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The Modified Analogy of Heat and Momentum Transfers for Turbulent Flows in Channels of Plate Heat Exchangers

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The modification of Von Karman analogy for turbulent flow in channels of Plate Heat Exchangers (PHEs) is proposed. The resulting equation enables to calculate film heat transfer coefficients in PHE channel on a data of hydraulic resistance of the channel main heat transfer field, accounting for the influence of channel geometry, flow velocity and fluid properties. The comparison with experimental data for water flow in models of PHE channels main corrugated fields is presented. It is shown the good accuracy of prediction for film heat transfer coefficients. In the limiting case, where corrugations are parallel to plate axis, the results of calculations by proposed Equation are in excellent agreement with Equation published for straight tubes and channels by Gnielinski in 1975. The analysis of the Prandtl number influence on heat transfer is performed. It explains the difference of Pr powers, which varies from 0.6 to 0.3 at correlations reported in different experimental papers on heat transfer. The proposed Equation can be used for modelling of PHEs heat transfer performance in a wide range of different applications in process industries.

1. Introduction

Plate Heat Exchangers (PHEs) are one of the efficient types of contemporary heat exchangers with intensified heat transfer. Construction and operation principles of PHEs are sufficiently well described in literature, see e.g. book by Wang et al. (2007). Their application in process industries save space and construction material, increase reliability and operability compare to conventional shell and tubes heat exchangers. The heat transfer processes in PHE takes place in the channels of complex geometry, formed by plates pressed from thin sheet metal. The form of plate corrugations strongly influences the heat and hydraulic performance and the whole heat transfer efficiency of the PHE. The investigations on heat transfer in PHE channels, available in literature, generalize data in form of empirical correlations. In these correlations the influence of fluid velocity and its properties is usually accounted by functions of Reynolds and Prandtl numbers. The forms of such functions and predicted by them character of this numbers influence are significantly varying and are specific for investigated channels geometries and the range of experimental conditions.
To generalise data on heat transfer in PHE channels Martin (1996) have used Leveque analogy equation, which was initially proposed for laminar flow. Later same approach was employed by Dović et al. (2009) and gave reasonable accuracy in generalising heat transfer data of different authors. Arsenyeva et al. (2012) have used modification of Reynolds analogy which has proved fairly accurate in comparison with experimental data of different investigations in PHE channels. One of the problems with mentioned generalisation attempts is that they have fixed power at Prandtl number in correlations (0.33 in Leveque equation and 0.4 taken in modified Reynolds analogy). In fact the power at Prandtl number in empirical correlations for specific PHE channels by different authors vary in rather wide range, mostly from 0.3 to 0.5. Attempt to use Gnielinski Equation for PHE channels give discrepancies up to 300 % with experimental results. In present paper the modification of Von Karman analogy for PHE channels is

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proposed, which can predict the heat transfer characteristics for turbulent flow based on the data for friction factor at the channel corrugated field in a wide range of Reynolds and Prandtl numbers Pr ≥1.

2. Theoretical background

One of the developments for Von Karman analogy in pipes was proposed by Lyon (1951). He has derived following equation:

$$\mathrm{Nu}^{-1} = 2 \cdot \int_0^1 \frac{\left(\int_0^{\xi} \omega \cdot \xi \cdot d\xi \right)^2}{\left(1 + \varepsilon \cdot \mathrm{Pr} \cdot {\nu_T}/{\nu} \right) \cdot \xi} d\xi \tag{1}$$

