

# Investigation on the RLS and Kalman Based Adaptive Order Tracking Techniques for Rotating Machinery Analysis

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This paper investigates the adaptive order tracking techniques, based on the RLS, the extended RLS, and the Kalman recursive algorithms, which all could be realized as real-time applications in the diagnosis of defects in rotating machinery. The numerical implementations of the considered methods through simulations on a representative noisy synthetic signal are performed. The results illustrate a possible degradation in the tracking performance of the RLS algorithm, and the effectiveness of the extended RLS, as well as Kalman based algorithms, for order tracking and distinguishing. Also, one example of practical implementation of the considered adaptive order-tracking methods is given. The analysis of a real technical system – a twisting machine for the polypropylene yarn production, justifies the implementation of the extended RLS algorithm, which is not used so far as order tracking tool for industrial purposes.

## 1. Introduction

Order tracking (OT) techniques are the art of extracting the amplitude and phase of the sinusoidal content of non-stationary noise or vibration signal measured on rotating machinery under the periodic loading. They are of great importance for the condition monitoring, troubleshooting, design and synthesis.

Each periodic loading produces sinusoidal overtones, or orders/harmonics, at the frequencies that are integer or fractional multiples of the fundamental rotational frequency corresponding to various rotating elements, such as gears, chains, belts and shafts for power transmission.

Unlike steady-state conditions, for OT analysis in non-stationary environments additional information is needed – the instantaneous angular shaft position, usually obtained by numerical integration of the angular frequency measured by the tachometer signal.

Conventional approaches based on fast Fourier transform (FFT) such as windowed Fourier transform (Potter, 1990) and computed order tracking (COT) methods (Fyfe and Munck, 1997) are characterized by smearing problems at low revolutions (Todorović et al., 2011), finite order resolution, and inability to analyze crossing orders induced by multiple independent shaft speeds.

Post-processing (off-line) OT methods which are able to fully detach close and/or crossing spectral components are: Vold-Kalman (VK) OT schemes, introduced in profit oriented papers (Vold et al., 1997; Herlufsen et al., 2000) with little delivery on method details, in contrast to the VK OT schemes presented in (Pan and Lin, 2006a and 2006b) and implemented in (Wang and Heyns, 2009; Pan and Wu 2010); as well as Gabor OT schemes for processing vibration signal without (Sh. Quian, 2003) or with order-crossing noise (Guoa and Tan, 2009). All these forms and realizations of the OT filters are characterized by time consuming, memory intensive, huge matrix manipulations, especially ineffective in the case of a long time record.

Real-time linear adaptive filtering based on the exponentially weighted recursive least-squares (RLS) algorithm (Haykin, 2002), for tracking single or multiple orders, was proposed by Bai et al. (2002), while Wu et al. (2009) implemented this method in an expert system for the diagnosis of faults in practical applications.

Recursive Kalman filtering algorithm (Brown and Hwang, 1997), with a two pole structural equation, labelled as the recursive VK OT filter, was introduced by Wu et al. (2004) for two practical experimental

implementations, while Pan and Wu (2007) tested the method on a various synthetic signals. It is shown that the recursive Kalman based OT schemes are very efficient, in comparison to the original off-line, having advantages in the discarding of the end effects that makes the terminal portion of the computed orders zero, and the capability to be realized as a real time scheme for practical purposes.

Here is performed investigation on the RLS, extended RLS (which is not used as OT tool so far), and VK based adaptive OT techniques for rotating machinery analysis, and comparative tracking results on a representative synthetic signal with the process and measurement noise, including close and crossing orders, as well as resonant order, is performed. In addition, an example of the analysis of a real technical system - a twisting machine for the polypropylene yarn production is shown, justifying the implementation of the proposed techniques as OT tools for industrial purposes.

