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Weak Magnetic Memory Signal Denoising Based on Cascaded Singular Value Decomposition

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Magnetic memory method is widely used in finding and locating stress concentration zone of ferromagnets, which is of great importance in early diagnosis of ferromagnetic structures and components. Magnetic memory sensing signal is weak and easy to be corrupted by noise and interference. In such situation, it is difficult to distinguish the characteristic of the magnetic sensing signal, and the stress concentration zone is not easy to be distinguished. Singularity value decomposition (SVD) is a nonlinear filter method useful for signal denoising and enhancement. But the singularity values are very sensitive to noise, and traditional SVD cannot process signals contaminated by heavy noise. As the denoising ability of single SVD system is limited, a novel method called cascaded SVD system (CSVD) is proposed in this paper. The noisy signal is processed by the first layer of SVD firstly. The output of the first layer of SVD is filtered by a second layer of SVD. And the output of the second layer SVD has a signal to noise ratio (SNR) gain over the first layer output. The principle of CSVD is proposed and applied to the enhancement of magnetic memory signal measured from gear tooth surface with a tiny defection. The characteristic of the signal for distinguishing the defect zone is obvious enhanced and the result has advantage over that getting from wavelet denoising method.

1. Introduction

Metal magnetic memory (MMM) technique is a novel magnetic testing method to find stress concentration zone, which provides early diagnosis for the ferromagnetic material. Metal magnetic memory method is firstly proposed by Russian experts and its special characteristic lies in what to be measured in MMM technique is the self-magnetic flux leakage signal of ferromagnets (Doubov, 2004, 2005). Wilson et al. (2007) applied the MMM technique to stress measurement, which can provide early indications of stress status and eventual failure of mechanical structures. Especially, the earth magnetic field instead of an external magnetic field is applied as the stimulus source and no special magnetizing device is required during tests.

The basic principle of magnetic memory can be expressed as: due to magnetostriction, under both of applied load and earth magnetic field existing, the direction and irreversible reorientation of the magnetic domain textures will take place around stress concentrations of the ferromagnetic materials. The irreversible change of magnetic domains shall preserve after loads, and has a relationship with maximum applied stress.

Due to the applied load, leakage magnetic field, Hp forms. As shown in Figure 1, around stress concentration zones, Hp(x) exhibits a peak and Hp(y) changes its polarity (passes through zero) where Hp(x) is the field strength parallel to the material surface and Hp(y) is the field strength perpendicular to the material surface. So stress concentration of ferromagnetic material could be inspected through measurement of Hp(y) signal (Wang et al., 2010).

Usually, the leakage magnetic signal is a kind of very weak aperiodic signal and high precision magnetic sensor is required here for sensing the magnetic memory signal. And the sensing signal is often influenced by noise and interference, such as the dithering of the sensor probe, the undesired magnetic signals from ambience. Thereby, the weak magnetic memory signal characterizing the stress concentration of ferrous

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material is often submerged in the heavy noise and the stress concentration zone is difficult to be located. Some signal denoising method, such as wavelet analysis has been applied to the processing of the magnetic memory sensing signal (Zhang and Wang, 2008). But the wavelet denoising effect is poor in heavy noise background. As regard to the wavelet-based denoising techniques, the selection of wavelet function is crucial for the noise reduction effect. However, up to date, because of the complexity of the real signals in composition, how to select a suitable wavelet function for a special signal is still a knotty problem expected to solve.

Singularity value decomposition (SVD) as a nonlinear filter method has received a considerable attention among researchers and is widely used for signal denoising and detection(Yang et al., 2003). In the work of Reninger et al. (2011) SVD is reported as an effective denoising tool for airborne time domain electromagnetic data. Using SVD, a time data matrix from the noisy signal is first constructed. Then this data matrix is divided into signal subspace and noise subspace using the SVD-based approach (Zhao and Ye, 2009). And the singular values of the matrix are obtained in a degressive order. The lower singular values can be categorized as the singular values of the noise subspace and hence should be set to zero. The left singular values, which are categorized as the main singular values of the useful signal, are used to reconstruct another data matrix, and the useful signal is recovered from the matrix and the weak useful signal is enhanced(Hassanpour, 2007).