where ξ = R/R$_0$ is the relative distance from tube centre; ω = w/W is the relative velocity; w is the local velocity, m/s; W is the average velocity, m/s; ν is the kinematic viscosity, m^2/s; ν_T is the turbulent viscosity (eddy diffusivity for momentum), m^2/s; $\varepsilon = \lambda_T/(c\rho)/\nu_T$ is the ratio of eddy diffusivities for heat and momentum; Pr is the Prandtl number Pr = $c\rho\nu/\lambda$; c is the specific heat, J/(kg·K); ρ is the density, kg/m^3; λ is the heat conductivity, W/(m·K).
As it was shown by Lyon (1951), Eq.(1) can be used regardless of the flow regime in pipe, with the proper estimation of velocity, ν_T and ε distributions. The method to use Eq.(1) for turbulent heat transfer in pipes based on Von Karman analogy is shown in a book by Kutateladze (1979). In our study the main assumptions of that method are taken in adaptation to turbulent flow in PHE channels.
Let's consider that the turbulent flow in direction perpendicular to PHE channel wall can be divided on viscous sub layer, buffer layer and turbulent main stream. For further analysis it is important to estimate the relative thickness of layers compare to the channel equivalent diameter De. For flows near smooth wall surface the distribution of velocities is closely related to shear stress on that wall τ_W. Using the coefficient of the total hydraulic resistance of the length unit of PHE channel ζ_S , for average shear stress on the wall it can be written:

$$\tau_W = \frac{\zeta_S \cdot \psi}{F_x} \frac{\rho \cdot W^2}{8} = \zeta_\tau \cdot \frac{\rho \cdot W^2}{8} \tag{2}$$

where ψ is the share of pressure losses due to friction in total pressure loss at channel main corrugated field; Fx is the coefficient of surface area enlargement because of corrugations; ζ_T is friction coefficient.
The correlations to calculate ζ_S and ψ in a wide range of corrugations geometrical parameters were reported by Arsenyeva et al. (2012a). The estimation of wall shear stress for water flowing in PHE channels in another paper by Arsenyeva et al. (2012b) have shown that in effectively working PHEs the shear stress on the wall can change from 10 Pa to 100 Pa and even more. Introducing dimensionless distance from the wall η the thickness of buffer layer (including viscous sub layer) can be estimated for such conditions from following relation (assuming η=η_2=30):

$$y = \eta \cdot \nu / (\tau_W / \rho)^{0.5} \tag{3}$$

For τ_W = 10 Pa and water at 50 °C y_2 = 0.17 mm diminishing to y_2 = 0.05 mm for τ_W = 100 Pa. It is from about 4 % to 1 % of De. The thickness of viscous sub layer is about 5 times even smaller, as by data of different researchers its dimensionless upper boundary estimated from η_1 = 5 to η_1 = 7. Counting on such small thickness of both layers it can be concluded that in this region:

1. Variable ξ is very close to 1, changing maximally from 0.9 to 1.
2. Compare to such dimensions the surface of the plates forming PHE channels can be considered as smooth. To be pressed from sheet metal of even 0.4-0.5 mm it should have curvature radius at least 1-1.5 mm not to jeopardize metal quality.
3. Counting that on integration of the inner integral of Eq.(1) most of its parts are outside of considered layers, it can be assumed ω = 1 in this region.

The right side of Eq.(1) can be presented as a sum of integrals corresponding to division of flow on turbulent main stream, buffer and viscous sub layers. Each of these integrals represents the influence on heat transfer of the specific region. As it is shown in literature (see eg. Lyon, 1951) for Pr >1 the main temperature change occurring in buffer and viscous sub layers. It becomes more close to the wall with increase of Prandtl number and the role of heat transfer in the main stream is diminishing. In view of this fact the estimation of the integral corresponding to turbulent main stream can be made in assumption, that its value is approximately equal to that in a flow core of the smooth tube with the same shear stress on the wall and diameter equal to equivalent diameter of the channel. Assuming also that there $\nu \ll \nu_T$, $\omega = 1$ and that eddy diffusivities of heat and momentum in all flow are equal ($\varepsilon=1$) we can rewrite Eq.(1) as follows:

$$\mathrm{Nu}^{-1} \approx \frac{1}{2} \cdot \left(\int_0^{\xi_2} \frac{\xi^3 d\xi}{\mathrm{Pr} \cdot \nu_T / \nu} + \int_{\xi_2}^{\xi_1} \frac{d\xi}{1 + \mathrm{Pr} \cdot \nu_T / \nu} d\xi + \int_{\xi_1}^{1} \frac{d\xi}{1 + \mathrm{Pr} \cdot \nu_T / \nu} d\xi \right) \tag{4}$$

