CHEMICAL ENGINEERING TRANSACTIONS
VOL. 33, 2013
A publication of AIDIC
The Italian Association of Chemical Engineering
Online at: www.aidic.it/cet
Guest Editors: Enrico Zio, Piero Baraldi

ISBN 978-88-95608-24-2; **ISSN** 1974-9791
DOI: 10.3303/CET1333004

Relationship between Delay Time and Gamma Process Models

Renyan Jiang

Faculty of Automotive and Mechanical Engineering, Changsha University of Science and Technology,
960, Section II, Wan Jia Li Nan Lu, Changsha, Hunan 410004, China
jiang@csust.edu.cn

The delay-time concept and gamma process model have been widely applied in CBM settings. This paper reveals a close relationship between these two models. That is, given one of them, the other can be approximately determined. This nature is very useful in reliability modeling and maintenance decision analysis based on the delay time model or gamma process model because some analysis is easier for one of them than for the other. The method to make mutual conversions between these two models is developed and illustrated by three examples. The potential applications and possible extensions of the method are also discussed.

1. Introduction

Condition-based maintenance (CBM) has been widely used to achieve a high reliability for critical components or major failure modes in a degrading system. The basic idea is to specify the degradation level by condition monitoring or inspections so that the component can be preventively repaired or replaced before it fails. In simple cases, the CBM decision is directly based on the observed state. In this case, the degradation and failure models are needed for optimizing the alarm limit, failure limit or/and the inspection scheme (Jiang, 2010). Two typical models used in this context are the delay time model and the gamma process model.

Similar to the concept of P-F interval of RCM (Moubray, 1992), where “P” means potential failure and “F” means functional failure, the delay time concept divides the degradation process into normal and defective phases. The normal phase is the time interval from the use beginning to defect initiation; and the defective phase is the time period from a defect initiation to failure and is termed as delay time. The delay time concept is applied to maintenance by implementing an inspection scheme to check whether the item is defective or not. If a defect is found, the item is preventively repaired or replaced. In such a way, the maintenance action can be arranged in a relatively timely way and the operational reliability gets improved.

Okumura (1997) presents a method to utilize the delay-time model for determining the time points of inspection for a deteriorating system under condition-based maintenance. Wang (2008) presents a review of the delay time inspection models. Jiang (2012) develops an inspection timeliness measure and shows that there exists a unique inspection interval where the timeliness measure achieves its minimum. This interval is called the timeliness-based optimal inspection interval. Wang (2012) overviews the recent advances in delay-time-based maintenance modelling with focus on new methodological developments and industrial applications.

The gamma process has been extensively used to model degradation processes due to wear (Grall et al, 2002), corrosion (Yuan and Pandey, 2008), and so on; and is suited to represent the slow, continuous and progressive degradation process (Abdelbaki et al., 2012). Noortwijk (2009) presents a detailed literature survey of the application of gamma processes in maintenance. While applying the gamma process model to CBM, two degradation limits (i.e., alarm and failure thresholds) are usually pre-specified. Referring to Figure 1, the intersection point between the degradation curve and the alarm limit can be viewed as the “P” point of RCM and the intersection point between the degradation curve and the failure limit can be viewed as the “F” point. In terms of the delay time concept, the time interval with the degradation level being

Please cite this article as: Jiang R., 2013, Relationship between delay time and gamma process models, Chemical Engineering Transactions, 33, 19-24 DOI: 10.3303/CET1333004

smaller than the alarm limit can be viewed as the "normal phase" and the time interval with the degradation level being between the alarm and failure limits can be viewed as the "defective phase".

There are differences between the delay time model and the gamma process model. For the delay time model, the defect is physically identifiable and usually results in a "hard" failure; and the "defect" status is represented in a yes-or-no way without quantitative information. Once the inspection scheme is specified, the maintenance decision is straightforward. The "lead" time (i.e., the time interval between the identification of defect and the time to failure) may be short, depending on the distribution of delay time and the time when the defect is found. For the gamma process model, the failure is a "defined" status and usually associated with a functional failure or "soft" failure. The degradation level is quantitatively measured and the item with the degradation between the two limits is not necessary to have a "physically observable defect". Once the inspection scheme, alarm and failure limits are specified, the maintenance decision is also straightforward, and the lead time can be controlled by appropriately adjusting the alarm limit.