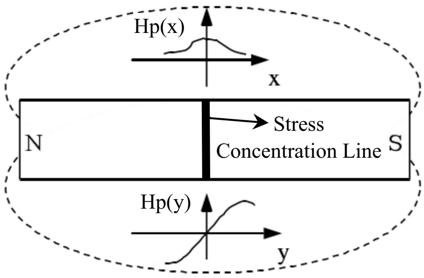

Figure 1: Principle of magnetic memory method

However, the singularity values are sensitive to noise, and SVD algorithm cannot process signals contaminated by heavy noise. As the denoising ability of single SVD system is limited, a novel method of cascaded SVD (CSVD) is proposed to enhance the weak magnetic memory sensing signal measured from gear tooth surface with a tiny defection.

In the paper, a two-layer cascaded CSVD diagram and algorithm is proposed to meet the enhancement of weak aperiodic signal. The CSVD method is applied to the enhancement of the magnetic memory sensing signal measured from the gear tooth surface with a tiny defection. And the result getting from CSVD method has advantage over that from wavelet denoising method.

2. Princple and algorithm of CSVD for weak signal denoising

SVD is a kind of nonlinear filter widely used in denoising and signal detection. Generally, the singular value decomposition of the $m \times n$ real matrix **A** is of the form

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \tag{1}$$

where $\mathbf{U}_{m\times m}$ and $\mathbf{V}_{n\times n}$ are orthogonal matrices and their columns are called the left and right singular vectors respectively. The matrix $\mathbf{\Sigma}$ is a $m \times n$ diagonal matrix of singular values and consequently can be expressed as

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{2}$$

The diagonal elements of matrix $\mathbf{\Sigma}$ are called the singular values of **A**, which are sorted in a degressive order. Therefore, $\mathbf{S} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots \lambda_r)$, with components such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > 0$, and *r* is the rank of matrix **A**.

Removing the zero singular values of **A,** the simple SVD form for matrix A can be expressed as below

$$\mathbf{A} = \sum_{i=1}^{r} \lambda_i \mathbf{u_i} \mathbf{v_i}^T \tag{3}$$

where $\mathbf{u_i}$, $\mathbf{v_i}$ are the *i*th column singular vectors of **U** and **V** respectively.
Let's suppose that the clean useful signal has been corrupted by an additive white Gaussian noise

$$x(t) = s(t) + n(t) \tag{4}$$

where $x(t)$, $s(t)$ and $n(t)$ respectively denote the noisy signal, clean useful signal and additive white Gaussian noise. For $x(i)$, i =1, 2, . . . ,N representing sampling points of noisy signal $x(t)$, the data matrix can be constructed as follows

$$\mathbf{A} = \begin{bmatrix} x(1) & x(2) & \cdots & x(n) \\ x(2) & x(3) & \cdots & x(n+1) \\ \vdots & \vdots & \vdots & \vdots \\ x(m) & x(m+1) & \cdots & x(N) \end{bmatrix} \tag{5}$$

where $n=N-m+1$, and $m > n$. Because there is white Gaussian noise in the constructed data matrix **A**, so matrix **A** is usually a full rank matrix in columns. We can get *n* nonzero singular values by performing SVD on matrix **A**. and the data matrix **A** can be divided into signal subspace and noise subspace.
According to the theory of SVD, the singular values reflect the energy concentration of the clean signal and noise. The desired signal mainly contributes to the precedent *k* larger singular values $\lambda_1, \lambda_2, \cdots \lambda_k, k \le n$.
And the lower singular values mainly reflect the influence of noise. So we must determine a threshold point where lower singular values from that point can be categorized as the singular values of the noise subspace and hence should be set to zero.
The key point during the procedure is the choice of *k*. To determine the value of *k*, we can plot the singular values with respect to their indexes. Usually a break point can be seen clearly, where the slope of singular value curve changes drastically. Thus this threshold point can be determined by calculating derivation of the curve in each point. Since the noise subspace is mainly related to those singular values that are lower than this threshold point, we suggest setting these singular values to zero for space division.
The matrix after denoising can be reconstructed as follows

$$\mathbf{A}' = \sum_{i=1}^{k} \lambda_i \mathbf{u_i} \mathbf{v_i}^T \tag{6}$$

According to the reconstructed matrix $\mathbf{A}'$ and equation (5) the denoised useful signal can be obtained.
However, the denoising ability of SVD is limited for the detection of weak useful signal immerged in strong background noise. To the problem, here we proposed the cascaded SVD to improve the denoising ability in heavy background noise.
A two-layer SVD diagram of CSVD is illustrated in Figure 2. The noisy signal inputs into the first layer of SVD to perform singular values decomposition and output of the first layer of CSVD is the denoised signal. The denoised signal $s_1(t)$ still contains some noise and acts as the input to the second layer of CSVD.