To estimate turbulent viscosity in the turbulent flow at the central part of the tube ($0 \le \xi \le \xi_2$) more accurate than $\omega = 1$ distribution of velocity is needed. The logarithmic velocity profile can be used:

$$w = w^* \left(C_* + \frac{1}{\chi} \cdot ln \frac{v^* \cdot y}{\nu} \right) \tag{5}$$

Here $w^* = (\tau_W/\rho)1/2$, m/s; C^* is the constant; χ is the constant determined by experimental data for turbulent flow in tubes. Taking that the local shear stress is equal to turbulent shear stress and is proportional to distance from the centre $\tau = \tau_W \cdot \xi$, the turbulent viscosity can be estimated as:

$$\nu_\mathrm{T} = \frac{\tau_\mathrm{T}}{\rho \cdot dw / dy} = \chi \cdot w^* \cdot y \cdot \xi \tag{6}$$

Accounting for Eqs.(2) and (3), in dimensionless form:

$$\nu_\mathrm{T} / \nu = \chi \cdot \mathrm{Re} \cdot \sqrt{\zeta_\tau / 32} \cdot (1 - \xi) \cdot \xi \tag{7}$$

Substituting this Eq.(7) in first integral of Eq.(4) after integration gives for heat transfer in the main turbulent stream:

$$I_\mathrm{T} = \int_0^{\xi_2} \frac{\xi^3 d\xi}{\mathrm{Pr} \cdot \nu_T / \nu} = \frac{\sqrt{32}}{\mathrm{Pr}\, \chi\, \mathrm{Re} \sqrt{\zeta_\tau}} \left[\ln\left(\frac{\mathrm{Re}}{\eta_2} \sqrt{\frac{\zeta_\tau}{32}} \right) - \frac{1}{2} \left(1 - \frac{\eta_2}{\mathrm{Re}} \sqrt{\frac{32}{\zeta_\tau}} \right)^2 - 1 + \frac{\eta_2}{\mathrm{Re}} \sqrt{\frac{32}{\zeta_\tau}} \right] \tag{8}$$

To estimate turbulent viscosity in the buffer layer of PHE channel ($\xi_2 \le \xi \le \xi_1$) the logarithmic velocity profile correlation established in experiments for tubes and turbulent flows near flat surfaces can be used:

$$w = w^* \left(C_*' + \frac{1}{\chi'} \cdot ln \frac{v^* \cdot y}{\nu} \right) \tag{9}$$

Here C^*'and χ' are the empirical constants for buffer layer; χ' is the constant determined by experimental data for turbulent flow in tubes. Taking that the local shear stress τ in this layer is the sum of viscous and turbulent τ_T shear stresses and approximately equal wall shear stress, the turbulent viscosity can be estimated as follows:

$$\nu_\mathrm{T} = \frac{\tau_\mathrm{W}}{\rho \cdot dw / dy} - \nu \approx \chi' \cdot w^* \cdot y - \nu \tag{10}$$

Accounting for Eqs.(2) and (3), in dimensionless form:

$$\nu_\mathrm{T} / \nu \approx \chi' \cdot \mathrm{Re} \cdot \sqrt{\zeta_\tau / 32} \cdot (1 - \xi) - 1 \tag{11}$$