## 2. Theoretical basis

The measured noise or vibration signal  $d(t)$ , generated by one rotating shaft, can be written as the superposition of  $K$  orders and some zero mean white Gaussian noise  $v(t)$ , as

$$d(t) = \sum_{k=1}^K y_k(t) + v(t) \quad (1)$$

while the  $k$ -th order can be expressed as

$$y_k(t) = A_k(t) \cos[k \int_0^t \omega(\tau) d\tau + \varphi_k(t)] \quad (2)$$

where  $\omega(t)$  denotes the instantaneous angular frequency of the reference shaft, while  $A_k(t)$  and  $\varphi_k(t)$  represent the amplitude and the phase of the order  $k$ , respectively.

## 3. Numerical simulation

The measurement signal for the synthetic two-axle vibration system comprises resonant, crossing and close orders, is simulated in this work by using VK state-space OT signal model (Pan and Lin, 2006a), with a sampling frequency of 4096 [Hz], and the states and measurement noise parameters:  $q_1 = 1$ ,  $Q_2 = 0.1$ , respectively. The time diagrams of the constant (resonant) frequency  $f_r$ , the frequencies of the first ( $f_1$ ) and second ( $f_2$ ) reference shaft, as well as the frequencies of the higher spectral components included in the signal: order 2  $f_1$  induced by the first reference shaft, and close order 1.2  $f_2$  induced by the second reference shaft are shown in Figure 1a. The time history of the synthetic measurement signal is shown in Figure 1b.

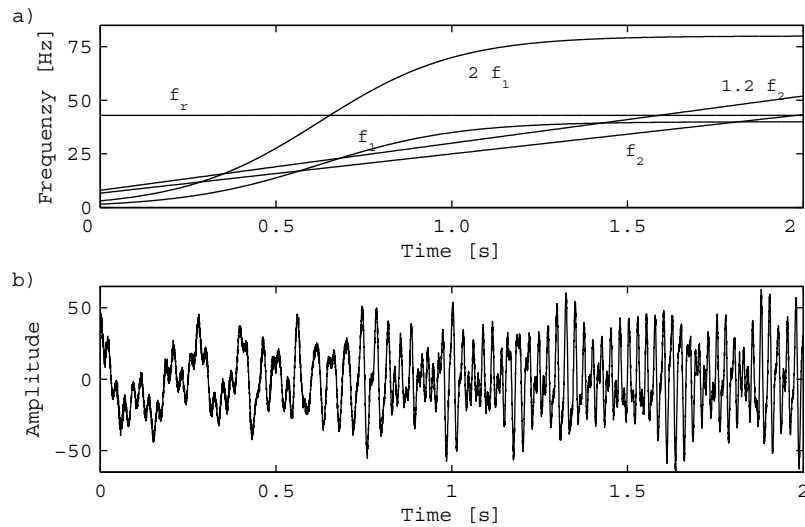


Figure 1. Illustration of two-axle synthetic vibration system: a) signal frequency components; b) the time history of the synthetic vibration signal with states and measurement noises.

The frequency of the first reference shaft, denoted as  $f_1$ , changes nonlinearly according to function  $f_1 = a/(1+bc^{-t/t_{\text{end}}})$ , where  $a = 60$  [Hz],  $b = 25$ ,  $c = 3 \cdot 10^4$ , while the frequency of the second reference shaft, denoted as  $f_2$ , increases linearly from 6.67 to 43.33 [Hz]. Resonant order amplitude is approximated with a constant order, while both sets of the order components consisting of one fixed order, and one linearly increasing order.

