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PDLF-Based Robust MPC of a Heat Exchanger Network

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The paper is focused on the case study of the advanced control of the heat exchanger network (HEN). The HEN is used for cooling petroleum produced by distillation. The robust model predictive control (RMPC) strategy is implemented to find the optimal control actions taking into account the boundaries on the control inputs. RMPC approach is also able to design the controller managing the process uncertainties. The aim is to demonstrate that the HEN robust model predictive control can be improved and the energy efficiency can be optimized using the parameter-dependent Lyapunov functions (PDLF). The simulation results confirmed also the energy savings.

1. Introduction

The heat exchangers (HEs) are often used in the chemical industry. Due to the fact that heat losses can rise up to 50 % (Čuček et al., 2013), there is the necessity to implement the advanced optimization-based control algorithms, e.g. model-based predictive control (MPC) (Bemporad and Morari, 1999). It was demonstrated in Pan et al. (2013) that the non-linear model of HEN can be found as the solution of an optimization problem. In the paper Walmsley et al. (2013) the optimization was utilized to determine optimal structure of a HEN. The HEN control using PID controllers was studied e.g. in Ipsakis et al. (2013). The uncertain HE control via H_2 and H_∞ approaches was studied in Vasičkaninová and Bakošová (2013). As the HENs belong to the key devices in the petroleum industry (González et. al, 2006) with high energy demands, it is important to find optimal control of the HEN. It was shown in Bakošová and Oravec (2013) that the robust MPC strategy decreased energy consumption during the HEN operation. The aim of this paper is to investigate further possibilities to increase the energy savings. In the presented case study the HEN was utilized to cool the petroleum. The hot petroleum was the product of distillation and the water was the cooling medium. Three robust MPC approaches were designed and the control performance of three counter-current shell-and-tube HEs in series was studied by simulations for two scenarios in each strategy, the worst and the best case scenarios.

2. Controlled heat exchangers

Based on the previous research, the controlled process was adopted from Bakošová and Oravec (2013) and is briefly described next. The simple HEN is composed of three identical counter-current shell-and-tube HEs in series. The feed of the HEN to be cooled down is the petroleum as a product of distillation in a refinery. Petroleum flows in the inner tubes and the cooling water in shell of every heat exchanger. The tubes of the HEs are made from steel. The controlled variable is the temperature of the outlet stream of the petroleum from the 3rd HE. The control input is the volumetric flow rate of the inlet cold water into the 3rd HE. The objective is to decrease the outlet temperature of the petroleum to the reference value 45 °C and to minimise the energy demands measured by the total consumption of cold water. The technological parameters and control conditions are the same as in Bakošová and Oravec (2013) and are summarized in Table 1, where n is the number of HE's tubes, l is the length of the HE, $d_{\text{in},1}$ is the inner diameter of the tube, $d_{\text{out},1}$ is the outer diameter of the tube, $d_{\text{in},2}$ is the inner diameter of the HE, A_h is the total heat transfer area, V is the volume, c_p is the thermal capacity, ρ is the density, T_in is the inlet temperature, q is the volumetric flow rate. The subscripts 1 and 2 refer to the water and petroleum, respectively. The

Table 1: Technological parameters and reference values of HEs

Parameter	Unit	Value	Parameter	Unit	Value
n	1	40	$T_{in,1}$	°C	20.0
l	m	6	$T_{in,2}$	°C	180.0
$d_{in,1}$	m	19×10^{-3}	$T_1^{(1),S}$	°C	75.8
$d_{in,2}$	m	414×10^{-3}	$T_1^{(2),S}$	°C	48.0
$d_{out,1}$	m	25×10^{-3}	$T_1^{(3),S}$	°C	30.8
A_h	m^2	16.6	$T_2^{(1),S}$	°C	113.0
V_1	m^3	91.2×10^{-3}	$T_2^{(2),S}$	°C	71.3
V_2	m^3	716.5×10^{-3}	$T_2^{(3),S}$	°C	45.3
q_2^S	$m^3\ s^{-1}$	5.8×10^{-3}	$T_1^{(1),0}$	°C	87.1
$c_{p,1}$	$J\ kg^{-1}\ K^{-1}$	4.186×10^3	$T_1^{(2),0}$	°C	55.7
$c_{p,2}$	$J\ kg^{-1}\ K^{-1}$	2.140×10^3	$T_1^{(3),0}$	°C	34.4
ρ_1	$kg\ m^{-3}$	980.0	$T_2^{(1),0}$	°C	118.4
ρ_2	$kg\ m^{-3}$	810.0±16.2	$T_2^{(2),0}$	°C	76.8
U	$J\ s^{-1}\ m^{-2}\ K^{-1}$	482.3±9.7	$T_2^{(3),0}$	°C	48.7

superscripts (1) – (3) denote individual HEs and the superscripts S and 0 denote the reference value and the initial value, respectively.

