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A Derivative Method for Minimising Total Cost in Heat Exchanger Networks through Optimal Area Allocation

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This paper presents a novel Cost Derivative Method (CDM) for finding the optimal area allocation for a defined Heat Exchanger Network (HEN) structure and stream data, without any stream splits to achieve minimum total cost. Using the Pinch Design Method (PDM) to determine the HEN structure, the approach attempts to add, remove and shift area to exchangers where economic benefits are returned. From the derivation of the method, it is found that the slope of the ε-NTU relationship for the specific heat exchanger type, in combination with the difference in exchanger inlet temperatures and the overall heat transfer coefficient, are critical to calculating the extra overall duty each incremental area element returns. The approach is able to account for differences in film coefficients, heat exchanger types, flow arrangements, exchanger cost functions, and utility pricing. Incorporated into the method is the newly defined “utility cost savings flow-on” factor, θ, which evaluates downstream effects on utility use and cost that are caused by changing the area of one exchanger. To illustrate the method, the CDM is applied to the distillation example of Gundersen (2000). After applying the new CDM, the total annual cost was reduced by 7.1 % mainly due to 24 % less HEN area for similar heat recovery. Area reduction resulted from one exchanger having a minimum approach temperature (ΔT_{min}) of 7.7 °C while the other recovery exchangers had larger ΔT_{min} values. The optimum ΔT_{min} for the PDM was 12.5 °C. The CDM solution was found to give a comparable minimum total area and cost to two recently published programming HEN synthesis solutions for the same problem without requiring the increased network complexity through multiple stream splits.

1. Introduction

Heat Exchanger Network (HEN) synthesis has been the focus of numerous studies (Furman and Sahinidis, 2002). The most significant contribution in this field over the past four decades has been the development of Pinch Analysis and the Pinch Design Method (PDM) (Linnhoff and Hindmarsh, 1983). Pinch is a holistic approach to network synthesis that now has a proven track record for achieving energy savings in a range of industries. Extensions to the original method, such as targeting total area, Heat Exchanger (HE) shells (Ahmad and Smith, 1989) and pressure drop (Polley et al., 1990), have been developed to improve its industrial relevance and profitability. Much less, however, has been published on improving the area allocation within HEN structures as an approach to reducing total cost.

Pinch Analysis uses thermodynamic principles to identify temperatures that constrain heat recovery of a process assuming a minimum approach temperature, ΔT_{min}. But the ΔT_{min} constraint for most problems results in non-cost optimal HEN area allocation partially due to differences in utility costs, stream heat transfer film coefficients, HE types and flow arrangements, HE capital costs, and approach temperatures. To an extent the ΔT contribution concept for individual streams, in place of a global ΔT_{min}, was developed to account for large differences in film coefficients. Simple methods for ΔT_{min} relaxation through dual minimum approach temperatures, where one ΔT_{min} is selected for heat recovery targeting and the other ΔT_{min} is applied to individual exchangers, have also been proposed (Shenoy, 1995).

Ait-Ali and Wade (1980) derived conditions to determine the optimal heat recovery area allocation in multi stage heat exchanger systems with any number of exchangers in series. The derivation was based on the

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Log-Mean Temperature Difference (LMTD) heat exchanger design method. Limitations of the method are that it only applies to multi-stage heat exchanger systems -- not HEN's in general -- and assumes counter flow heat transfer. Focus is directed towards achieving maximum heat recovery for a given total area, which does not necessarily equate to minimum total cost. The method of Ait-Ali and Wade (1980) is one of the few published methods on optimising area allocation in literature other than pure programming based approaches, e.g. (Gorji-Bandpy et al., 2011).