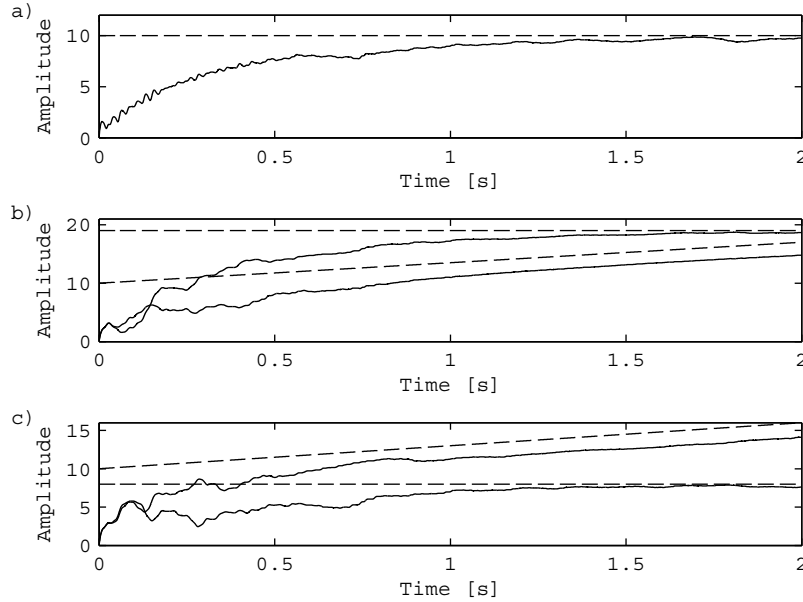


Figure 2. Estimated order amplitudes by using RLS adaptive algorithm ('--' are actual values): a) resonant frequency component; b) orders 1 and 2 of the first shaft; c) close orders 1 and 1.2 of the second shaft.

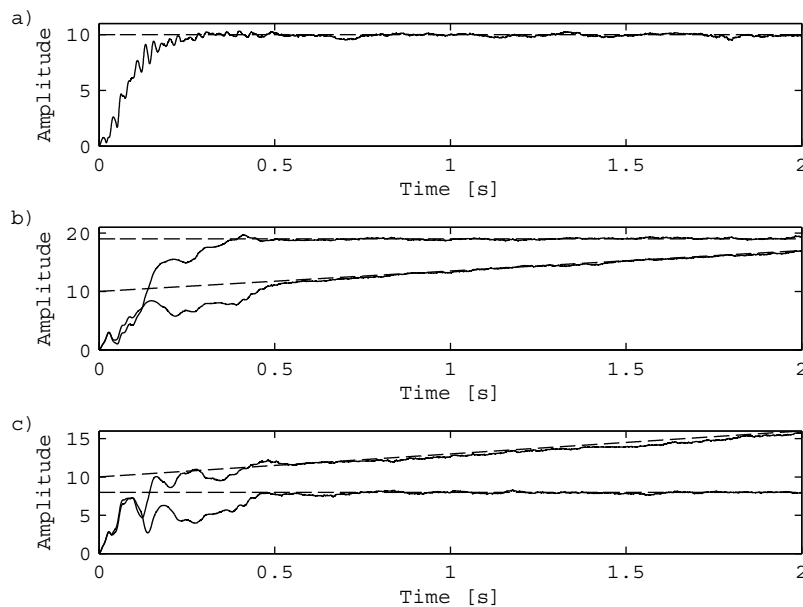


Figure 3. Estimated order amplitudes by using extended RLS algorithm ('--' are actual values): a) resonant frequency component; b) orders 1 and 2 of the first shaft; c) close orders 1 and 1.2 of the second shaft.

Estimated order amplitudes computed by using the standard RLS (Haykin, 2002; Bai et al., 2002) and extended RLS OT adaptive algorithms (Haykin et al., 1997), are given in Figure 2 and Figure 3, respectively. Actual values of the order amplitudes are denoted with dashed lines in all cases. As can be

seen in Figure 4, the extended RLS adaptive OT algorithm can simultaneously discriminate close and crossing order components effectively, while the amplitudes of orders extracted by the RLS adaptive algorithm, Figure 3, is not so well estimated. Furthermore, there is a discrepancy between the estimated and actual values of linearly increasing orders, whereas the estimated value of the fixed order is good, but transient time is long. For concision, we don't present here the results for the Vold-Kalman adaptive OT filter (Wu et al., 2004), since the identified orders are almost the same as for the values obtained by using extended RLS OT filter, which is not used as OT tool so far.

#### 4. Practical engineering implementation

The twisting machine, which operates in a production line for polypropylene binder, is used as example of practical engineering implementation of the considered adaptive OT methods. From the aspect of kinematics and machine dynamics, the twisting machine is a complex mechanical system. More details about the results pertaining to technical condition diagnostics, redesign, and analysis of its effects for a polypropylene yarn twisting machine are given by Tadić et al. (2011).