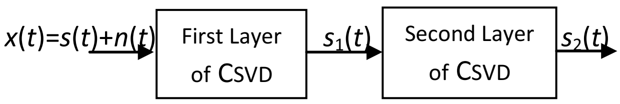

Figure 2: The denoising diagram using CSVD

The first layer of CSVD and the second layer of CSVD are performed as follows

$$\begin{aligned} \mathbf{A}_1 &= \sum_{i=1}^{n} \lambda_{1i} \mathbf{u_{1i}} \mathbf{v_{1i}}^T \Rightarrow \mathbf{A}_1' = \sum_{i=1}^{k_1} \lambda_{1i} \mathbf{u_{1i}} \mathbf{v_{1i}}^T \\ \mathbf{A}_2 &= \sum_{i=1}^{n} \lambda_{2i} \mathbf{u_{2i}} \mathbf{v_{2i}}^T \Rightarrow \mathbf{A}_2' = \sum_{i=1}^{k_2} \lambda_{2i} \mathbf{u_{2i}} \mathbf{v_{2i}}^T \end{aligned} \tag{7}$$

In CSVD, the output of the first layer $s_1(t)$ is obtained from the matrix $\mathbf{A}_1{}'$,which is reconstructed using the k_1 precedent singular values of matrix $\mathbf{A}_1$ according to equation (7). And $s_1(t)$ is used to construct the data matrix $\mathbf{A}_2$ again according to equation (5). Then SVD of $\mathbf{A}_2$ is performed. The matrix $\mathbf{A}_2{}'$ is reconstructed using the k_2 main singular values of $\mathbf{A}_2$ according to equation (7). The output of the second layer $s_2(t)$ is obtained from $\mathbf{A}_2{}'$. The output signal $s_2(t)$ has a higher SNR than signal $s_1(t)$. And noise can be further filtered and the useful signal emerges from the background noise.

The selection of the values of k_1 and k_2 in CSVD is of great importance. Because of the influence of heavy background noise, the value of k_1 in first layer of CSVD can be much larger than the value of the threshold point determined by mean of the way discussed above. And the singular values versus the indexes plot for matrix $\mathbf{A}_1$ and $\mathbf{A}_2$ will diverge when the index of the singular values is becoming larger (seeing in Figure 4). The singular value index corresponding to the diverge point of the two plots can be selected to be the value of k_2.

3. Experiment on enhancing magnetic memory sensing signal of gear tooth defect

Gear transmission need to comply high standards as for their reliability and durability. Even a small defect in a gear transmission makes its operation impossible. Usually the damage in a gear transmission occurs when a part of one or more teeth breaks. So it is very important to find the defects in gear tooth as early as possible. The metal magnetic memory method was applied to the investigations of stress concentration zone or the location of tiny defect along the gear tooth. The magnetic memory sensing signal, the magnetic field component *H* on the surface of the investigated part is recorded using hall sensor with high sensitive. The gear wheel with a tiny tooth defect is shown in Figure 3.

Figure 4(a) shows the distribution of the normal component of the magnetic field, which was measured along the width of the gear tooth with defect. The defect locates between 40mm and 50mm along the tooth. Due to heavy noise, it is difficult to determine the position of *H* passing zero, and the location of the defect cannot be verified. Figure 4(b) is the output result of wavelet denoising method. The Daubechies wavelet is used as a wavelet function with three series，and the threshold is calculated using the adaptive method. Though the SNR of the magnetic memory sensing signal after denoising is improved, the position of the *H* passing zero is not obvious.

The CSVD method is used for denoising the normal component of the magnetic signal shown in Figure 4(a), wherein the sample points *N* is 2200. In the first layer of CSVD, the data matrix $\mathbf{A}_1$ is constructed according to equation (5) with m=1470, n=730. The singular values are shown in Figure 5 in solid-line style. And the precedent 400 larger singular values are selected to reconstruct the matrix $\mathbf{A}_1{}'$, that is k_1 is assigned to be 400. The noise-deduced signal from matrix $\mathbf{A}_1{}'$ is shown in Figure 6(a). It is seen that the signal has an SNR improvement against the noisy magnetic signal shown in Figure 4(a). But there is much noise still resident in it. So the second layer of CSVD is performed on the data matrix $\mathbf{A}_2$, which is constructed from the signal shown in Figure 6(a).