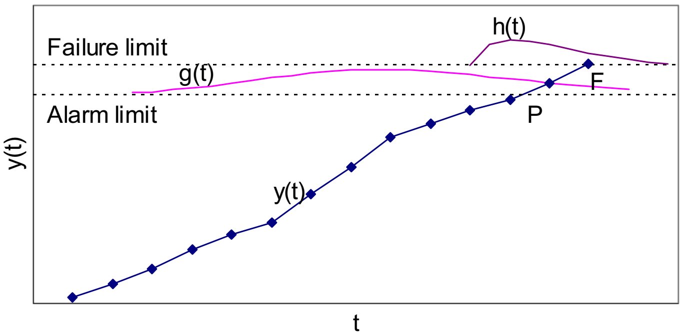

Figure 1: Links between P-F interval, gamma process and delay time

In this paper, we will further examine the relationship between the delay time and gamma process models, and show how they are mutually converted. The usefulness of such mutual conversions is obvious. For example, if one wants to fit a gamma process model but the data available is only enough for fitting a delay time model, it will be possible to derive the gamma process model from the fitted delay time model. Another potential application is to utilize the existing results from one model to analyze the other model if it is more difficult to analyze the latter model. Such an example is to optimize the inspection interval of CBM with the gamma degradation process using the delay-time model. This is because the relationship between item failure and inspection interval can be explicitly modelled by the delay-time model (see Wang, 2012).

The paper is organized as follows. Section 2 presents the details of the two models. The conversion method between the two models is presented and illustrated in Section 3. The paper is concluded in Section 4.

2. Delay time and gamma process models

2.1 Delay time model

Consider a single item with a major failure mode (e.g., fatigue). The failure process can be represented by the delay time concept. Let U and Z denote the initial and delay times, respectively. Assume that U and Z are mutually independent and their distributions are known and denoted as $G(u)$ and $H(z)$, respectively. Time to failure is a random variable given by $T=U+Z$. Let $F(t)$ denote the distribution of T. Given $G(u)$ and $H(z)$, $F(t)$ can be derived through a convolution operation.

To avoid the convolution operation, a simple way to get $F(t)$ is to use simulation. First, randomly generate a large sample of T ($=U+Z$), and then fit the empirical distribution of T obtained from the random sample to an appropriate distribution function. We illustrate the approach as follows.

Example 1: Assume that $G(t)=W(t;2.5,10)$ and $H(z)=Gamma(z;2,1)$, where $W(t;\beta,\eta)$ is a Weibull distribution with the shape parameter β and scale parameter η, and $Gamma(z;s,b)$ is a gamma distribution with the shape parameter s and scale parameter b.

Randomly generate a sample of T with sample size 5000, and fit the random sample to the Weibull, normal, lognormal and gamma distributions, respectively. It is found that the Weibull distribution with $\beta = 2.86$ and $\eta = 12.19$ provides the best fit. Figure 2 shows the empirical and fitted CDFs, which are close to each other.

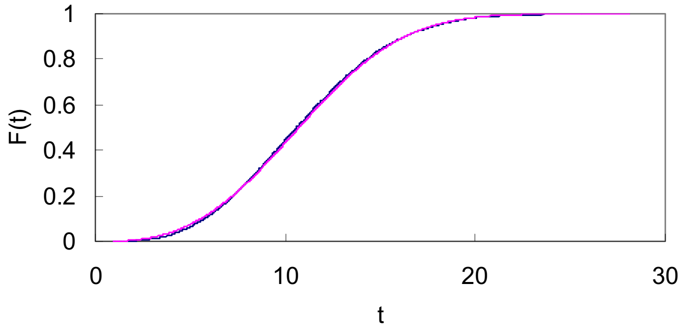

Figure 2: Empirical and fitted CDFs for Example 1

2.2 Stationary gamma process model

Suppose that a non-decreasing degradation quantity Y follows the stationary gamma process. In a given time t, $Y(t)$ is a random variable and follows the gamma distribution with shape parameter $s = at$ and scale parameter b. As such, the CDF of $Y(t)$ is given by $Gamma(y; at, b)$. An important property of the stationary gamma process is that increment ΔY is independent and has the distribution $Gamma(\Delta y; a\Delta t, b)$.

Referring to Figure 1, let y_a and y_0 denote the alarm and failure limits. The time to the alarm limit (similar to U in the delay time model) is a random variable and has the distribution function given by

$$F_a(t) = 1 - Gamma(y_a; at, b) \tag{1}$$

The time to the failure limit (similar to T in the delay time model) is a random variable and has the distribution function given by

$$F_0(t) = 1 - Gamma(y_0; at, b) \tag{2}$$

The duration that Y is between the alarm and failure limits (similar to Z in the delay time model) is a random variable and has the distribution function given by

$$F_z(z) = 1 - Gamma(y_0 - y_a; az, b) \tag{3}$$

2.3 Discussion

1. The delay time model can be fully specified by two distributional models and hence usually has four parameters. Given the distributions of two of U, Z and T, the other distribution can be derived. But the derivation is usually mathematically intractable.
2. The stationary gamma process model also has four parameters (a, b, y_a, y_0). The distributions of the three random variables (similar to U, Z and T) are given directly by (1) through (3), which can be easily evaluated using a spreadsheet program such as Microsoft Excel.