The vibrations of twisting machine are measured with B&K 4391 piezoelectric charge accelerometer, which have a frequency range from 0.1 [Hz] to 10 [kHz] and a charge sensitivity of 9.8 [pC/g]. Acceleration signal integration was done with a vibration signal conditioner with appropriate anti-aliasing low pass filter and a velocity vibration signal was obtained. The key phasor was placed on the drive shaft, i.e. reference shaft. Sampling frequency was 5 [kHz] with 16-bit resolution data acquisition module. In Figure 4a the profile of the estimated reference shaft rotational frequency, is shown, while in Figure 4b the measured time history of the vibration signal is presented.

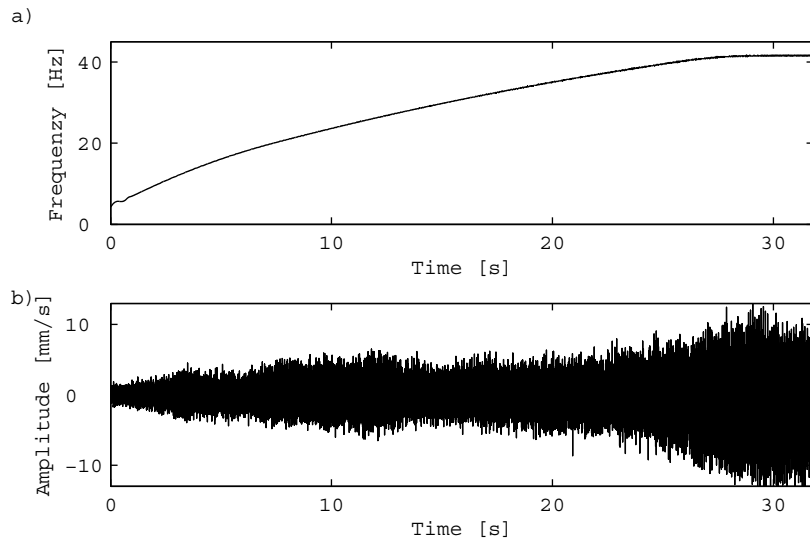


Figure 4. Example of signal for real vibration system: a) the estimated rotational frequency change of reference shaft; b) the measured time history of the vibration signal.

The amplitude of the first order, estimated by: VK recursive OT filter, the RLS OT algorithm, and the extended RLS OT algorithm, are shown in Figure 5a, 5b, and 5c, respectively, while in Figure 6 are presented the estimated amplitude of resonant frequency order, obtained by using the same adaptive OT methods, respectively. As can be seen, all three presented recursive schemes give practically identical results in this case, due to proper parameter tuning, and a relatively simple vibration system, in contrast with complex synthetic vibration signal, presented in previous chapter, illustrating a possible degradation in tracking capabilities of the RLS adaptive OT algorithm as a result of a model mismatch, and the neglecting of the states noise components (Haykin et al., 1997), which are present here in the generated synthetic vibration signal.

