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Numerical Key Performance Indicators for the Validation of PHM Health Indicators with Application to a Hydraulic Actuation System

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In order to perform Prognostic and Health Management (PHM) of a given system, it is necessary to define some relevant variables sensitive to the different degradation modes of the system. Those variables are named Health Indicators (HI) and they are the keystone of PHM. However, they are subject to a lot of uncertainties when computed in real time and the stochastic nature of PHM makes it hard to evaluate the efficiency of a HI set before the extraction algorithm is implemented. This document introduces Numerical Key Performance Indicators (NKPI) for the validation of HI computed only from data provided by numerical models in the upstream stages of a PHM system development process. In order to match as good as possible the reality, the multiple sources of uncertainties are quantified and propagated into the model. After having introduced the issue of uncertain systems modeling, the different NKPI are defined and eventually an application is performed on a hydraulic actuation system of an aircraft engine.

1. Introduction

In recent years, increasing availability has become the main purpose of many industries and particularly in aeronautics because a great amount of the average airlines costs is attributable to Maintenance, Repair and Overhaul (**MRO**) and Delays and Cancellations (**D&C**). In order to increase availability, advanced maintenance strategies based on failure anticipation and real-time optimization of MRO plan are being developed. Most of these strategies are based on Prognostics and Health Management (**PHM**), and the most used and proven one is Conditioned Based Maintenance (**CBM**).
A PHM system can be seen as an entity linked on the one hand to the monitored complex system by an extraction process and on the other hand to the maintenance system by a supervision process. The extraction process' purpose is to furnish relevant variables named Health Indicators (**HI**) to the PHM system and the supervision process' purpose is to forecast the health assessment to the maintenance system. In an industrial application scope, both of these processes need some key performance indicators (**KPI**) in order to perform their validation.
Whereas the supervision validation has been the subject of many papers, see (Saxena, et al., 2009) or (Baraldi, et al., 2013) for examples, the extraction validation is rarely addressed, because the issue of HI definition is often underestimated. Indeed, even if some research have been conducted in order to define some generic methods for HI construction, such as structured residuals and parity space (Gertler, 1997), when it comes to real operating complex systems, those methods are not adapted to overcome the following issues: random uncertainties, imposed sensors number and location, limited computation capabilities and prohibitive controller retrofit costs. Among those issues, the most restrictive one is the controller retrofit cost because it imposes that HI are definitively chosen before the system entry into service. However, PHM processes are inherently stochastic problems and it is obviously difficult to validate something stochastic before the availability of measured data.
In order to overcome this lack of measured data for the validation of HI, numerical modeling associated to a complete management of parameters uncertainties is used to simulate their distributions with or without

degradations. Eventually, some Numerical Key Performance Indicators (**NKPI**) are defined in order to quantify the quality of the HI in terms of detection and identification potential. In the aeronautic industry, those NKPI could be a great progress because data storage is very expensive and there is currently no way to validate the relevance of extracted data. In order to illustrate the potential of these NKPI, they are computed for a set of HI defined for the PHM of a hydraulic actuation system.
The remainder of the document is organized as following: In a first part, a new formalism for uncertain systems modeling is addressed. Then the definition of Numerical Key Performance Indicators will be introduced. The third and fourth parts are dedicated to the application on a hydraulic actuation system.

2. Uncertain System Modeling

2.1 Definitions and formalism

In this document, a numerical model $\mathscr{f}$ is defined as following:

$$(\eta_1, \dots, \eta_h) = \mathscr{f}(\boldsymbol{U}, \beta_1, \dots, \beta_p) \tag{1}$$

with $\boldsymbol{U} \in \mathbb{R}^{n \times k}$ matrix of the model inputs, k number of samples, $\eta_1, \dots, \eta_h$ values of HI and $\beta_1, \dots, \beta_p$ model parameters.
Parameters are variables that are considered constant during a single simulation but can vary between two different runs. When a variable is not constant during a run, it is classified as an input.
The parameters $\beta_1, \dots, \beta_p$ are divided into two types: context parameters $\beta_1^{ctx}, \dots, \beta_c^{ctx},\ c \leq p$ and structure parameters $\beta_1^{str}, \dots, \beta_s^{str},\ s \leq p$. A parameter cannot be both contextual and structural so $s + c = p$. The structure parameters are sub-divided into epistemic parameters $\beta_1^{epi}, \dots, \beta_e^{epi},\ e \leq s$ and degradation parameters $\beta_1^{deg}, \dots, \beta_d^{deg},\ d \leq s$. A parameter can be simultaneously of epistemic and degradation types. The parameters classification is schematized on figure 1.