3. Conversions between the two models

3.1 Convert the delay time model into the gamma process model

Suppose that the delay time model is given by $H(z)$ and $F(t)$. The approach is similar for other combinations. Let

$$p = 0.01(0.01)0.99, \; z_p = H^{-1}(p), \; t_p = F^{-1}(p) \tag{4}$$

The problem is to find the parameters set (a , b , y_a , y_0) so that SSE achieves its minimum:

$$SSE = \sum_{p=0.01}^{0.99} \{[p - F_0(t_p)]^2 + [p - F_z(z_p)]^2\} \tag{5}$$

where $F_0(t)$ and $F_z(z)$ are given by (2) and (3), respectively.

Example 2: $H(z)$ and $F(t)$ are the same as those given in Example 1. Using the above approach, we have

$$(a,\ b,\ y_a,\ y_0) = (0.6100,\ 1.2801,\ 6.9693,\ 7.9108) \tag{6}$$

Ideally, $F_z(z_p) = F_0(t_p) = p$. That is, the plots of $F_z(z_p)$ and $F_0(t_p)$ versus p are a straight line through the points (0, 0) and (1, 1). Figure 3 shows the plots of $F_z(z_p)$ and $F_0(t_p)$ versus p . As seen, the fitted gamma process model roughly meets the desired property.

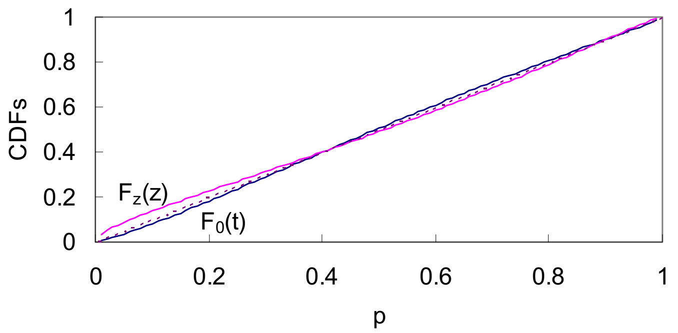

Figure 3: Goodness of fit of the gamma process model for Example 2

It is difficult to derive the distribution of U from $H(z)$ and $F(t)$ but the distribution of U can be easily obtained from the fitted gamma process model. Using the parameters given by (6) to (1), $G(t)$ can be approximated by $F_a(t)$, which can be further approximated by the Weibull distribution $W(t;2.81,10.84)$. Figure 4 shows the true distribution $W(t;2.5,10)$ and its approximations. As seen, the approximations are fairly close to the true distribution. This illustrates the usefulness of the conversion.

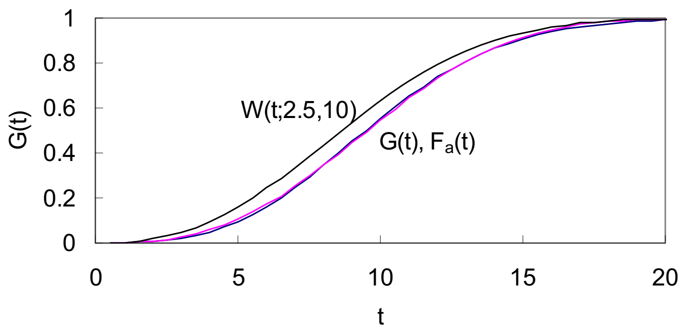

Figure 4: Plots of $W(t;2.5,10)$, $G(t)$ and $F_a(t)$ for Example 2

3.2 Convert the gamma process model to the delay time model

The conversion is straightforward by letting

$$F(t) = F_0(t)\ ,\ \ G(t) = F_a(t)\ ,\ \ H(z) = F_z(z) \tag{7}$$

As illustrated earlier, these distributions can be further approximated by common distributions such as the Weibull, gamma and lognormal distributions using the least square method.

Example 3: Assume that the parameters set of the gamma process model is

$$(a,\ b,\ y_a,\ y_0) = (0.6, 1.0, 7.0, 8.0) \tag{8}$$

The problem is to find the parameters of the delay time model. Without loss of generality, assume that T follows the Weibull distribution and Z follows the gamma distribution. Using the above approach, we obtained the parameters of the delay time model:

(β, η, s, b) = (3.38, 15.59, 1.98, 1.29) 9)

It is expected that the plot of $F_z(z)$ versus $H(z)$ or $F_0(t)$ versus $F(t)$ is a straight line through the points (0, 0) and (1, 1). Figure 5 shows these plots. As seen, the fitted delay time model meets the desired property well.

