boldsymbol{\vartheta})] \in \mathbb{R}^{H\times d}$ where H is the number of the last samples of the chain $\{\boldsymbol{\vartheta}_{t-H+1}, \boldsymbol{\vartheta}_{t-H+2}, \dots, \boldsymbol{\vartheta}_t\}$ and it is called the memory parameter, while d is the number of parameters to estimate. So the matrix $[K(\boldsymbol{\vartheta})]$ contains H samples of the parameter vector $\boldsymbol{\vartheta} \in \mathbb{R}^{1\times d}$. The matrix $[\widetilde{K}(\boldsymbol{\vartheta})]$ contains the residuals of the chain (4) and it is used to build the covariance matrix of the proposal (5).

$$\left[\widetilde{K}(\boldsymbol{\vartheta})\right] = [K(\boldsymbol{\vartheta})] - E([K(\boldsymbol{\vartheta})]) \tag{4}$$

$$COV(\boldsymbol{\vartheta}) = \frac{c_d^2}{H-1}\left[\widetilde{K}(\boldsymbol{\vartheta})\right]^T\left[\widetilde{K}(\boldsymbol{\vartheta})\right] \tag{5}$$

The scaling factor c_d depends only on the number of parameters to estimate d (Gelman et al., 1996). The proposal updating can be done every $U \geq H$ samples, where U is the frequency parameter. U and H are taken equal in this work. Thanks to this algorithm, the necessity to tune the variance of the proposal PDF to produce good results disappears. However, the parameters U, H and the number of samples have to be properly selected (Haario et al., 1998). Considering a set of parameters normally distributed, the consolidated MH algorithm becomes:

1. Initialize $\boldsymbol{\vartheta}_* = \boldsymbol{\vartheta}_0$ and $\mathrm{COV}(\boldsymbol{\vartheta}) = \mathrm{COV}(\boldsymbol{\vartheta}_0)$
2. for $\mathrm{i} = 1 : \mathrm{N_s}$
   - Draw a sample $\boldsymbol{\vartheta}_\mathrm{i} \sim \mathrm{MVNPDF}\left(\boldsymbol{\vartheta}_*, \mathrm{COV}(\boldsymbol{\vartheta})\right)$
   - Accept the sample ϑ_i with probability $\alpha = \min\left(1, \frac{\pi(\boldsymbol{\vartheta}_\mathrm{i}|\boldsymbol{z}_{0:k}) \cdot \mathrm{f}_{\boldsymbol{\Theta}}(\boldsymbol{\vartheta}_\mathrm{i})}{\pi(\boldsymbol{\vartheta}_{\mathrm{i}-1}|\boldsymbol{z}_{0:k}) \cdot \mathrm{f}_{\boldsymbol{\Theta}}(\boldsymbol{\vartheta}_{\mathrm{i}-1})}\right)$
   - If ϑ_i is accepted → $\boldsymbol{\vartheta}_* = \boldsymbol{\vartheta}_\mathrm{i}$, else → $\boldsymbol{\vartheta}_* = \boldsymbol{\vartheta}_{\mathrm{i}-1}$
   - If remainder of i/U is null
     - Build the $[\mathrm{K}]$ matrix and the residuals $\left[\widetilde{\mathrm{K}}\right]$
     - Update the covariance matrix of the proposal $\mathrm{COV}(\boldsymbol{\vartheta})$

   end for
3. Extract a vector of uncorrelated samples of the chain to produce the parameter PDFs.

Nevertheless, this algorithm can be applied to parameters with whatever distributions. The prior knowledge of parameter PDFs represented by $f_{\boldsymbol{\Theta}}(\boldsymbol{\vartheta})$ on the pseudo-code guarantees the refusing of improbable parameter values when few observations are available. The estimated average values of the posterior parameter PDFs become the prior values $\boldsymbol{\vartheta}_0$ at the next iteration besides.

3.2 Particle Filtering with Adaptive DSS

During the PF operation, the well-known degeneracy phenomenon of the weights appears after few measures, so a resampling technique is required. There are many resampling methods and they can be applied at every iteration (SIR) or when the number of effective particles (i.e. particles with weights markedly different from zero) drops under a pre-determined threshold (called On-Demand Resampling here). Thus, the APMH algorithm can be run during the resampling procedure. The APMH algorithm estimates the parameter distributions of the law prior the resampling operation. The new parameter PDFs

will be used inside the Stochastic DSS according to the methodology proposed by Corbetta et al. (2013). This algorithm allows reducing the uncertainties about the parameter of the DSS model in real time. So, the sequential samples of the system state $\boldsymbol{x}_{i,k}$ self-adjust their trajectories improving the prediction performance. Figure 1 shows a scheme about PF with APMH operating during the resampling.