In this paper, a novel Cost Derivative Method (CDM) is developed and applied to produce near optimal area allocation within a defined HEN to achieve minimum total cost. The intention is for this method to be applied after HEN structures are developed by the PDM or some other approach. The approach attempts to add and shift area to recovery exchangers (RE) to where the greatest cost benefit is returned. Effectiveness–Number of Transfer Units (ε-NTU) relationships for sizing exchangers (Kays and London, 1998) form an essential part of the method. The CDM is able to account for differences in film coefficients, HE types and arrangements, HE cost functions, and utility pricing. The method is applied to the distillation example of Gundersen (2000) to illustrate the potential cost savings, although results are case specific.

2. Derivate method for cost optimal area allocation

2.1 Conditions for optimal heat exchanger sizing before detailed mechanical design

Consider, at the design stage, a process containing hot stream "j" and cold stream "a" with a HEN, which consists of a single heat recovery exchanger (RE) and hot and cold utility exchangers (UE), where the area, A, of RE_{aj} is increased by ΔA (Figure 1a).

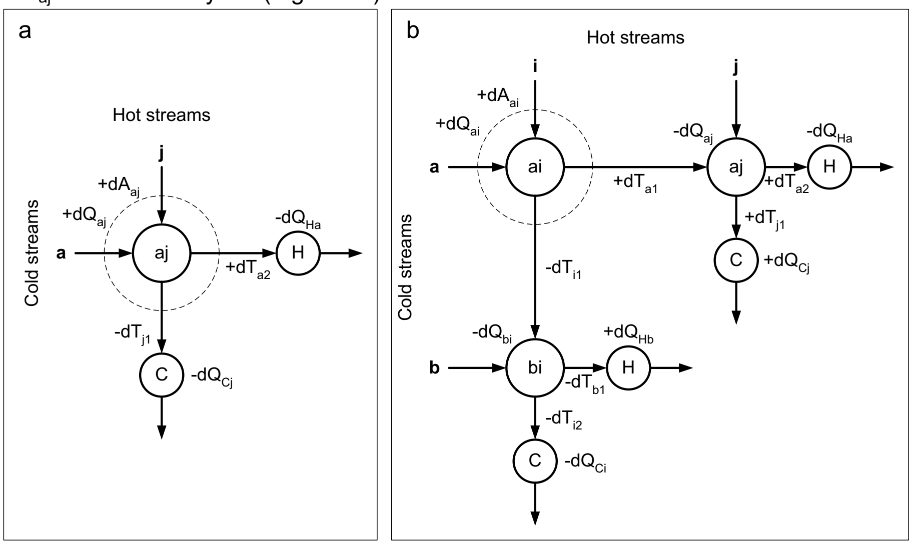

Figure 1: Effect of increasing area for a heat exchanger (a) and a simple heat exchanger network (b)

The resulting change in total annual cost, ΔTC, is

$$\Delta TC = \Delta CC_{RE} - \Delta S - \sum \Delta CC_{UE} + \sum \Delta PC \tag{1}$$

where CC is the annualised exchanger capital cost, S is annual utility savings due to heat recovery and PC is the pumping (or fanning) cost. If the pumping and utility exchanger capital costs are ignored and the change in area is very small, i.e. dA, then after dividing by dA_{aj}, Eq. 1 may be re-written

$$\frac{d(TC)}{dA_{aj}} = \frac{d(CC_{RE})}{dA_{aj}} - \frac{dS}{dA_{aj}}, \quad \text{where } CC_{RE} = FC + kA^n \text{ and } S = Q_r\left(p_a + p_j\right)t \tag{2}$$

Where FC is the fixed cost for a HE, A is exchanger area, k is a positive constant, n is a positive constant less than unity, Q_r is the duty, p is the hot and cold utility prices for streams a and j and t is the annual hours of plant production. By setting dTC/dA_{aj} to zero in Eq.(2) and rearranging we obtain

$$\frac{dS}{dA_{aj}} = \left(knA^{n-1}\right)_{aj} \tag{3}$$

Eq. 3 is true for global and local minima and is a criterion for optimality. Using the definition of S from Eq. 2, an expression for dS/dA_{aj} is obtained

$$\frac{dS}{dA_{aj}} = \left(\varphi \frac{dQ_r}{dA}\right)_{aj}, \quad \text{where } \varphi_{aj} = \left(p_a + p_j\right)t \tag{4}$$