Furthermore, two interval parametric uncertainties are considered – the heat-transfer coefficient U changes as the flow rate of the cooling medium changes, and the density of the petroleum ρ_2 depends on the temperature in the HEs (Table 1).

3. Robust MPC

For the robust MPC design, the mathematical model of the heat exchangers was derived using the enthalpy balances. The linearized time-invariant state-space model in the discrete-time domain is given by

$$\begin{aligned} x(k+1) &= A^{(v)}x(k)+B^{(v)}u(k),\ x(0)=x_0 \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where k represents the discrete time. The used sampling period was t_s = 25 s. Further, $x(k)$ is the vector of states represented by the temperatures $T_1^{(1)–(3)}$ and $T_2^{(1)–(3)}$ (Table 1), $u(k)$ is the control input represented by the volumetric flow rate of the cooling medium q_1, $y(k)$ is the vector of the system outputs. The matrices $A^{(v)}$, $B^{(v)}$, C have appropriate dimensions. The model in Eq(1) is an uncertain system with interval polytopic uncertainty. For the uncertain model of the HEN one can obtain four vertices computed as the combination of boundary values of uncertain parameters. Hence, the matrices $A^{(v)}$, $B^{(v)}$, v = 1,…,4, describe the vertex systems of the uncertain system Eq(1). The 5th considered system is the nominal system calculated for the mean values of the uncertain parameters (Table 1). Then the robust static state-feedback control problem in the discrete-time domain can be formulated as follows: find the state-feedback control law

$$u(k)=F_k x(k) \tag{2}$$

for the system described by Eq(1). The matrix F_k in Eq(2) represents the static state-feedback robust controller for the k-th control step.

The quality of the control performance can be described using the quadratic cost function

$$J=\sum_{k=0}^{n_k}\left(x(k)^T W_x x(k)+u(k)^T W_u u(k)\right) \tag{3}$$

where n_k is the total number of control steps. For the design purposes the infinity control horizon is assumed, and W_x, W_u are the real square symmetric positive-definite weight matrices of the states $x(k)$ and the system inputs $u(k)$. The aim is to design the controller F_k that ensures robust stability of all considered vertex systems and minimizes the cost function J in Eq(3). The control performance can be improved by taking into account the symmetric constraints on the system outputs $y(k)$ and inputs $u(k)$ in the form

$$\|y(t)\|^2 \le y_{max}^2,\ \|u(t)\|^2 \le u_{max}^2 \tag{4}$$

Following conditions hold for the symmetric positively defined Lyapunov matrix P_k and the feedback controller F_k

$$P_k = \gamma_k Q_k^{-1}, Y_k = F_k Q_k, \Rightarrow F_k = Y_k Q_k^{-1} \tag{5}$$

where γ_k is the auxiliary optimization parameter, Q_k is the symmetric positively defined matrix, and Y_k represents the auxiliary matrix enabling the evaluation of the robust feedback controller F_k (Cuzzola et al., 2002).

Several strategies were used to investigate the robust MPC of the HEN. *$RMPC_1$* denotes the control strategy described in the paper (Kothare et al., 1996). The algorithm for the controller design by the *$RMPC_1$* was presented in the paper Bakošová and Oravec (2013).