Configurations:
We construct the configuration space $\mathcal{C}$ as a Euclidian vector space of dimension p provided with canonical base $(\boldsymbol{e_1}, \dots, \boldsymbol{e_p})$ and norm $\|\cdot\|_{\mathcal{C}}$. A configuration $\boldsymbol{\Gamma}$ is defined as a vector of $\mathcal{C}$ whose components are the parameters values: $\boldsymbol{\Gamma} = \beta_1 \boldsymbol{e_1} + \dots + \beta_p \boldsymbol{e_p}$ also written $\boldsymbol{\Gamma} = (\beta_1, \dots, \beta_p)$. The nominal configuration $\boldsymbol{\Gamma^{nom}}$ is the configuration with the nominal values of the parameters $\boldsymbol{\Gamma^{nom}} = (\beta_1^{nom}, \dots, \beta_p^{nom})$.

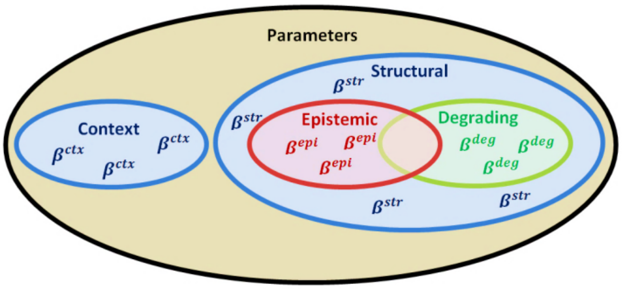


Figure 1: Classification of the different types of parameters for a system modeling

Conditions and Degradation Modes:
The condition space $\mathcal{D}$ is defined as a subspace of the configuration space $\mathcal{C}$ of dimension d with canonical base $(\boldsymbol{b_1}, \dots, \boldsymbol{b_d})$. A condition is defined as a vector of $\mathcal{D}$ whose components are the degradation parameters values: $\boldsymbol{\Delta} = \beta_1^{deg} \boldsymbol{b_1} + \dots + \beta_d^{deg} \boldsymbol{b_d}$ also written $\boldsymbol{\Delta} = (\beta_1^{deg}, \dots, \beta_d^{deg})$. The nominal condition $\boldsymbol{\Delta^{nom}}$ is the configuration with the nominal values of the degradation parameters.
A degradation mode is defined as a one dimension subspace of the condition space. A degradation mode is defined by a unitary generator $\boldsymbol{G}$ indicating its direction. A degradation $\boldsymbol{\Omega}$ is a vector of a degradation mode, defined by its magnitude Mgn so that $\boldsymbol{\Omega} = Mgn\ \boldsymbol{G}$.
A generator can be any type of unitary vector but if only simple degradation modes are considered, which is the case in this document, generators are vectors of the canonical base so there are d types of degradation mode. For instance, a degradation $\boldsymbol{\Omega}$ of magnitude Mgn corresponding to the j[th] degradation mode can be written $\boldsymbol{\Omega} = Mgn\ \boldsymbol{b_j}$.
The system cannot overcome all the magnitudes for a given degradation modes, at some point a failure will appear. The Maximal Admissible Magnitude (**MAM**) of a degradation mode is defined the as the magnitude for which this failure occurs.

Health Indicators:
We construct the health indicator space $\mathcal{S}$ as a Euclidian vector space of dimension h provided with canonical base $(\boldsymbol{v_1}, \dots, \boldsymbol{v_h})$ and norm $\|\cdot\|_{\mathcal{S}}$. A syndrome $\boldsymbol{\Sigma}$ is defined as a vector of $\mathcal{S}$ whose components are the parameters values: $\boldsymbol{\Sigma} = \eta_1 \boldsymbol{v_1} + \dots + \eta_h \boldsymbol{v_h}$ also written $\boldsymbol{\Sigma} = (\eta_1, \dots, \eta_h)$. The nominal syndrome is the syndrome obtained for the nominal configuration: $\boldsymbol{\Sigma^{nom}} = \oint(\boldsymbol{U}, \boldsymbol{\Gamma^{nom}})$. A fault is defined as a syndrome different from the nominal syndrome. The diagnostic problem can be seen as an identification of the relation between degradations and faults.

