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On Stability and Controllability of Processes with Internal Recycle

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In the presented paper dynamic state analysis, stability and controllability aspects of recycle processes are discussed, in general. A comparison of process dynamic behaviour and investigation of influence of process recycle loop parameters are carried out, applying simulation experiments on linear transfer function models. Stability and controllability considerations are provided in both, open and closed loop mode.

1. Introduction

In most cases, recycle leads to positive feedback effects. For example, increasing the concentration of a chemical species in a process stream will normally increase the amount of this species in the recycle stream, and, thus, lead to a reinforcement of the original increase. It refers to a self-reinforcing mechanism associated with the recycle. This positive feedback will usually increase the plant time constant, and also increase the process sensitivity to slow disturbances. This is because recycle will tend to “store” material or energy within some part of the plant. An example is a high purity distillation column where extremely long time constants can be observed.
The overall dynamics of chemical processing plants with material recycle or heat integration can be very different from the dynamics of the individual processing units. Material recycle and heat integration may dramatically alter the overall gain and time constants of the plant, and may give rise to oscillatory or instable behaviour, even when the individual processing units are stable by themselves. Moreover, plant interconnections may introduce fundamental limitations in the achievable performance of any control system. The knowledge of such phenomena is important for controller design, and their effects may even pose a threat to plant safety if not foreseen. There has been lot of works devoted to the problem of investigating and handling chemical processes with recycle. Luyben (1993) shows that, by changing the gain of the recycle process independently from the other process parameters, the open loop response can become slow, oscillating and even unstable. Morud and Skogestad (1994) analyse the effects of different elements on the dynamics. Scali and Antonelli (1995) investigate the performance of different regulators (PI, IMC) for plants with recycle. Taiwo and Krebs (1996) show how a robust control system can be designed and cope with recycle plant problems.
The aim of this paper is to give a comprehensive picture on dynamic behaviour of processes with internal recycle (PIR) in both, open loop and closed loop configuration. A comparison of process dynamic behaviour and investigation of influence of process recycle loop parameters is carried out, applying simulation experiments on linear transfer function models. Stability and controllability conditions are derived and influence of recycle parameters are shown for open loop and closed loop cases.

2. Open-loop system with internal recycle

Let us consider a heating plant process identified in the form of simple linear system consisting of two forward paths and a recycle unit as depicted in Figure 1. G_M, G_D and G_R stand for plant forward unit, disturbance and recycle unit *u, d* and *y* are the input, load disturbance and output. Assuming $G_M = G_D$, we get the simplified structure of the recycle process, shown as Figure 2.

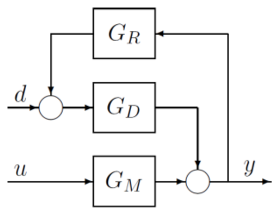

Figure 1: Open-loop process with recycle

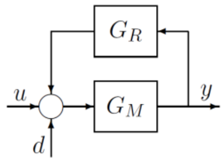

Figure 2: Simple open-loop process with recycle

Let the first object of our study be the recycle system in Figure 2, assuming *d* = 0*.* This can be described by the open loop transfer function (*s* being the derivation operation)

$$G_S(s) = \frac{y(s)}{u(s)} = \frac{G_M}{1 - G_M G_R} \tag{1}$$

In dynamic analysis of the recycle system Eq(1), we will focus on investigation of influence of the recycle path (G_R) parameters onto the overall plant behaviour. For open-loop and closed-loop stability considerations and simulation experiments, G_M and G_R will be considered as units of first order dynamics. It can be done without significant loss of generality, as to the results expected, because lot of real plants dynamics can be identified, in a close neighbourhood of the operation regime, as a first order lag with steady-state gain and dead time (if necessary). Simulation experiments could prove that increasing order of either G_M or G_R does not basically alter the tendency of recycle parameters influence onto overall process dynamics.