Substituting definitions for Q_r and A from the ε-NTU HE design method and simplifying, Eq(4) becomes Eq. 5a. By recognising that the minimum heat capacity flow rate (C_{min}), the maximum difference between inlet temperatures for RE_{aj} (ΔT_{max}) and the overall heat transfer coefficient (U) are constant, then Eq. 5a simplifies to Eq.(5b).

$$\frac{dS}{dA_{aj}} = \left(\varphi \frac{d\left(\varepsilon C_{\min}\Delta T_{\max}\right)}{d\left(C_{\min}NTU/U\right)}\right)_{aj}, \quad \text{where } \Delta T_{\max} = T_{a,1} - T_{j,1} \text{ and } C_{\min} = \min\left(C_a, C_j\right) \tag{5a}$$

$$\frac{dS}{dA_{aj}} = \left(\varphi U \Delta T_{\max} \frac{d\varepsilon}{d(NTU)}\right)_{aj} \tag{5b}$$

Eq. 5 assumes U is independent of area, which at the pre-detailed mechanical design stage is a fair assumption. The derivative function for the counter flow ε-NTU relationship is

$$\frac{d\varepsilon}{d(NTU)} = \frac{\left(1 - C^*\right)^2 \exp\left(-NTU(1 - C^*)\right)}{\left(1 - C^* \exp\left(-NTU(1 - C^*)\right)\right)^2}, \quad \text{where } C^* \equiv \frac{C_{\min}}{C_{\max}} \tag{6}$$

C_{max} is the maximum heat capacity flow rate. The condition in Eq. 3 (and the subsequent equations) assumes there are no downstream RE's, which is not normally the case for all RE's in HEN's.

2.2 Conditions for optimal area allocation in a simple HEN

When the HEN in Figure 1b is considered, increasing the area of RE_{ai} increases its duty, but lowers the temperature driving force and duty of RE_{aj} and RE_{bi}. As a result the overall effect on utility savings is

$$\sum dS = \varphi_a\left(dQ_{ai} - dQ_{aj}\right) - \varphi_j dQ_{aj} + \varphi_i\left(dQ_{ai} - dQ_{bi}\right) - \varphi_b dQ_{bi} \tag{7}$$

Following the ε-NTU method, the change in duties for RE_{bi} and RE_{aj} are

$$dQ_{aj} = \left(\varepsilon C_{\min}\right)_{aj} dT_{a1} \tag{8a}$$

$$dQ_{bi} = \left(\varepsilon C_{\min}\right)_{bi} dT_{i1} \tag{8b}$$

where dT_{a1} and dT_{i1} are respectively the incremental changes in ΔT_{max} for RE_{aj} and RE_{bi}. Substituting Eq.(8a) and (8b) into Eq.(7) and rearranging gives

$$\begin{gathered}\sum dS = \varphi_{ai} dQ_{ai} - \varphi_{aj}\left(\varepsilon C_{\min}\right)_{aj} dT_{a1} - \varphi_{bi}\left(\varepsilon C_{\min}\right)_{bi} dT_{i1} \\ \text{where } \varphi_{ai} = \varphi_a + \varphi_i, \ \ \varphi_{aj} = \varphi_a + \varphi_j, \ \ \varphi_{bi} = \varphi_b + \varphi_i\end{gathered} \tag{9}$$