2.2 Uncertainties Management
When it comes to the modeling of multi-physic complex systems subject to real operating conditions, to manage the parameters uncertainties is of paramount importance. In this paper, two types of uncertainties are considered: random uncertainties derived from environment variations affecting context parameters and systematic uncertainties derived from manufacturing variations affecting epistemic parameters.
Those uncertainties are random variables that can be characterized by their probability density functions (**pdf**). A pdf is defined by its type (uniform, normal, exponential…) and its parameters vector $\boldsymbol{\Theta} = (\theta_1, \dots, \theta_m)$. For the remainder of the document, without loss of generality, it is supposed that uncertainties are of Gaussian type so their pdf are completely defined by the maximum likelihood estimation (**MLE**) of their parameter vector $\boldsymbol{\Theta} = (\mu, \sigma)$.
After having determined the pdf of uncertainty sources, configurations are propagated into the model. In a PHM scope, the purpose of this propagation is to compute the MLE of $\boldsymbol{\Theta}$ for simulated distribution of HI under uncertainties for both nominal condition and MAM degradations.
For the simulated healthy distribution of the i[th] HI, the parameter vector is written $\boldsymbol{\Theta_i^0} = (\mu_i^0, \sigma_i^0), i \in [\![1; h]\!]$.
Likewise, the simulated faulty distribution of the i[th] HI, with j[th] degradation mode of magnitude Mgn, the parameter vector of the distribution is $\Theta_i^{(j,Mgn)} = \left(\mu_i^{(j,Mgn)}, \sigma_i^{(j,Mgn)}\right),\ (i,j) \in [\![1;h]\!] \times [\![1;d]\!], Mgn \in \mathbb{R}$.
Many tools are available for uncertainties propagation but the most famous and proven one is the Monte-Carlo algorithm (Metropolis & Ulam, 1949) which consists in a random sampling of the parameters in accordance with their parameter vector. In the applicative part, this algorithm will be applied.

3. Numerical Key Performance Indicators

3.1 Detection NKPI
Typically, detection specifications give a maximum False Positive Rate (**FP**) and a minimum True Positive Rate (**TP**) for detection. The basics of detection theory can be found in (Wickens, 2002). Detection NKPI (**D-NKPI**) are based on Receiver Operating Characteristic (ROC) curves (Fawcett, 2005).Two types of D-NKPI are used: Global Detectability matrix and Compliant Detectability matrix.
ROC curves contain a lot of information about the relative positioning of two pdf. The ROC curve between two pdf of parameters vectors $\boldsymbol{\Theta}$ and $\boldsymbol{\Theta'}$ is written $ROC(\boldsymbol{\Theta}, \boldsymbol{\Theta'})$**.**
The ROC curve between two simulated distributions is defined as the ROC curve between their estimated pdf. Thus, for the i[th] HI, the ROC curve between healthy distribution and faulty distribution of degradation mode j with magnitude Mgn is $ROC(\boldsymbol{\Theta_i^0}, \boldsymbol{\Theta_i^{(j,Mgn)}})$.

Global Detectability:
For a given ROC curve, Global Detectability (**GD**) is defined as a function calculating from the Area Under the Curve (**AUC**) (Bradley, 1997). The closer to one the value is, the higher the detection potential. GD does not depend on the detection specifications, so it is robust to specification changes.

$$GD(\boldsymbol{\Theta}, \boldsymbol{\Theta'}) = 2 \times (AUC(ROC(\boldsymbol{\Theta}, \boldsymbol{\Theta'})) - 0.5) \tag{2}$$

Finally, GD are computed for each couples $\left(\boldsymbol{\Theta_i^0}, \boldsymbol{\Theta_i^{(j,MAM)}}\right),\ (i,j) \in [\![1;h]\!] \times [\![1;d]\!]$ to construct the following NKPI: the Global Detectability matrix $\boldsymbol{GDx}$.

$$\boldsymbol{GDx} = \begin{vmatrix} GD(\boldsymbol{\Theta_1^0}, \boldsymbol{\Theta_1^{(1,MAM)}}) & \dots & GD(\boldsymbol{\Theta_1^0}, \boldsymbol{\Theta_1^{(d,MAM)}}) \\ \vdots & \ddots & \vdots \\ GD(\boldsymbol{\Theta_h^0}, \boldsymbol{\Theta_h^{(1,MAM)}}) & \dots & GD(\boldsymbol{\Theta_h^0}, \boldsymbol{\Theta_h^{(d,MAM)}}) \end{vmatrix} \in [0;1]^{h \times d} \tag{3}$$