Then, the forward path is described by a simple linear transfer function consisting of a steady-state gain, K_M, and a first order lag with time constant, τ_M. The unit in the recycle path also has a simple gain and lag transfer function (with K_R and τ_R), as follows

$$G_M = \frac{K_M}{\tau_M s + 1}, \qquad G_R = \frac{K_R}{\tau_R s + 1} \tag{2}$$

Inserting (2) into (1), we get the overall transfer function

$$G_S = \frac{\dfrac{K_M}{\tau_M s + 1}}{1 - \dfrac{K_M}{\tau_M s + 1}\dfrac{K_R}{\tau_R s + 1}} = \frac{K_M(\tau_R s + 1)}{\tau_M \tau_R s^2 + (\tau_M + \tau_R)s + 1 - K_M K_R} \tag{3}$$

From characteristic equation of system (3),

$$\tau_M \tau_R s^2 + (\tau_M + \tau_R)s + 1 - K_M K_R = 0 \tag{4}$$

the necessary and sufficient condition for recycle plant stability is

$$K_M K_R < 1 \qquad \text{or} \qquad K_R < \frac{1}{K_M} \tag{5}$$

2.1 Influence of recycle loop gain and time constant

In order to show the effect of recycle parameters, K_R and τ_R, we express the overall process steady-state gain, K_S, and time constant, τ_S, using Eq(3)

$$K_S = \frac{K_M}{1 - K_M K_R} \tag{6}$$

$$\tau_S = \sqrt{\frac{\tau_M \tau_R}{1 - K_M K_R}} \tag{7}$$

and calculate the limits of G_S for boundary values of K_R and τ_R, from the same equation

$$\lim_{K_R \to 0} G_S = G_M \,, \qquad \lim_{K_R \to \infty} G_S = 0 \tag{8}$$

$$\lim_{\tau_R \to 0} G_S = \frac{\dfrac{K_M}{1 - K_M K_R}}{\dfrac{\tau_M}{1 - K_M K_R} s + 1} \,, \qquad \lim_{\tau_R \to \infty} G_S = G_M \tag{9}$$

From Eq(8) and Eg(9) it is obvious that the recycle effect is diminishing for “very small” values of the gain as well as “very large” values of the time constant. Large values of K_R cause instability and “very large” values stop the whole process operation while “very small” values of τ_R reduce the order of process.
To give a complete picture on recycle effect, step-wise response dynamic simulations have been carried out and evaluated with respect to recycle loop parameter changes. In Figure 3 and Figure 4, a comparison of the plant output responses is given for various values of K_R and τ_R. The other process parameters during the two courses of simulation kept the following values: $K_M = \tau_M = \tau_R = 1$ and $K_M = \tau_M = 1$, $K_R = 0.8$. It can be concluded that the effect of K_R is more straightforward because it influences both, overall gain and dynamics, while τ_R changes system behaviour in transient stage only.
From theoretical and simulation results it is obvious that one of the most important effects of recycle is to slow down the response of the process, i.e. increase the process overall time constant.

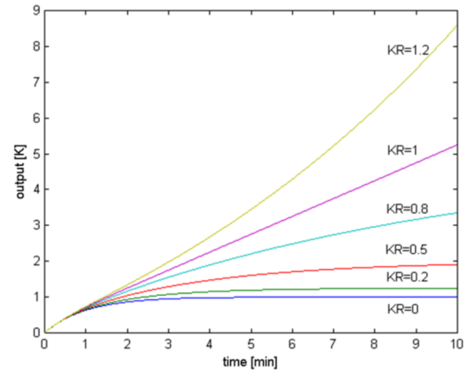

Figure 3: Influence of recycle loop gain onto process dynamics

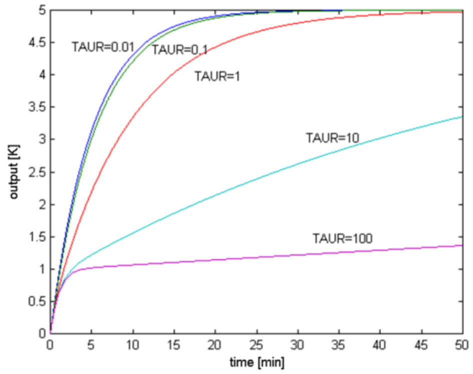

Figure 4: Influence of recycle loop time constant onto process dynamics

3. Closed-loop system with recycle effect

Considering the recycle process with closed loop control as depicted in Figure 5, the system characteristic equation takes the form