The approach denoted *$RMPC_2$* was introduced in Cuzzola et al. (2002), and refined in Mao (2003). The robust stabilization problem can be solved as the robust MPC convex optimization problem based on the LMIs as follows (Cuzzola et al., 2002)

$$\min_{\lambda_k, X_k, Y_k} \gamma_k \tag{6}$$

subject to

$$\begin{bmatrix} 1 & x_k^{\mathrm{T}} \\ * & X_k^{(v)} \end{bmatrix} \geq 0, \begin{bmatrix} Q_k + Q_k^{\mathrm{T}} - X_k^{(v)} & \left(A^{(v)} Q_k + B^{(v)} Y_k\right)^{\mathrm{T}} & Q_k^{\mathrm{T}} \sqrt{W_{\mathrm{x}}} & Y_k^{\mathrm{T}} \sqrt{W_{\mathrm{u}}} \\ * & X_k^{(v)} & 0 & 0 \\ * & * & \gamma_k I & 0 \\ * & * & * & \gamma_k I \end{bmatrix} \geq 0 \tag{7}$$

where v = 1,…, n_{v}. The symbol * denotes a symmetric structure of the matrix, and I, 0 are the identity and zero matrices of appropriate dimensions. $X_k^{(v)}$ are the symmetric positively defined matrices. The symmetric constraints on control inputs and outputs in the form of Eq(4) can be added to the optimisation problem Eq(6) – Eq(7) in the following LMI form

$$\begin{bmatrix} u_{\max}^2 I & Y_k \\ * & Q_k + Q_k^{\mathrm{T}} - X_k^{(v)} \end{bmatrix} \geq 0, \begin{bmatrix} Q_k + Q_k^{\mathrm{T}} - X_k^{(v)} & \left(A^{(v)} Q_k + B^{(v)} Y_k\right)^{\mathrm{T}} C^{\mathrm{T}} \\ * & y_{\max}^2 I \end{bmatrix} \geq 0 \tag{8}$$

where $v = 1,\ldots, n_{\mathrm{v}}$.

The algorithm for the *$RMPC_2$* can be formulated in following eight steps (Cuzzola et al., 2002).
Step 1: Set parameter $k = 0$.
Step 2: Set number of control steps N, initial conditions of states $x(0)$, values of the symmetric constraints on control input $u_{\max}$ and output $y_{\max}$.
Step 3: Set parameter $k = k + 1$.
Step 4: Set the values of states $x(k)$.
Step 5: Solve optimization problem described by Eq(6) – Eq(8) to evaluate Q_k, $X_k^{(v)}$ and Y_k.
Step 6: Design the matrix F_k of the feedback controller using Eq(5).
Step 7: Calculate the control input $u(k)$ using the control law Eq(2).
Step 8: If the parameter $k < N$ then go to the *Step 3* else *Stop*.

The third considered strategy, denoted as *$RMPC_3$*, is the robust MPC approach presented in Cao et al. (2005). In this approach, the single Lyapunov function is considered and the maintenance of input constraints is modified. This procedure reduces the conservativeness of control input evaluation and all at once ensures the robust stability. On the other hand, the additional saturation of computed values of control inputs is necessary. In the optimisation problem in Eq(6) – Eq(8) the LMIs presented in Eq(7) are replaced using

$$\begin{bmatrix} 1 & x_k^{\mathrm{T}} \\ * & X_k \end{bmatrix} \geq 0$$

$$\begin{bmatrix} X_k & \left(A^{(v)}X_k + B^{(v)}\left(E_jY_k + E_j^{-}U_k\right)\right)^{\mathrm{T}} & X_k^{\mathrm{T}}\sqrt{W_{\mathrm{x}}} & \left(E_jY_k + E_j^{-}U_k\right)^{\mathrm{T}}\sqrt{W_{\mathrm{u}}} \\ * & X_k & 0 & 0 \\ * & * & \gamma_k I & 0 \\ * & * & * & \gamma_k I \end{bmatrix} \geq 0 \tag{9}$$

Instead of LMIs in Eq(8) the constraints are handled by following LMIs

$$\begin{bmatrix} u_{\max}^2 I & U_k \\ * & X_k \end{bmatrix} \geq 0, \begin{bmatrix} X_k & \left(A^{(v)}Q_k + B^{(v)}\left(E_jY_k + E_j^{-}U_k\right)\right)^{\mathrm{T}} C^{\mathrm{T}} \\ * & y_{\max}^2 I \end{bmatrix} \geq 0 \tag{10}$$

for $v = 1,\dots, n_{\mathrm{v}}$, $j = 1,\dots, n_{\mathrm{u}}$. The matrices E_j are the diagonal matrices with all variations of 1 and 0 on principal diagonal and zeroes elsewhere. Then the E_j^{-} are the complement matrices obtained as $E_j^{-} = I - E_j$. The idea of this extension is to take into account all variations of constrained and unconstrained control inputs. Then the algorithm for the $RMPC_3$ can be formulated in following eight steps (Cao et al., 2005).