The terms dT_{a1} and dT_{i1} are related to the increase in duty of dQ_{ai}

$$dT_{a1} = \frac{dQ_{ai}}{C_a} \tag{10a}$$

$$dT_{i1} = \frac{dQ_{ai}}{C_i} \tag{10b}$$

Substituting Eq.(10a) and b into Eq.(9) gives

$$\sum dS = \varphi_{ai} dQ_{ai} - \varphi_{aj}\left(\varepsilon C_{\min}\right)_{aj} \frac{dQ_{ai}}{C_a} - \varphi_{bi}\left(\varepsilon C_{\min}\right)_{bi} \frac{dQ_{ai}}{C_i} \tag{11}$$

By dividing by dA_{ai} and substituting in the ε-NTU definitions for Q_r and A, Eq.(11) simplifies to

$$\frac{\sum dS}{dA_{ai}} = \left(\varphi U \Delta T_{\max} \frac{d\varepsilon}{d(NTU)} \theta\right)_{ai}, \quad \text{where } \theta_{ai} = 1 - \frac{\varphi_{aj}}{\varphi_{ai}} \left(\frac{\varepsilon C_{\min}}{C_a}\right)_{aj} - \frac{\varphi_{bi}}{\varphi_{ai}} \left(\frac{\varepsilon C_{\min}}{C_i}\right)_{bi} \tag{12}$$

The difference between Eq.(4) and Eq(12) is an additional term θ_{ai} called the utility cost savings flow-on factor and is defined as the ratio of the actual change in utility savings to the apparent change in utility savings as a result of dA_{ai}. For RE_{aj} and RE_{bi}, θ is unity because there are no downstream recovery exchangers.

2.3 Generalised conditions for optimal area allocation in HEN's without stream splits

The results of the derivation may be generalised by setting Eq.(3) and Eq(12) equal and applying the result to a heat recovery exchanger between streams x and y. Hence the condition for optimal RE area sizing is

$$\left(\varphi U \Delta T_{\max} \frac{d\varepsilon}{d(NTU)} \theta\right)_{x,y} = \left(knA^{n-1}\right)_{x,y} \tag{13}$$

It is important to note that φ may be divided by k in Eq.(13) to form a ratio of utility to capital cost. This ratio essentially determines the total HEN area that is economic. The network structure dependent term in Eq.(13) is the flow-on factor, θ. Additional work, not presented in this paper, has shown the generalised θ for any RE in a HEN without stream splits is

$$\theta_{x,y} = 1 - \frac{\varphi_{x,y+1}}{\varphi_{x,y}} \left(\frac{\varepsilon C_{\min}}{C_x} \theta\right)_{x,y+1} - \frac{\varphi_{x+1,y}}{\varphi_{x,y}} \left(\frac{\varepsilon C_{\min}}{C_y} \theta\right)_{x+1,y} \tag{14}$$

Applying Eq.(14) to RE_{ai} in Figure 1 gives the same θ as expressed in Eq.(12) because the value of θ for the downstream recovery exchangers is unity. As stated after Eq.(1), the problem was simplified by ignoring changes to pumping and utility exchanger capital costs that result from adding $dA_{x,y}$. As a result the solution to Eq.(13) is actually near optimal, rather than truly optimal.

2.4 Application of the method using an Excel™ Spreadsheet

The CDM has been implemented using an Excel™ Spreadsheet to optimise HEN area allocation for minimum total cost. To apply the CDM to a HEN, all ΔT_{min} constraints are removed and heaters and coolers are placed on all process streams to ensure target temperatures are met. Heat exchanger and energy continuity formulas are inputted to find a solution to Eq.(13) for each heat recovery exchanger by varying the area. In most HEN's, the heat exchangers are inter-dependent and so the complete minimum cost HEN solution requires a few iterations to solve. Besides the cost rate balance as expressed in Eq.(13), stream target temperatures may restrict the maximum duty of a recovery exchanger. Where stream target temperatures in the solution are met utility exchangers may be discarded.