Substituting this Eq.(7) into the second integral of Eq.(4) after integration gives for heat transfer in the buffer layer:

$$I_{\mathrm{B}} = \int_{\xi_2}^{\xi_1} \frac{d\xi}{1+\mathrm{Pr}\cdot {}^{\nu_T}\!/_{\nu}} = \frac{\sqrt{32}}{\mathrm{Pr}\,\chi'\,\mathrm{Re}\sqrt{\zeta_\tau}} \ln\left(\frac{1+\mathrm{Pr}(\chi'\cdot\eta_2-1)}{1+\mathrm{Pr}(\chi'\cdot\eta_1-1)}\right) \tag{12}$$

The third integral in Eq.(4) characterise the heat transfer in viscous sub layer. At Prandtl numbers much higher than unity the biggest temperature gradient occurs in this area and its parts closest to the wall. It requires accounting the influence of intruding from outer layers turbulent pulsations. Following Kutateladze (1979) it can be done by introducing following relation:

$${}^{\nu_{\mathrm{T}}}\!/_{\nu} \approx \beta\cdot\eta^3 \tag{13}$$

where β is the empirical proportionality coefficient, which value is estimated as $\beta \approx 0.03$. According to Kutateladze (1979) in the 3rd integral of Eq.(4) for heat transfer another empirical coefficient β_{T} should be used and this integral can be presented as follows:

$$I_{\mathrm{L}} = \int_{\xi_1}^{1} \frac{d\xi}{1+\mathrm{Pr}\cdot {}^{\nu_{\mathrm{T}}}\!/_{\nu}} = \frac{1}{\mathrm{Re}}\sqrt{\frac{\xi_\tau}{32}} \int_0^{\eta_1} \frac{d\eta}{1+\mathrm{Pr}\cdot\beta_{\mathrm{T}}\cdot\eta^3} \tag{14}$$

The analytical expression for the integral of the function type $1/(a^3+x^3)$ are rather cumbersome and long. It is more convenient to perform numerical integration on a computer. The following values for the turbulent velocity profile parameters and empirical coefficients in equations were assumed: η_2 = 30; η_1 = 6.8; χ = 0.37; χ' = 0.2; $\beta_{\mathrm{T}} = \beta/\eta_1^2$. As a result for calculation of Nusselt number the following expression was obtained:

$$\mathrm{Nu} = \frac{0.131\cdot R_\xi\cdot \mathrm{Pr}}{\ln\left(\frac{R_\xi}{760}\right) - \frac{14450}{R_\xi^{\,2}} + \frac{340}{R_\xi} + 1.85\ln\left(\frac{1+5\,\mathrm{Pr}}{1+0.36\,\mathrm{Pr}}\right) + 2.52\,\mathrm{Pr}\cdot\varphi(\mathrm{Pr})} \tag{15}$$

where $R_\xi = \mathrm{Re}\cdot\sqrt{\frac{\zeta_S\cdot\psi}{F_x}}$;

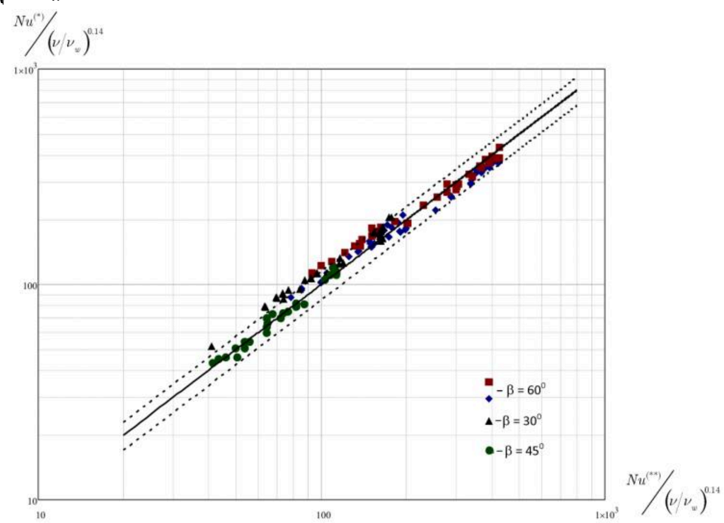

Figure 1: The comparison of Nu(*) (solid line) calculated by Eq.(15) with experimental Nu(**): dashed lines corresponds to error ±15 %.