Step 1: Set parameter $k = 0$.
Step 2: Set number of control steps N, initial conditions of states $x(0)$, values of the symmetric constraints on control input $u_{\max}$ and output $y_{\max}$.
Step 3: Set parameter $k = k + 1$.
Step 4: Set the values of states $x(k)$.
Step 5: Solve optimization problem described by Eq(6), Eq(9), Eq(10) to evaluate X_k, Y_k and U_k.
Step 6: Design the matrix F_k of the feedback controller using Eq(5).
Step 7: Calculate the control input $u(k)$ using the control law Eq(2).
Step 8: If the parameter $k < n_{\mathrm{k}}$ then go to the *Step 3* else *Stop*.

4. Results and discussion

The designed robust MPC strategies $RMPC_1$ – $RMPC_3$ were investigated via simulations of control of the non-linear model of HEN using 2.8 GHz CPU and 4 GB RAM in the MATLAB-Simulink environment using the toolbox YALMIP (Löfberg, 2004) and the solver SeDuMi (Sturm, 1999). The robust state-feedback controllers were designed using the weight matrices W_{x}, W_{u} in the cost function described by Eq(3) in the form $diag(W_{\mathrm{x}})=[100,100,100,100,100,100]^{\mathrm{T}}$, $diag(W_{\mathrm{u}})=[100]$, where *diag* denotes the diagonal matrix with the given elements on the principal diagonal and zero elsewhere. These weight matrices were considered in all $RMPC_i$ algorithms to make the obtained results fully comparable. The $RMPC_i$ strategies were analyzed by evaluating the offset of the petroleum temperature $\Delta T_2^{(3)}$, and consumption of the cooling medium V_C. The aim of control was to cool down the petroleum temperature from 118.4 °C to 45.3 °C during the control running 2,100 s.
The Figures 1, 2 show just first 1,500 s to show the dynamics clearly. Figure 1 presents the control performance of the outlet petroleum temperature assured by RMPC1 (dotted line), RMPC2 (solid line), RMPC3 (dashed line) strategies in the worst-case (●) and the best-case (□) scenarios. The reference is denoted by the dashed-dotted line. The worst-case scenario represents the vertex system with the maximal value of analyzed criterion Eq(3). The best-case scenario is the vertex system with the minimal value of analyzed criterion Eq(3). Figure 2 shows the associated control inputs.
The computed values of the cost function are presented in Table 2. As can be seen, the PDLF-based $RMPC_2$ approach assured better value of temperature in the best-case scenario in comparison with the original $RMPC_1$ strategy. But the worst-case scenario was not very efficient. Although $RMPC_3$ results were not the best, this strategy ensured the tightest range of the temperature offset. Contrary to the best-case behaviour, the worst-case temperature trajectories indicate slight overshoot at the beginning. The fastest convergence to the reference temperature was assured by the $RMPC_3$ procedure. The PDLF-based

Table 2: Results of the RMPC approaches of HEN control

method	scenario	$\Delta T_2^{(3)}$ [°C]	V_C [m^3]
$RMPC_1$	*best case*	0.49	0.64
	worst case	0.57	0.74
$RMPC_2$	*best case*	0.12	0.66
	worst case	1.01	0.71
$RMPC_3$	*best case*	0.16	0.66
	worst case	0.18	0.76

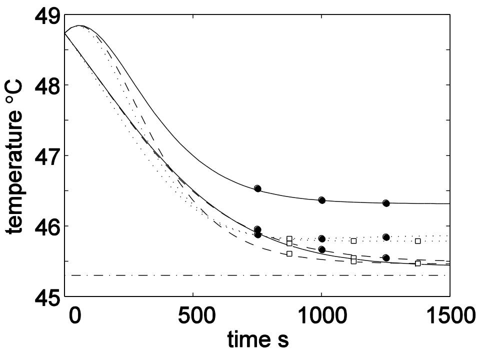

Figure 1: Control performance of the outlet petroleum temperature assured by $RMPC_1$ (dotted), $RMPC_2$ (solid), $RMPC_3$ (dashed) strategies in the worst-case (●) and the best-case (□) scenarios.