2.5 Limitations and possible future extensions of the method

The CDM has several limitations, not all of which can be mentioned in this section. The CDM focuses only on the variable cost components, which can be a limitation at times when fixed costs may be eliminated by removing an exchanger unit and/or shell (if applicable). This problem can be partially overcome by taking advantage of loops and paths that may exist (after solving the CDM exchanger duties) to eliminate exchanger units. The method does not alter the placement of RE's but may add/remove UE's. A few other limitations of the CDM are it does not account for stream splits, heat loss, film coefficient variations with exchanger design, and ignores pumping and utility exchanger area costs/savings. These are viewed as areas for future improvement.

3. Application to a simple distillation process

Process and utility stream data for the distillation process from Gundersen (2000) are given in Table 1. Heat exchangers are assumed to be counter flow and exchanger capital costs are estimated using $CC = 4000 + 500A^{0.83}$. To select an initial (pre-design) value of ΔT_{min}, Gundersen applied super targeting to calculate ΔT_{min} = 10 °C. As a result the HEN structure in Figure 2a is based on ΔT_{min} = 10 °C. This study also calculates a post-design optimisation of ΔT_{min} (12.5 °C) to further minimise total cost.

The CDM is applied to the HEN structure in Figure 2 to solve for near cost optimal area allocation. The CDM solution achieved a similar level of heat recovery, 5.0 MW, to the PDM for ΔT_{min} = 12.5 °C. As shown in Figure 2b, the cooler on stream H1 is not required in the CDM solution, but a heater on C1 is added compared to the PDM solution (Figure 2a). To further reduce the total cost of the CDM solution, the heater load on C2 of 33 kW may be shifted to the heater on C1 by taking advantage of the path indicated. The CDM focuses on finding the minimum of the variable cost component. As a result for some cases, such slight modifications to reduce units (or shells) for minimising the fixed cost component of the total cost may be possible. Table 2 compares the duty, area, LMTD and dTC/dA Eq. (2) for each heat exchanger. Specifically, the relaxed CDM* increases the duty and area on RE_A due to its high LMTD. The duty of RE_B is constrained by the target temperature of stream H1 and as a result the dTC/dA_B is negative. The values of dTC/dA for the other RE's in the CDM are near zero, rather than zero, because the network has been relaxed using the path shown in Figure 2b. Negative values for dTC/dA indicate that area could be added to reduce TC.

Table 1: Process and utility stream data for a simple distillation process from Gundersen (2000).

Stream	Code	T_s [°C]	T_t [°C]	CP [kW/°C]	Q [kW]	h [kW/m^2 °C]	φ [$/y/kW]
Reactor outlet	H1	270	160	18	1980	0.5	
Product	H2	220	60	22	3520	0.5	
Feed	C1	50	210	20	3200	0.5	
Recycle	C2	160	210	50	2500	0.5	
Steam	HU	250	249			2.5	200
Cooling water	CU	15	20			1.0	20

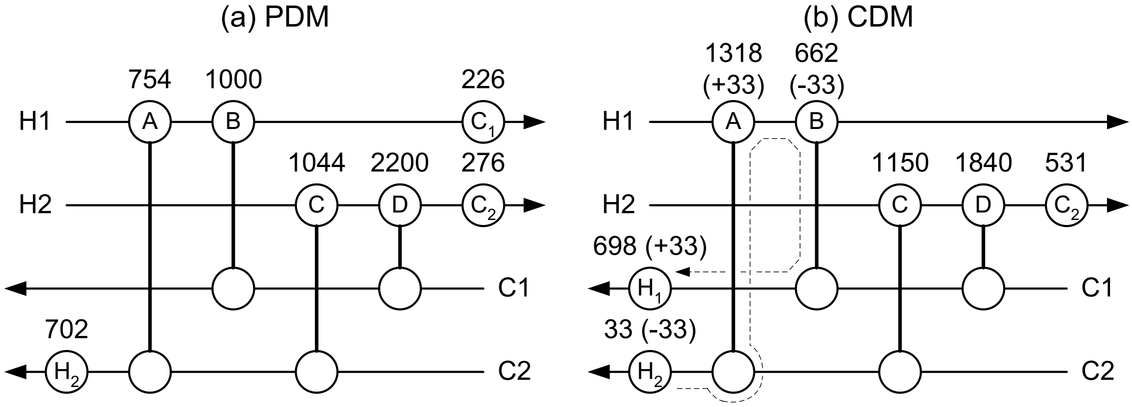

Figure 2: Heat Exchanger Network for a distillation process using the Pinch Design Method with ΔT_{min} = 12.5 °C (a) and the Cost Derivative Method (b). Duties are in kW