$$\varphi(\mathrm{Pr}) = \frac{1}{\eta_1}\cdot\int_0^{\eta_1}\frac{d\eta}{1+\mathrm{Pr}\cdot\beta_{\mathrm{T}}\cdot\eta^3} \approx \frac{1.14}{\eta_1}\cdot\mathrm{Pr}^{-0.04}\cdot arctg\left(\eta_1\cdot\sqrt[3]{\mathrm{Pr}\cdot\beta_{\mathrm{T}}}\right) \tag{16}$$

The above approximation of the integral solution deviate from numerical one not more than ±2 % for $3 \le Pr \le 10^4$ and not more than -8 % for Pr as low as 0.69. Accounting that for such low Pr the share of the viscous sub layer in total resistance to heat transfer becomes relatively smaller than of other parts, this approximation can be used to save computing time.

3. Comparison with experiments and discussion of the results

To validate Eq.(15) and check the limits of its application it should be compared to experimental data on heat transfer in channels of different geometrical form. On Figure 1 the experimental results for models of PHE channels of different corrugation size and inclination angle described in paper of Arsenyeva et al. (2012a) are compared with prediction by Eq.(15). The coefficient ζ_S and the share of pressure losses due to friction in total pressure loss were determined by generalized correlations presented in that paper. The Pr numbers are taken from experiments. The deviation of calculated Nu numbers from experimental ones not exceeded 15 % with mean squire error 6.5 %.

The experiments presented in Figure1 were made with water as tested fluid in a limited range of Pr numbers from 1.9 to 7. The available in literature data on heat transfer in PHE channels are presented as empirical correlations with mentioning just the range of Pr numbers, with no data on Pr in individual experiments. Arsenyeva et al. (2012a) has presented generalised correlation for heat transfer based on modification of Reynolds analogy:

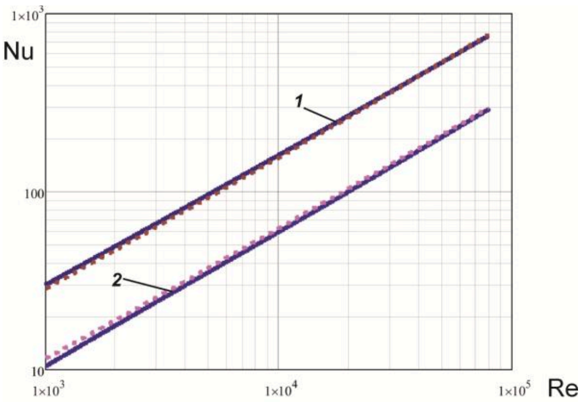

Figure 2: The comparison of $Nu_{(10)}$ (solid lines) calculated by Eq.(15) with calculated by Eq.(12) $Nu_{(12)}$ (dashed lines) for $\nu/\nu_W=1$ and Pr=1: 1 – $\beta=65^o$; 2 - $\beta=30^o$.

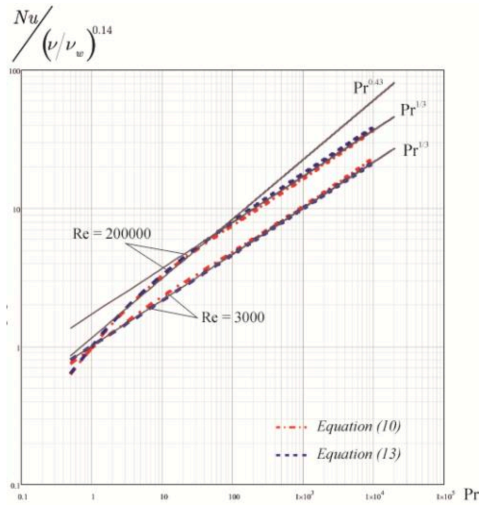

Figure 3: The comparison of Eq.(15) with calculations by Gnielinski (1975) Equation and correlations with fixed Pr powers.