Compliant Detectability:
For a given ROC curve, the compliance point is defined as the point of coordinates $(FPspec, TPspec)$ with $FPspec$ the specified maximal false positive rate and $TPspec$ the specified minimal true positive rate. The Compliant Detectability (**CD**) is defined as following:

$$CD(\mathbf{\Theta}, \mathbf{\Theta}') = \begin{cases} 1 \; if \; ROC(\mathbf{\Theta}, \mathbf{\Theta}') \; is \; above \; the \; compliance \; point \\ 0 \; if \; ROC(\mathbf{\Theta}, \mathbf{\Theta}') \; is \; under \; the \; compliance \; point \end{cases} \quad (4)$$

If specifications on FP are very restrictive, for example less than $1e^{-3}$, it is hard to see the compliance point on the curve. In this case, it is possible to use the semi-logarithmic ROC curve with a logarithmic scale in abscissa for FP in order to give more clarity to the curve.

Finally, CD are computed for each couples $\left(\mathbf{\Theta_i^0}, \mathbf{\Theta_i^{(j,MAM)}}\right)$, $(i, j) \in [\![1; h]\!] \times [\![1; d]\!]$ to construct the following NKPI: the compliant Detectability matrix $\boldsymbol{CDx}$.

$$\boldsymbol{CDx} = \begin{vmatrix} CD(\mathbf{\Theta_1^0}, \mathbf{\Theta_1^{(1,MAM)}}) & \dots & CD(\mathbf{\Theta_1^0}, \mathbf{\Theta_1^{(d,MAM)}}) \\ \vdots & \ddots & \vdots \\ CD(\mathbf{\Theta_h^0}, \mathbf{\Theta_h^{(1,MAM)}}) & \dots & CD(\mathbf{\Theta_h^0}, \mathbf{\Theta_h^{(d,MAM)}}) \end{vmatrix} \in \{0,1\}^{h \times d} \quad (5)$$

3.2 Identification NKPI

The classical identification process aims at finding the most probable Degradation Mode of the system. It is based on the classification of the current syndrome relatively to a reference database of different syndromes corresponding to degradations. In this section, one Identification NKPI (**I-NKPI**) is defined based on signature vectors: Cross Identifiability matrix.

Signature and distinguishability:

The signature vector of degradation mode j ($\boldsymbol{Sns^j}$) indicates the level of similarity between a set of healthy distributions of HI and a set of faulty distributions of HI computed for the MAM of degradation mode j. It is function of the global detectability and the sign of the difference between distributions means. For example, the k[th] component of the signature vector for degradation mode j is:

$$\boldsymbol{Sgn^j}(k) = sign(\mu_k^0 - \mu_k^{(j,MAM)}) \times \; GD\left(\mathbf{\Theta_k^0}, \mathbf{\Theta_k^{(j,MAM)}}\right) \in \; [-1; 1]^h \quad (6)$$

The Distinguishability index (Dst) is defined as the angle between two signature vectors. For degradation modes j and k:

$$Dst\left(\boldsymbol{Sns^j}, \boldsymbol{Sns^k}\right) = arccos\left(\frac{\boldsymbol{Sns^j}(i) \cdot \boldsymbol{Sns^k}(i)}{\|\boldsymbol{Sns^j}(i)\| \|\boldsymbol{Sns^k}(i)\|}\right) \quad (7)$$

Cross Identificability:

Cross Identificability matrix CIx is defined as following:

$$\boldsymbol{CIx} = \begin{vmatrix} Dst\left(\boldsymbol{Sns^1}, \boldsymbol{Sns^1}\right) & \dots & Dst\left(\boldsymbol{Sns^k}, \boldsymbol{Sns^1}\right) \\ \vdots & \ddots & \vdots \\ Dst\left(\boldsymbol{Sns^1}, \boldsymbol{Sns^k}\right) & \dots & Dst\left(\boldsymbol{Sns^k}, \boldsymbol{Sns^k}\right) \end{vmatrix} \in \; \mathbb{R}^{d \times d} \quad (8)$$

4. Application to a Hydraulic Actuation System

In this paper, the model is built on the AMESim© software which is based on the Bond Graph theory (Thoma, 1975). In this section, an application demonstrating the potential of NKPI is presented for the PHM of a hydraulic actuation system.