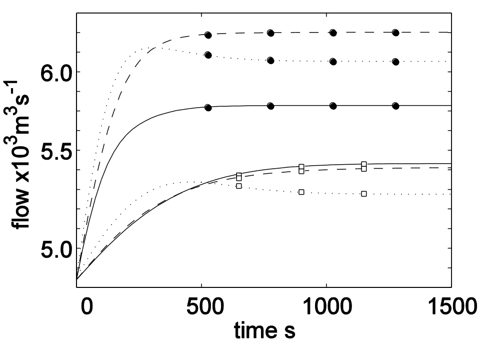

Figure 2: Control-input trajectories of the volumetric flow-rate generated by $RMPC_1$ (dotted), $RMPC_2$ (solid), $RMPC_3$ (dashed) strategies in the worst-case (●) and the best-case (□) scenarios.

approach led to the minimal consumption of cooling medium in the worst-case scenario and to the satisfactory consumption in the best-case scenario (Table 2). Hence, the $RMPC_2$ is the most suitable strategy for minimisation of the energy demands and the $RMPC_3$ is the most suitable strategy for the precise temperature control.

5. Conclusion

The paper demonstrates on the simulation case-study of the non-linear HEN control the possibility to implement various robust MPC strategies. The obtained results were analysed according to the control trajectories and consumption of cooling medium. In comparison with the other investigated procedures, the PDLF-based approach ensured the highest energy savings of the worst-case scenario, meanwhile providing the satisfying control performance. The tightest range of the worst-case and best-case scenarios was ensured by the robust MPC with improved handling of control inputs.

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References

Bakošová, M., Oravec, J., 2013, Robust Model Predictive Control of Heat Exchanger Network. Chemical Engineering Transactions 35, 241-246, DOI:10.3303/CET1335040.

Bakošová, M. and Oravec, J., 2012, Robust Model Predictive Control of Heat Exchangers. Chemical Engineering Transactions 29, 1465-1470, DOI: 10.3303/CET1229245.

Bemporad, A. and Morari, M., 1999, Robust Model Predictive Control: A survey. Robustness in Identification and Control, 207-226, Springer, London.

Cao, Y. Y., Lin, Z., 2005, Min–max MPC algorithm for LPV systems subject to input saturation. IEE Proceedings-Control Theory and Applications, 152, 266.

Čuček,L., Varbanov, P. S., Klemeš, J. J., Kravanja, Z., 2013, Multi-Objective Regional Total Site Integration. Chemical Engineering Transactions 35, 241-246, DOI:10.3303/CET1335016.

González A. H., Odloak D., and Marchetti J. L., 2006, Predictive control applied to heat-exchanger networks. Chemical Engineering and Processing 45, 661-671, DOI: 10.1205/cherd05052.

Ipsakisa, D., Papadopouloua, S., Voutetakis, S., Seferlis, P., 2013, Analysis and Implementation of a Plant-Wide Control System for an LPG Reforming-Fuel Cell Power System. Chemical Engineering Transactions 35, 241-246, DOI:10.3303/CET1335159.

Kothare M. V., Balakrishnan V., and Morari, M., 1996, Robust constrained model predictive control using linear matrix inequalities. Automatica 32, 1361-1379.

Mao, J. W., 2003, Robust Stabilization of Uncertain Time-Varying Discrete Systems and Comments on “An Improved Approach for Constrained Robust Model Predictive Control”, Automatica, 39, 1109-1112, DOI: 10.1016/S0005-1098(03)00069-4.

Löfberg, J., 2004, Yalmip: A toolbox for modelling and optimization in Matlab. Proc. of the CACSD Conference, Taipei, Taiwan, 284-289.

Pan M., Bulatov I., Smith R., 2013, Heat transfer intensified techniques for retrofitting heat exchanger networks in practical implementation, Chemical Engineering Transactions, 35, 1189-1194 DOI:10.3303/CET1335198

Sturm, J. F., 1999, Using SeDuMi 1,02, a MATLAB toolbox for optimization over symmetric cones. Optimization Methods Software, 625-653.

Vasičkaninová, A. and Bakošová, M., 2013, Application of H2 and H∞ Approaches to the Robust Controller Design for a Heat Exchanger. Chemical Engineering Transactions 35, 263-468, DOI:10.3303/CET1335077.

Walmsley T.G., Walmsley M.R.W., Morrison A.S., Atkins M.J., Neale J.R., 2013, A derivative approach to minimising total cost in heat exchanger networks through optimal area allocation, Chemical Engineering Transactions, 35, 1171-1176, DOI:10.3303/CET1335195.