$$\mathrm{Nu} = 0.065 \cdot \mathrm{Re}^{6/7} \cdot \left(\psi \cdot \zeta_s / \mathrm{F_x}\right)^{3/7} \cdot \mathrm{Pr}^{0.4} \cdot \left(\nu / \nu_W \right)^{0.14} \tag{17}$$

In this correlation the power of Pr number was fixed to 0.4 and comparison with quite a number of literature data was made by adjusting results accounting for the value of Pr power at correlation under comparison and the range of Pr specified in the papers. The discrepancies not exceeded 15%. In Figure 2 is presented the comparison of calculations by Eqs.(15) and (17) at $(\nu/\nu_W)=1$ and Pr=1. The discrepancies not exceed 5 %. It can be concluded that Eq.(15) predicts the influence of corrugation geometry on heat transfer with practically same accuracy as Eq.(17).

The accuracy of Eq.(15) in accounting of Prandtl number influence on heat transfer was estimated by comparison with calculations on Gnielinski (1975) Equation for flows inside straight smooth tubes:

$$\mathrm{Nu} = \frac{\zeta \cdot \mathrm{Pr} \cdot \left(\mathrm{Re} - 1000\right)}{8 \cdot \left[1 + 12.7 \cdot \sqrt{\zeta / 8 \cdot \left(\mathrm{Pr}^{2/3} - 1 \right)} \right]} \tag{18}$$

Here ζ is the friction factor in smooth tube calculated by correlation:

$$\zeta = \left(0.79 \cdot \ln \mathrm{Re} - 1.64\right)^{-2} \quad (19)$$

To compare the calculations it was assumed $\zeta_s = \zeta$; $F_x = 1$; $\psi = 1$. The discrepancies of the results (see Figure 3) not exceed 6 %. The accuracy of Eq(18) suitable for practical applications in wide range of Prandtl numbers (0.5 ÷ 100,000) was confirmed in papers of Gnielinski (2009) and by authors of Perry's Chemical Engineers Handbook (2008). The good agreement of the results by both Equations allows to recommend It for calculations in the same range of Prandtl numbers not only for smooth tubes but also for PHE channels.

The influence of Prandtl number on heat transfer can be analysed using graphs in Figure 3. For fully developed turbulent flow (the curves for Re = 200,000 in Figure 3) for 1.5≤Pr≤12 the curves can be approximated by using Pr in power 0.4, as it is made in Nusselt Equation (see Kutateladze, 1979). In known Equation of Micheyev for this range of Pr the power 0.43 was used. When the Prandtl number becomes bigger than 20, the power 0.33 can be used, which is usually apply for laminar flow. It can be explained by shifting of the main part of thermal resistance to viscous sub layer - 3rd Integral in Eq(4).

For relatively low Reynolds numbers (the curves for Re=3,000 in Figure 3, which correspond to transition flow regime in smooth tube) the power at Pr number can be taken as 0.33 for all regarded range of Prandtl numbers. It can be explained by lower intensity of turbulence in the main stream, which causes smaller influence on regions with effects of laminar heat transfer (buffer and viscous sub layers). Consequently, the influence of Prandtl number on heat transfer depends not only on its value, but also from Reynolds number characterizing the flow hydrodynamics in channel. The accurate prediction of Prandtl number influence in wide range of Prandtl and Reynolds numbers is not possible by introducing it as multiplier in some fixed power. The calculations must be made with Eq(15) accounting for different flow conditions and Reynolds number influence.

4. Conclusions

The proposed Eq.(15), based on Von Karman analogy of heat and momentum transfer, gives good prediction of the influence on heat transfer of Prandtl number in channels of PHEs. It allows extrapolation of empirical correlations obtained for a limited range of Prandtl numbers for wider applications and can be used for optimization of PHEs in different process conditions.

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