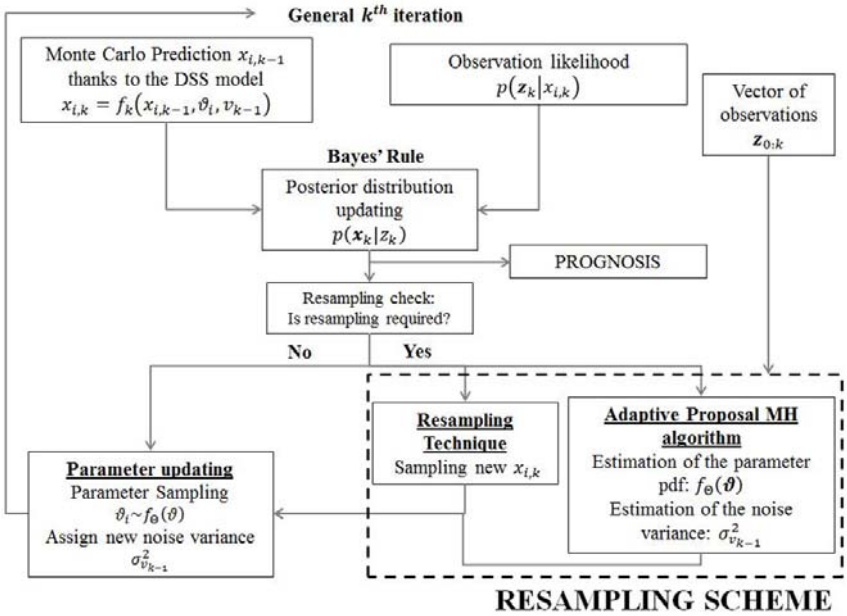

Figure 1: Scheme of PF with sequential updating of the parameter PDFs through APMH algorithm.

4. Application of Adaptive Particle Filters into a simulated fatigue crack growth

Let us consider a simulated thin plate subjected to Fatigue Crack Growth (FCG) problem. The algorithm presented in the above sections is used to estimate the residual lifetime of the plate now, starting from $1.8\ mm$ centre-crack and considering a failure crack length of $140\ mm$. The Residual Useful Life (RUL) of the system is calculated as the difference between the estimated time to reach the failure crack length and the current time. The crack dynamic is well-described by NASGRO equation (6), representing the state of the art of lifetime prediction for structural components (NASA J. S. Centre, 2002). The DSS model of the crack evolution is built through equation (6) and artificial noise defined by a log-Normal distribution v_{k-1}, considering small increment of cycles ΔN directly related to the time operation (7), (Corbetta et al., 2013).

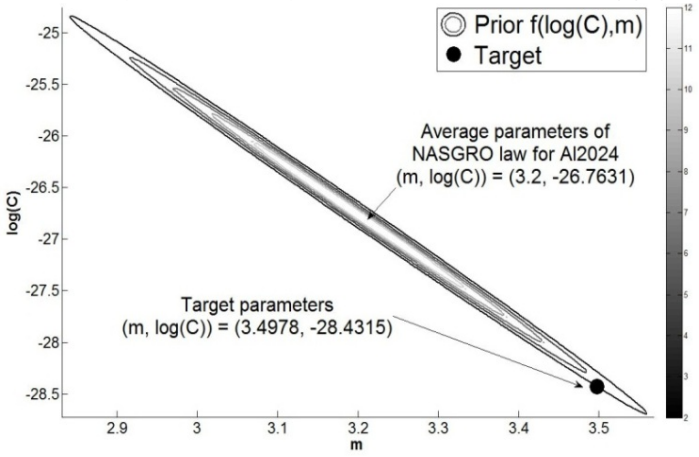

Figure 2: Multivariate normal distribution for DSS parameters. Prior PDF and target values used to simulate the FCG

$$\frac{da}{dN} = C\left(\frac{1-f}{1-R}\Delta K\right)^m \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{max}}{K_c}\right)^q} \tag{6}$$

$$a_k = a_{k-1} + 2\,\Delta N\,\frac{da}{dN}\,v_{k-1} \tag{7}$$

The measurement system is simulated through normally-distributed measures with an uncertainty of $\pm 1\ mm$ centred to the exact crack length (measure variance: $\sigma_v^2 = 0.11$). The measures are provided to the algorithm every 2000 load cycles up to 70 mm crack length. The parameters C and m are random variables; their mean is set according to the aluminium Al2024 parameters for NASGRO equation, while their covariance matrix is set according with Virkler's results (Virkler et al., 1978). The parameters are sampled from these initial distributions and associated to the samples of the crack length $a_{i,k}$ (Corbetta et al., 2013).The PDFs to draw the parameter samples and the noise variance σ_v^2 are updated through the APMH algorithm during the resampling stage thanks to the available measures. This technique allows accounting for all the possible evolutions of the crack firstly, and then filtering the most probable evolution in time improving the RUL prediction. In order to highlight the prediction capability of the algorithm, the parameters C and m used to draw the FCG simulation are selected far away from the average of the prior parameter PDF inserted in the Adaptive PF. In particular, $C = 4.4912e-13$ and $m = 3.4978$ instead of the averages $C = 2.382e-12$; $m = 3.2$. Figure 2 shows the prior combined multivariate normal distribution of $\log(C)$ and m parameters for the Al2024 alloy and the values used during the simulation.