$$1 + \frac{G_C G_M}{1 - G_M G_R} = 0 \tag{10}$$

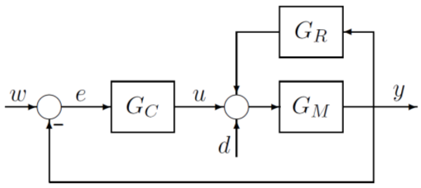

Figure 5: Recycle system with closed loop control

3.1 Dynamic Analysis and Stability Considerations

Case I. Proportional controller

The controller transfer function is

$$G_C = K_C \tag{11}$$

With respect to (1), (2) and (3), characteristic eq. (10) results in

$$\tau_M \tau_R s^2 + (\tau_M + \tau_R + K_C K_M \tau_R)s + 1 - K_M K_R + K_C K_M = 0 \tag{12}$$

The necessary and sufficient condition of closed loop system stability is

$$K_C > \frac{K_M K_R - 1}{K_M} \tag{13}$$

For boundary values holds

$$K_C \to 0 : \; K_R < \frac{1}{K_M} \tag{14}$$

$$K_C \to \infty : \; K_R < K_C \tag{15}$$

where Eq(14) corresponds to the open loop case Eq(5).
To give a picture on recycle effect, set-point step-wise response dynamic simulations have been carried out and evaluated with respect to recycle loop parameter changes. In Figure 6 and Figure 7, a comparison of the controlled plant output responses is given for various values of K_R and τ_R. The other process parameters during the two courses of simulation kept the following values: d = 0, $K_M = \tau\tau_M = \tau_R = 1$, $K_C = 5$ and d = 0, $K_M = \tau_M = 1$, $K_R = 0.8$, $K_C = 5$.

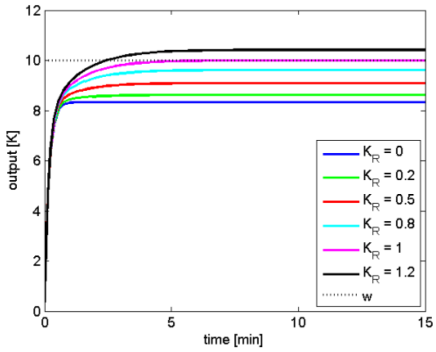

Figure 6: Influence of recycle loop gain onto controlled process dynamics

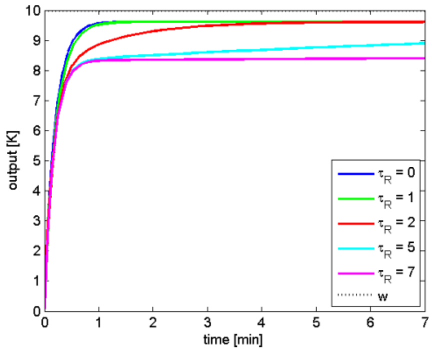

Figure 7: Influence of recycle loop time constant onto controlled process dynamics

Simulation results confirmed the theoretical stability analysis. It is straightforward from (13) that the closed loop system remains stable only for the values $K_R < 6$. It can be concluded that the effect of K_R is more straightforward because it influences both, overall gain and dynamics, while τ_R changes system behaviour in transient stage only.

Case II. Proportional-integral controller

The controller transfer function is

$$G_C = \frac{K_C(\tau_i s + 1)}{\tau_i s} \tag{16}$$

Characteristic eq. (10) takes the form

$$\tau_M \tau_R \tau_i s^3 + \tau_i(\tau_M + \tau_R)s^2 + \tau_i(1 - K_M K_R)s + K_C K_M(\tau_R \tau_i s^2 + (\tau_R + \tau_i)s + 1) = 0 \tag{17}$$

Applying the Hurwitz criterion, the necessary and sufficient condition of closed loop system stability is

$$K_C > \frac{\tau_i(K_M K_R - 1)}{K_M(\tau_R + \tau_i)} \tag{18}$$

For boundary values of τ_i holds

$$\tau_i \to 0: \ K_C > 0 \tag{19}$$

$$\tau_i \to \infty: \ K_C > \frac{K_M K_R - 1}{K_M} \tag{20}$$

where the latter corresponds to the case of P-control - Eq (13).