4.1 System Presentation

The system is a control loop regulating the position of a hydraulic cylinder. It is composed of a servovalve, a cylinder, a PID corrector, a Linear Variable Differential Transformer (**LVDT**) sensor and some harnesses as presented in Fig.2. Co-simulation between AMESim© and Matlab/Simulink© is used in order to manage the runs of the Monte-Carlo algorithms.

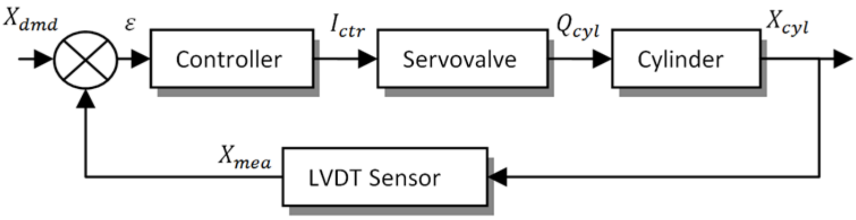


Figure 2: Actuation Loop Scheme. Where X_{dmd} is the position demand, X_{mea} the measured position, ε the error, I_{ctr} the control current, Q_{cyl} the actuation flow sent to the cylinder and X_{cyl} the real position of the cylinder

4.2 Parameters

The different degradation modes of this system have already been addressed in a previous paper (Lamoureux, et al., 2012). The model of the system is normally composed of about 34 parameters including 2 context parameters, 26 epistemic parameters and 15 degradation parameters. In this document, the list is limited to the ones presented in Tab.1. It means that only 2 degradation modes are considered here: Increase of the cylinder cooling flow due to internal leakage and positive drift of the null bias current of the servovalve.

Table 1: Model Parameters

Name	Uncertainty Quantification	Name	Uncertainty Quantification
Fuel Temperature β_1^{ctx}	$(\mu,\sigma) = (15\,°C, 10\,°C)$	Chamber Length β_3^{epi}	$(\mu,\sigma) = (54mm, 0.3mm)$
Aircraft Pressure β_2^{ctx}	$(\mu,\sigma) = (2\,bar, 0.2\,bar)$	Offset LVDT sensor β_4^{epi}	$(\mu,\sigma) = (0mm, 0.05mm)$
Feedback stiffness β_1^{epi}	$(\mu,\sigma) = (1.5N/m, 0.01N/m)$	Cooling Flow Diameter β_1^{deg}	$MAM = 6.5mm$
Cylinder Mass β_2^{epi}	$(\mu,\sigma) = (1.55kg, 0.01kg)$	Servo Null Current β_2^{deg}	$MAM = 16.5mA$

4.3 Health Indicators

HI are defined from the velocity gain curve, i.e. the curve representing the cylinder velocity versus control current (c.f.Fig.3). From the smoothed curve, 9 HI are constructed but as the problem is simplified to 2 degradation modes for the sake of the example, only 3 of them are sufficient here: The idle current of the loop **CIdle**, the abscissa difference between slope changes 2 and 1 **X21** and the slope of the null region **NSlope** (c.f.Fig.4). They are respectively numbered 1 to 3.