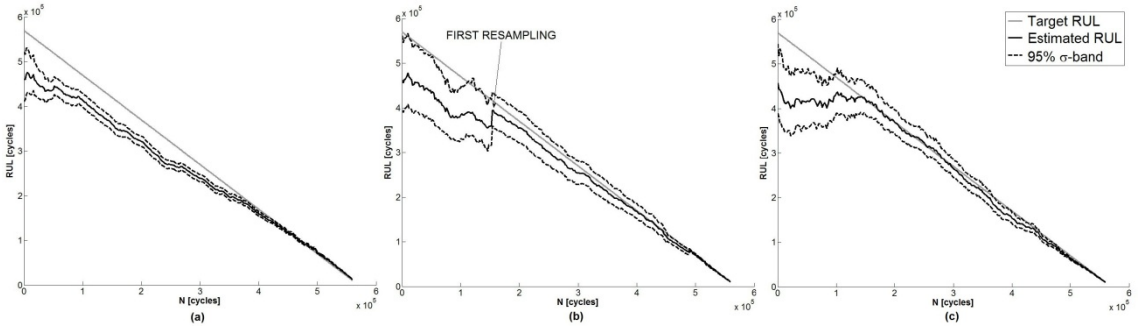

Figure 3: RUL estimation by PF algorithms. (a) refers to the standard PF with Systematic Resampling, (b) refers to Adaptive PF with Resampling under the effective particle threshold and (c) refers to Adaptive PF with Resampling made at each iteration

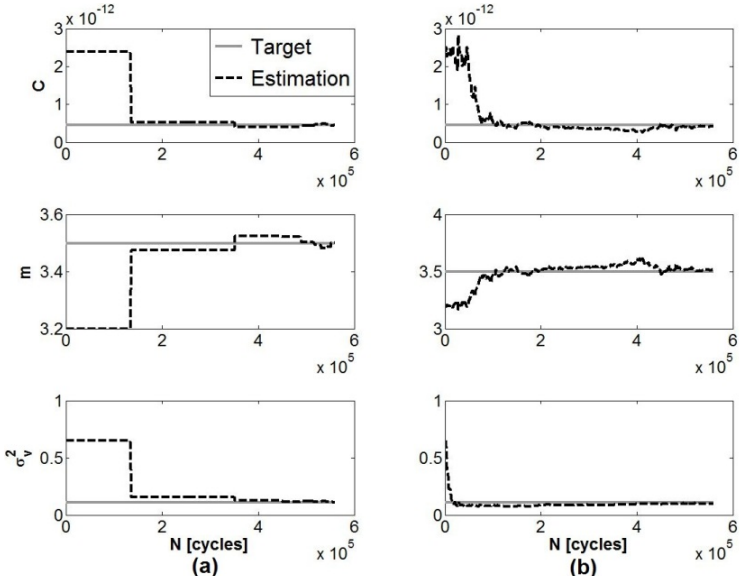

Figure 4: Parameter estimation through APMH algorithm during the Resampling stage. The two cases refer to the On-Demand Resampling (a) and the one made at each iteration (b)

4.1 Results

Figure 3 shows the RUL prediction with Standard PF algorithm (a), Adaptive PF with On-Demand Resampling (b) and Sequential Importance Resampling (c). The capability to self-centre the estimation thanks to the parameter updating is clearly visible. Moreover, the reduction of the parameter variances narrows the σ-band of the RUL prediction. Figure 4 shows the estimation of C, m and σ_v^2 over time for the cases of On-Demand Resampling and SIR. The parameters governing the FCG simulation are well-estimated by the APMH algorithm.

5. Conclusions

An Adaptive Dynamic State Space model for filtering problems has been proposed in this paper. The proposed PF algorithm shows good prediction capability inside the Fatigue Crack Growth estimation context. For example, it can be applied to on-line Structural Health Monitoring systems for diagnosis and prognosis of aeronautical structures (See Colombo et al. 2007, Giglio et al. 2006, 2008 and Viganò et al. 2012 for crack problem on aeronautical components or Giglio et al. 2011 for cracks after ballistic damage). Although the procedure can be extended to other dynamic degradation problems, further investigations are mandatory. First of all, the bias related to the posterior parameter PDFs mentioned by Haario et al. (1998) has to be studied and proved inside this context. Although it doesn't affect the RUL estimation appreciably, it can produce inefficiency of the Adaptive PF method in other applications. A clear disadvantage is the higher computing time needed to run the APMH inside the resampling. Even though this is a minor issue in the On-Demand Resampling, it might become of prime importance in the SIR algorithm. In addition, a quantitative analysis of the estimation error of the Adaptive PF has to be made in order to underline merits and disadvantages of the method, comparing it with other Resampling techniques.

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