Table 2: A detailed comparison of PDM (ΔT_{min} = 12.5 °C) and CDM (relaxed) solutions.*

HE	Q [kW]		A [m^2]		LMTD [°C]		dTC/dA [$/m^2/y]	
	PDM	CDM*	PDM	CDM*	PDM	CDM*	PDM	CDM*
A	754	1350	51	181	59.6	29.8	-1945	17
B	1000	630	264	128	15.2	19.7	-23	-220
C	1044	1150	179	246	23.4	18.7	-278	3
D	2200	1840	516	247	17.1	29.8	17	-4
H_1	0	731	0	31	--	55.9		
H_2	702	0	36	0	46.2	--		
C_1	226	0	5	0	148.7	--		
C_2	276	531	17	29	48.7	54.0		

Table 3 summarises the results including three solutions from Escobar and Trierweiler (2013) for the same problem generated by Superstructure, Hyperstructure and Synheat programming HEN synthesis methods. Compared to the PDM with ΔT_{min} = 12.5 °C, the relaxed CDM solution uses 24 % less area, recovers a similar quantity of heat, and saves 7.1 % of the total cost. The programming synthesis methods developed new HEN's that reduced total area and cost. Even so, for this example the CDM obtained the lowest total cost using the same structure as PDM. It is important to note that the other programming methods increased network complexity as indicated by the number of stream splits and exchanger units, whereas the CDM focuses solely on optimal area allocation for a defined HEN. The programming methods were constrained to obtain the same heat recovery target as the PDM with ΔT_{min} = 10 °C. As a result, the total costs of programming methods do not necessarily represent the minimum total cost solution the method could find without such a constraint.

Future work will look at application of the method to optimal area allocation in HEN's for milk spray drying (Walmsley et al., 2012).

Table 3: Comparison of heat integration solutions for the distillation process. Superstructure, Hyperstructure and Synheat solutions were taken from Escobar and Trierweiler (2013).

	CDM	CDM* (relaxed)	PDM	PDM (ΔT_{min} opt.)	Super-structure	Hyper-structure	Synheat model
ΔT_{min} (°C)	7.7	7.7	10.0	12.5	10.0	6.9	9.9
RE	4	4	4	4	4	4	5
UE	3	2	3	3	2	2	2
Splits	0	0	0	0	2	4	3
ΣA (m^2)	855	863	1244	1067	1148	1105	1065
Q_r (MW)	4969	4969	5100	4998	5100	5100	5100
CC ($/y)	206,322	203,119	261,486	235,242	247,842	232,927	239,047
UC ($/y)	156,790	156,790	128,000	150,396	128,000	128,000	128,000
TC ($/y)	**363,111**	**359,908**	**389,486**	**385,638**	**375,842**	**360,927**	**367,047**

4. Conclusions

The novel Cost Derivative Method (CDM) derived in this paper has significant potential for improving the area allocation in Heat Exchanger Networks (HEN) to reduce total cost without increasing network complexity. Unlike the Pinch approach, the CDM (near) optimally accounts for differences in film coefficients, heat exchanger types and arrangements, exchanger cost functions, and utility pricing. Successful application of the CDM to the literature problem of Gundersen (2000) has decreased total cost by 7.1 % compared to the best Pinch Design Method solution. Future work and development is required to make the concept applicable to all HEN's, especially those networks with stream splits.

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