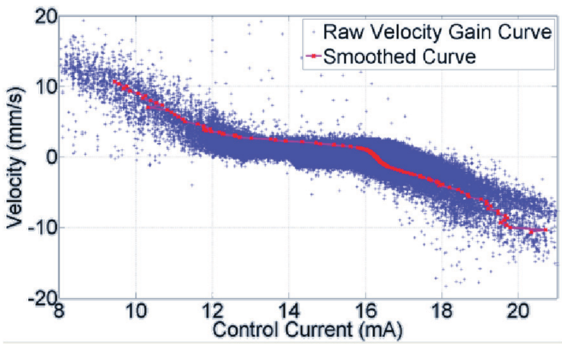


Figure 3: Velocity gain curve: raw and smoothed

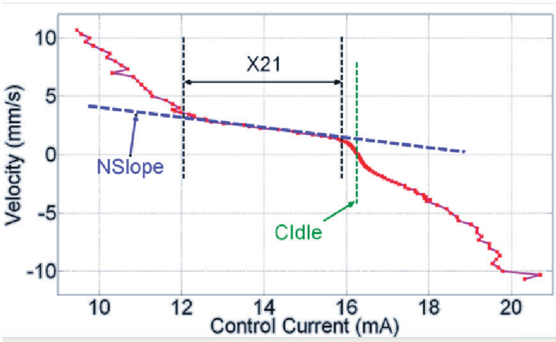


Figure 4: Health Indicators Construction

4.4 Simulations

Three Monte-Carlo algorithms are run to compute distributions of the HI for respectively healthy state, increased cooling flow diameter and positive null bias current drift. For each algorithm, only 100 simulations are run because the model is expensive in time: one simulation lasts 20 minutes. Eventually, 9 MLE parameters are computed: $\boldsymbol{\Theta}_{\mathbf{i}}^{\mathbf{0}}, i = 1, \dots, 3$, $\boldsymbol{\Theta}_{\mathbf{i}}^{(\mathbf{1,MAM})}, i = 1, \dots, 3$ and $\boldsymbol{\Theta}_{\mathbf{i}}^{(\mathbf{2,MAM})}, i = 1, \dots, 3$.

5. Results

5.1 Results Presentation

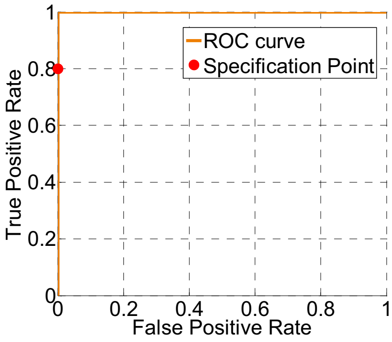
The specifications for detection are the following ones: $(FPspec, TPspec) = (0.8, 1e^{-8})$. The specification on TP is very stringent because airliners want to avoid an excess of alarm messages. So it is suitable to use semi-logarithmic ROC curves. The difference between simple ROC curve and semi-logarithmic ROC curve and the position of compliance point are presented in Figures 5 and 6.

5.2 Detection NKPI

Both global detectability and compliant detectability matrices are computed:

$$\boldsymbol{GDx} = \begin{vmatrix} 1 & 1 \\ 0 & 0.93 \\ 0 & 0.91 \end{vmatrix} ; \boldsymbol{CDx} = \begin{vmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{vmatrix} \tag{9}$$

Detection NKPI show that both considered degradation modes are detectable within the specifications because both columns of $\boldsymbol{CDx}$ are non-null. It is also obvious that is this case, only the first HI is useful for detection and so that the detection efficiency will not be robust to the loss of this HI. The global detectability matrix traduces that the first degradation mode has more impact on the set of HI.



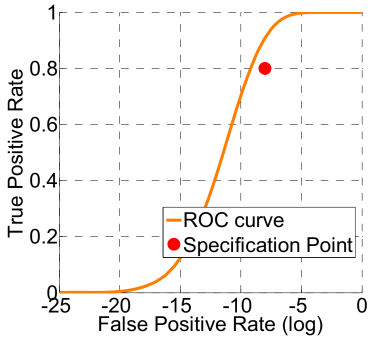


Figure 5: ROC curve of HI2 for degradation mode 1

Figure 6: Same curve in semi-logarithmic scale

5.3 Identification NKPI

The computation of the cross Identifiability matrix gives the following result:

$$\boldsymbol{CIx} = \begin{vmatrix} 0 & 52.46 \\ 52.46 & 0 \end{vmatrix} \quad (10)$$

Identification NKPI shows that the two degradation modes signature are separated by an angle of 52.46°. If it is considered that two signatures are different enough if their relative angle is more than 45°, the identification criterion is verified.

6. Conclusion

In this paper, a new kind of key performance indicator for PHM has been defined. Contrary to most of the others, focalized on the supervision process, the purpose of these performance indicators is to evaluate the efficiency of a health indicator set in order to perform the validation of the extraction process. The method is primarily intended for industries like aeronautics suffering from prohibitive retrofit costs because it is based only on simulated data from modeling. In the course of the document, a new formalism for uncertain systems modeling has been set and numerical key performance indicators has been defined. Eventually, the efficiency and interest of the method has been tested on a real industrial application aimed at performing PHM of a hydraulic actuation system and has shown good result to validate the potential of health indicators both for detection and identification. For further applications, the objective is to generalize the method first to the complete actuation loop with all the parameters and all the degradation modes and then to the whole fuel system of an aircraft engine.

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