For a given delay time model, the optimal decision models and their solutions (e.g., optimization of inspection schemes) have been well documented in the literature (Wang, 2008). This implies that converting the gamma process model into the delay time model can facilitate the maintenance decision analysis.

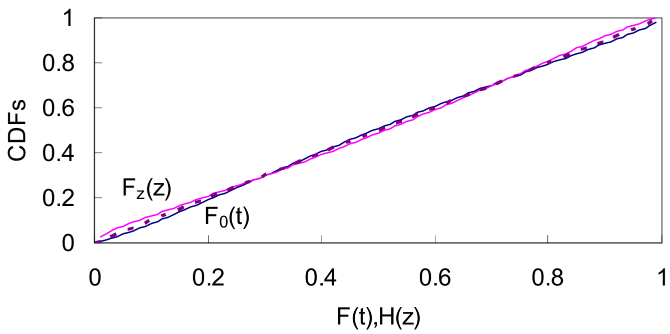

Figure 5: Goodness of fit of the delay time model for Example 3

4. Conclusions

In this paper, we have shown the links and differences between the delay time and gamma process models, and presented the approach to convert one model into the other model. The approach has been illustrated by three examples.

The approach presented in this paper is potentially useful for reliability modeling and maintenance decision analysis. Two possible applications are as follows:

1. To simplify the CBM decision optimization. For example, the optimization of the alarm limit and inspection scheme of CBM can be conducted in the framework of the delay-time model.
2. To simplify the spare part inventory control analysis in CBM setting. This is because the time to ordering a spare part is similar to U of the delay-time concept and the random lead time is similar to the delay time (e.g., see Segerstedt, 1994). As such, the spare part inventory control analysis can be also conducted in the framework of the delay-time model.

The method can be extended in the following several directions:

1. For the gamma process model in CBM setting, a topic for future study is to take the maintenance lead time into account for determining the alarm limit.
2. it is natural to extend the method to other stochastic processes such as the Wiener process (e.g., see Lindqvist and Skogsrud, 2009) rather than the gamma process.
3. It is more practical to extend the method to the non-stationary gamma process (e.g., see Nicolai et al., 2009) rather than the stationary gamma process.
4. For the delay time model, a topic for future study is to extend the approach to the case where U and H are correlated.

Acknowledgement

This research was supported by the National Natural Science Foundation (grant 71071026).

References

Abdelbaki N., Bouali E., Gaceb M., Bettayeb M., 2012, Analysis of the predictive models of oil and gas transportation ducts' rehabilitation. Chemical Engineering Transactions, 29, 889-894.

Grall A., Berenguer C., Dieulle L., 2002, A condition-based maintenance policy for stochastically deteriorating systems. Reliability Engineering and System Safety, 76, 167–180.

Jiang R., 2010, Optimization of alarm threshold and sequential inspection scheme. Reliability Engineering and System Safety, 95, 208–215.

Jiang R., 2012, A Timeliness-Based optimal inspection interval associated with the delay time model, 2012 Prognostics & system health management conference, 23 May - 25 May 2012, Beijing, 1-5.

Lindqvist B.H., Skogsrud G., 2009, Modelling of dependent competing risks by first passage times of Wiener processes. IIE Transactions, 41(1), 72-80.

Okumura S., 1997, An inspection policy for deteriorating processes using delay-time concept. International Transactions in Operational Research, 4(5–6), 365-375.

Moubray J., 1992, Reliability-Centred Maintenance. Industrial Press, New York, the USA.

Nicolai R.P., Frenk J.B.G., Dekker R., 2009, Modelling and optimizing imperfect maintenance of coatings on steel structures. Structural Safety, 31(3), 234-244.

Segerstedt A., 1994, Inventory control with variation in lead times, especially when demand is intermittent. International Journal of Production Economics, 35(1–3), 365-372.

Van Noortwijk J.M., 2009, A survey of the application of gamma processes in maintenance. Reliability Engineering and System Safety, 94, 2–21.

Wang W., 2008, Delay time modelling, in: Complex System Maintenance Handbook, Eds. Murthy DNP., Kobbacy AKS., Springer, Germany.

Wang W., 2012, An overview of the recent advances in delay-time-based maintenance modelling. Reliability Engineering and System Safety, 106, 165-178.

Yuan X.X., Pandey M.D., Bickel G.A., 2008, A probabilistic model of wall thinning in CANDU feeders due to flow-accelerated corrosion. Nuclear Engineering and Design, 238(1), 16–24.