The singular values are shown in Figure 5 in dotted-line style. From Figure 5, it can be found that the dotted line almost coincides with the solid line when the index of singular values is less than 18. Then the two lines diverges when the index become larger. And the dash-line is below the solid line, which shows that the noise in matrix $\mathbf{A}_2$ is much less than that in matrix $\mathbf{A}_1$. The precedent k_2 larger singular values are selected to reconstruct the matrix $\mathbf{A}_2{}'$ according to equation(7), where k_2 is selected to be 20. The noise reduction signal deduced from matrix $\mathbf{A}_2{}'$ is illustrated in Figure 6(b). And the signal has a higher SNR than the signal obtained using wavelet method. It can be seen from Figure 6(b) that the magnetic normal components of the memory sensing signal emerges from noise background. There is an obvious crossing zero point, and the location is nearby 46mm to left side of the tooth. The result coincides with the actual situation.

Figure 3: Gear wheel with a tiny tooth defect

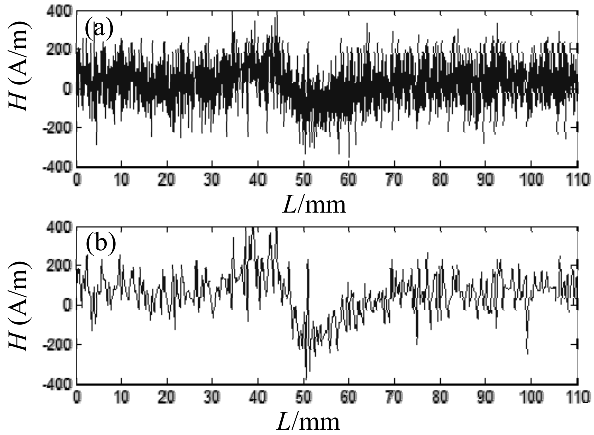

Figure 4: Magnetic memory sensing signal with heavy noise and the results processed by wavelet method

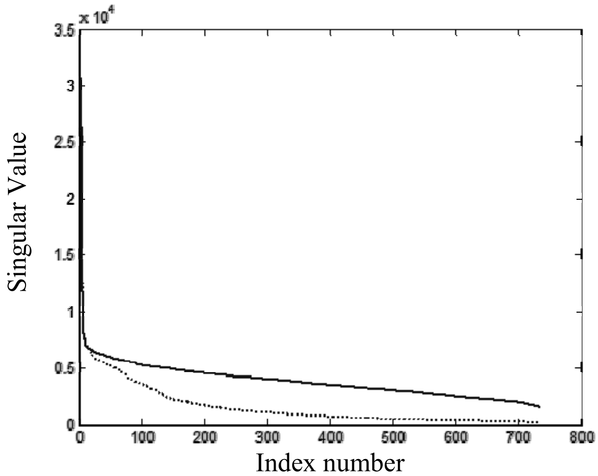

Figure 5: The Singular Values of CSVD for the magnetic memory signal

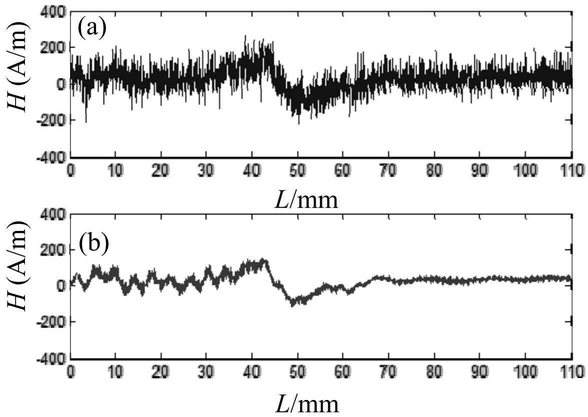

Figure 6: The denoised output of the first layer and second layer of CSVD for noisy magnetic memory signal

4. Conclusion

The magnetic memory method is widely used in early diagnosis of ferrous material. But the magnetic memory signal is easy to be disturbed by various sources of noises, and the detecting ability of defects is greatly lowered. SVD is a kind of nonlinear filter used in denoising and signal detection. But the denoising ability of traditional SVD is limited. The CSVD method proposed in the paper is a new attempt for signal enhancement immerged in heavy noise. The diagram of two layers of SVD is proposed. The experiment results show that CSVD method can improve the SNR of weak aperiodic signal immerged in heavy noise, and provide a novel means to the enhancement of weak magnetic memory sensing signal.

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