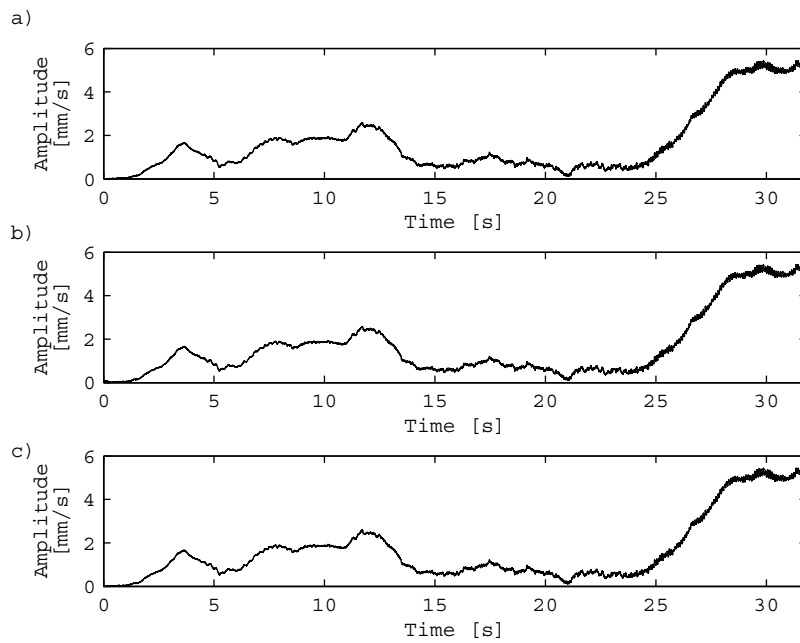


Figure 5. Estimated 1<sup>st</sup> order amplitude by using: a) VK OT filter; b) RLS OT filter; c) extended RLS OT filter.

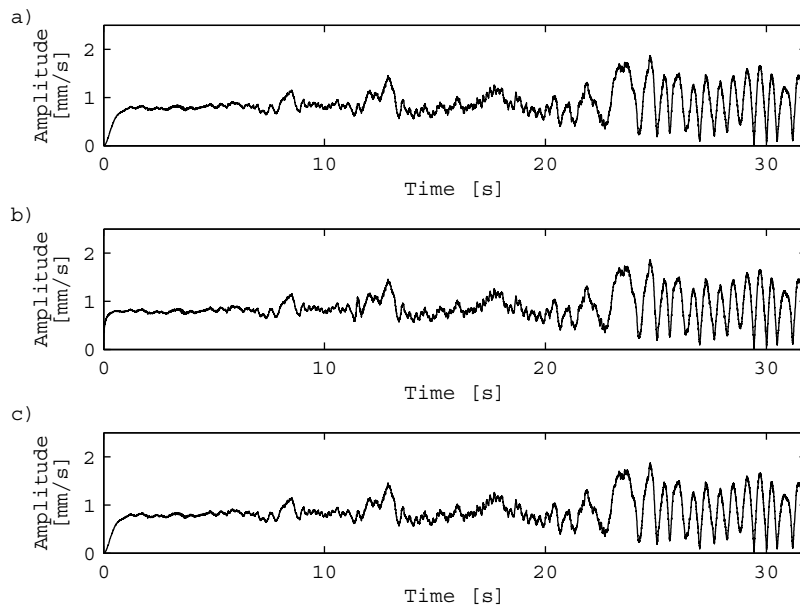


Figure 6. Estimated resonant ( $f_r = 48.5$  Hz) order amplitude by using: a) VK OT filter; b) RLS OT filter; c) extended RLS OT filter.

## 5. Conclusion

The adaptive estimation of the multiple orders magnitude from noisy non-stationary vibration or sound emission signals, for condition monitoring, troubleshooting, design and synthesis on rotating machinery, by using the RLS, extended RLS, and Vold-Kalman based recursive filtering algorithms were investigated in this paper. Comparisons and discussion of the implemented recursive OT schemes are accomplished through processing one representative synthetic signal, including crossing, close and resonant order components. Also one practical implementation of considered recursive OT tools is presented. All

investigated recursive OT methods are characterized by high resolution and extremely efficient computation, and could be realized as real-time applications in the diagnosis of rotating machinery defects for industrial purposes. Effectiveness of the extended RLS OT method, which is not used as OT tool so far, is demonstrated, together with the possible degradation of tracking possibilities of the standard RLS OT algorithm, when complex vibration system is under consideration. The presented results indicate that the extended RLS algorithm can be used as an alternative for VK recursive OT schemes, since the steady-state error and settling time characteristics of identified orders, can be tuned to be very similar for both methods, while the decreasing of the steady-state error for standard RLS OT method significantly prolongs settling time. As in the case of the off-line VK OT algorithms, practical implementation of the adaptive OT algorithms introduced here, still require some experience in parameter tuning, for obtaining accurate tracking of the orders amplitude.

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