As above, set-point step-wise response dynamic simulations have been carried out and evaluated with respect to recycle loop parameter changes. In Figure 8 and Figure 9, a comparison of the controlled plant output responses is given for various values of K_R and τ_R. The other process parameters during the two courses of simulation kept the following values: d = 0, $K_M = \tau_M = \tau_R = 1$, $K_C = 5$, $\tau_i = 0.2$ and $K_M = \tau_M = 1$, K_R = 0.8, $K_C = 5$, $\tau_i = 0.2$.

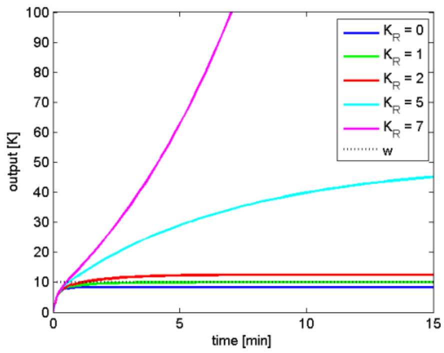

Figure 8: Influence of recycle loop gain onto controlled process dynamics

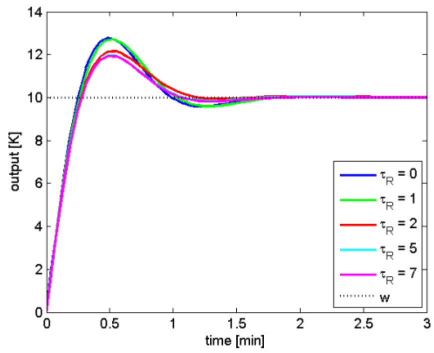

Figure 9: Influence of recycle loop time constant onto controlled process dynamics

It can be concluded that the effect of both, K_R and τ_R is well compensated by PI controller and it is in accordance with the theoretical stability results. The closed loop system for given parameters remains stable for values $K_R < 31$. However, larger values of the recycle parameters may cause more oscillatory behaviour of the overall system.

4. Controllability of PIR

For controllability analysis, let us consider the PIR identified in the form of transfer functions (1), (2) and (3). Then, the input-output process model takes the form a differential equation of second order

$$\tau_M \tau_R y''(t) + (\tau_M + \tau_R) y'(t) + (1 - K_M K_R) y(t) = K_M \tau_R u'(t) + K_M u(t) \tag{21}$$

Introducing state variables $y=x_1$, $y'=x_2$, $u_1=u$, $u_2=u'$,
the PIR state space model

$$\frac{d\overline{x}(t)}{dt} = A\overline{x}(t) + B\overline{u}(t)$$
$$y = C\overline{x}(t) \tag{22}$$

is composed by the following matrices

$$A = \begin{pmatrix} 0 & 1 \\ \dfrac{K_M K_R - 1}{\tau_M \tau_R} & -\dfrac{\tau_M + \tau_R}{\tau_M \tau_R} \end{pmatrix},\ B = \begin{pmatrix} 0 & 0 \\ K_M & K_M \tau_R \end{pmatrix},\ C = (1 \quad 0)$$

Applying the Controllability Theorem in state space context (Mikleš and Fikar, 2000), the Controllability Matrix of the system can be constructed as

$$Q_C = (B \mid AB) \tag{23}$$

The PIR system is then completely controllable if the rank of (23) is equal to two. This leads to the following condition of controllability

$$-K_M \tau_R \neq 0 \tag{24}$$

The above results refer to a significant condition: a PIR system can be completely controlled only if the recycle loop contains a first or higher order lag with non-zero time constants. A static positive feedback can make the overall system uncontrollable.

5. Conclusion

In this paper, dynamic state properties and control aspects of recycle processes, in general, have been studied. Assuming first order dynamics, comparison of process dynamic behaviour and investigations of influence of process recycle loop parameters were carried out, applying simulation experiments on linear transfer function models. A comprehensive picture of dynamic behaviour of processes with internal recycle (PIR) in both, open loop and closed loop configuration was given. Stability and controllability conditions were derived and influence of recycle parameters were shown for open loop and closed loop cases. Theoretical as well as simulation results have shown serious limitations as to PIR system stability and controllability conditions.

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